

Reglerteknik 3

Kapitel 7



Köp bok och övningshäfte på kårbokhandeln

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Lektion 3 kap 7

- Modelling
- Identifiering

Teoretisk modellering

Man använder grundläggande fysikaliska naturlagar och deras ekvationer för att härleda överföringsfunktionen.

Tex

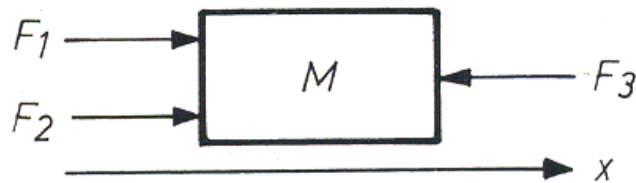
- Kraftekvationen
- Kirchhoffs lagar
- Energibalansprincipen
- Massbalansprincipen
- Volymbalansprincipen

Teoretisk modellering används när enkla modeller som kan ställas upp med måttlig möda är tillräckliga.

Genomgång av Processtyper

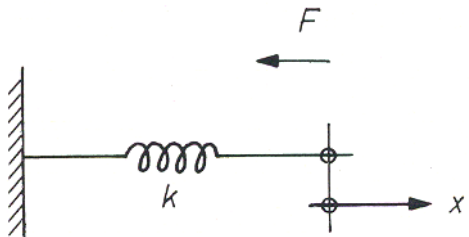
- Mekaniska system
- Elektriska system
- Temperatursystem
- Nivåsystem

Mekaniska system, grundelement



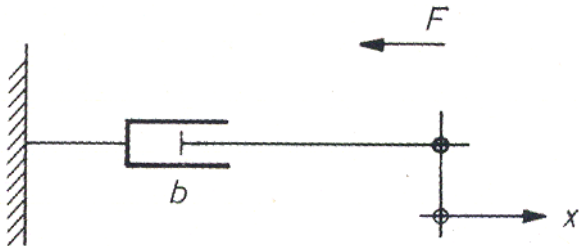
$$\sum F = M \cdot \ddot{x}$$

Newtons
kraftekvation.



$$F = k \cdot x$$

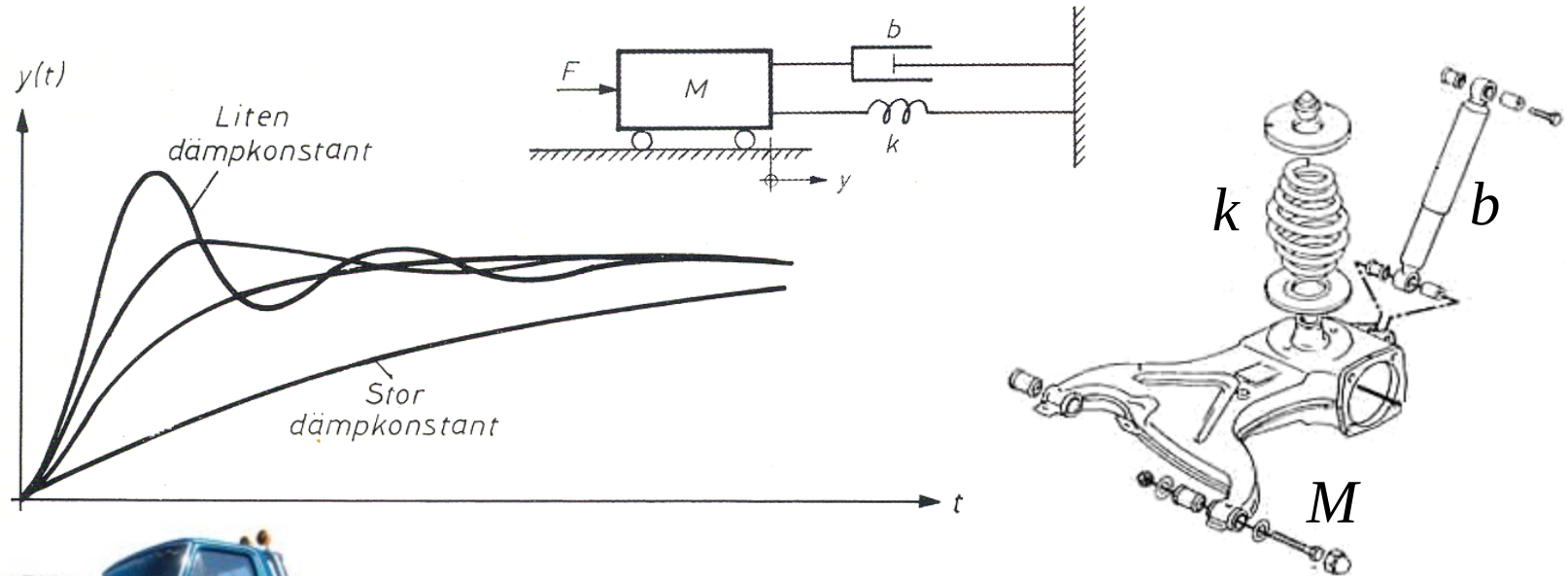
Hook's lag.
 F proportionell mot
fjäders förlängning.



$$F = b \cdot \dot{x}$$

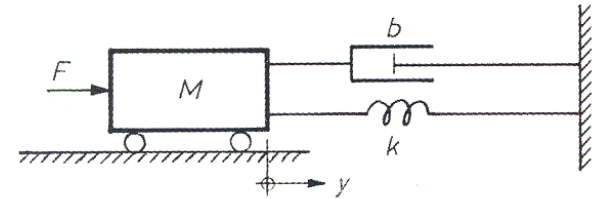
Hastighets-
dämpare.

