Why tensors in this course?

- A key concept in this couse is “the response of a media to electromagnetic fields”
- Given an electric field $\mathbf{E}$ this response can be described in term of the relation with the induced current $\mathbf{J}$.
- For linear media the Fourier transformed relation can often be described by a conductivity tensor $\boldsymbol{\sigma}$:

$$\mathbf{J} = \sigma \bullet \mathbf{E} \quad \text{or} \quad J_i = \sigma_{ij} E_j$$

- Example: In isotropic media the tensor I diagonal (no tensor description needed)

$$\sigma_{ij} = \sigma_0 \delta_{ij} \quad \Rightarrow \quad \mathbf{J} = \sigma_0 \mathbf{E} \quad \text{or} \quad J_i = \sigma_0 E_i$$

- Example: Uniaxial crystal conducts differently along the perpendicular to the crystal-plane. If the normal to the crystal-plane is in the z-directions then

$$\sigma_{ij} = \begin{bmatrix} \sigma_{\parallel} & 0 & 0 \\ 0 & \sigma_{\parallel} & 0 \\ 0 & 0 & \sigma_{\perp} \end{bmatrix} = \sigma_{\parallel}\delta_{ij} + \left(\sigma_{\perp} - \sigma_{\parallel}\right)\delta_{i3}\delta_{j3} \quad \Rightarrow \quad J_{1,2} = \sigma_{\parallel} E_{1,2} \;\; \& \;\; J_3 = \sigma_{\perp} E_3$$

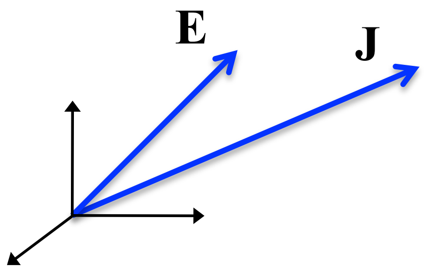
- In magnetised media the response often has off-diagonal components

$$\sigma_{ij} = \begin{bmatrix} \sigma_{11} & \sigma_{21} & 0 \\ \sigma_{12} & \sigma_{22} & 0 \\ 0 & 0 & \sigma_{33} \end{bmatrix}$$

Why tensors in this course?

- Example: Consider a crystal with the following conductivity

$$\sigma = \sigma_0 \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



and an electric field

$$\mathbf{E} = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^T$$

The current is then

$$\mathbf{J} = \sigma_0 \begin{bmatrix} 2 & 2 & 1 \end{bmatrix}^T$$

i.e. NOT in the same direction as the E-field

Rough guide to this course

- Description of the dielectric response : $\mathbf{J} = \sigma \bullet \mathbf{E}$
- Wave equation for dispersive media, e.g. crystals and plasmas
- Dissipation from e.g. collisions or Landau damping (Plemej formula)
  - Study damping: Assume small damping and expand
    - 0^{th} order: undamped plane wave (no dissipation)
    - 1^{st} order: calculate damping rate of the wave
- Next: insert the response $\mathbf{J}^* \bullet \mathbf{E} = \mathbf{E}^* \bullet \sigma \bullet \mathbf{E}$
  in the energy conservation equation to study
  - spatial and temporal damping
  - non-homogeneous media
  - emission processes