Sammanfatt system



Man måste välja k b och M för bästa köregenskaper ...

Sammanfatt system

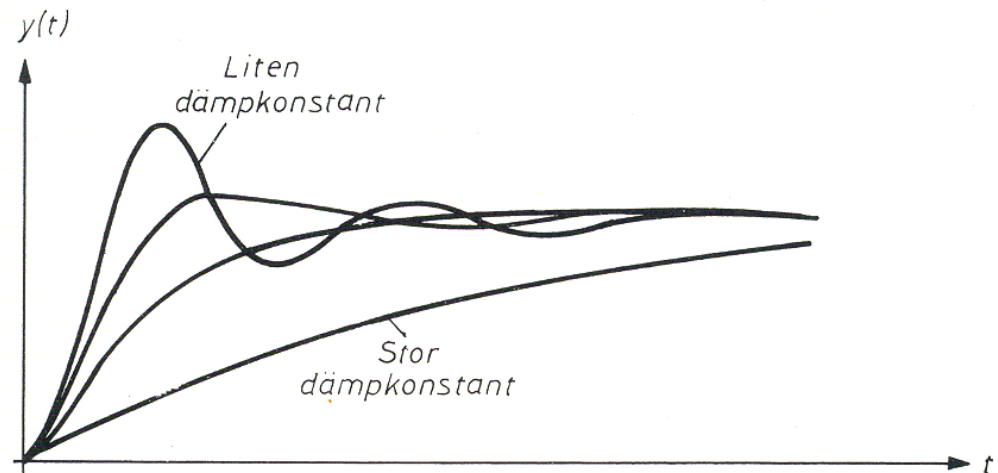


$$M \cdot \ddot{y} = F - ky - b\dot{y} \Rightarrow M \cdot \ddot{y} + b\dot{y} + ky = F$$

$$\Leftrightarrow \stackrel{L}{:} Y(Ms^2 + bs + k) = F \Rightarrow G(s) = \frac{Y}{F} = \frac{1}{Ms^2 + bs + k}$$

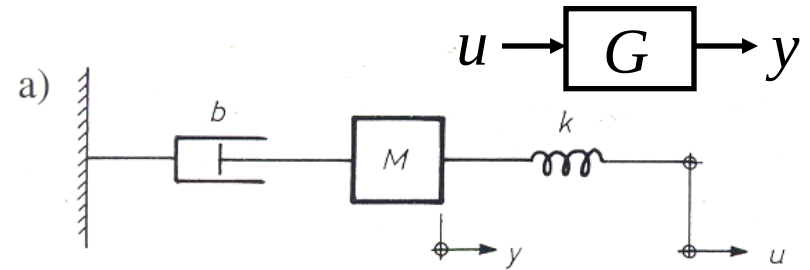


Stegsvar då man
”trycker” på massan
med en konstant kraft,
och följer med rörelsen.



7.1 a Mekanisk modell

Ställ upp differentialekvation
och överföringsfunktion.



7.1 a lösning Mekanisk modell

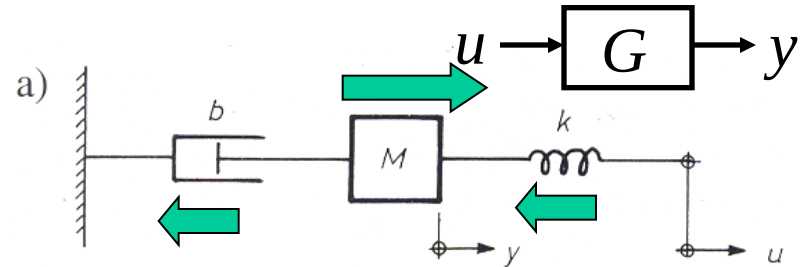
Fjäder:

$$\overleftarrow{F} = k(\overrightarrow{u} - \overrightarrow{y})$$

Dämpare:

$$\overleftarrow{F} = -b\overrightarrow{\dot{y}}$$

Kraftekvationen: $\sum \overrightarrow{F} = M \cdot \overrightarrow{\ddot{y}}$

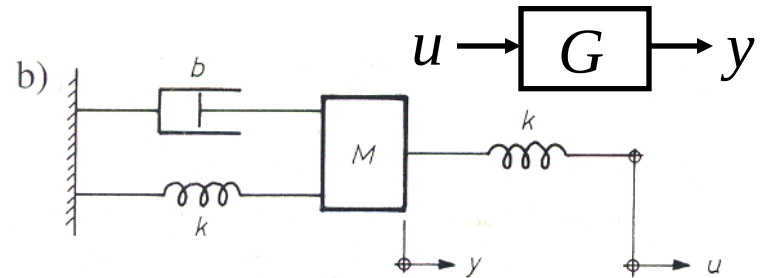


$$M \cdot \ddot{y} = k(u - y) - b\dot{y} \quad \overset{L:}{\Leftrightarrow} \quad Ms^2Y = kU - kY - bsY$$

$$G(s) = \frac{Y}{U} = \frac{k}{Ms^2 + bs + k}$$

7.1 b Mekanisk modell

Ställ upp differentialekvation
och överföringsfunktion.



7.1 b lösning Mekanisk modell

Fjäder 1:

$$F = -k\vec{y}$$

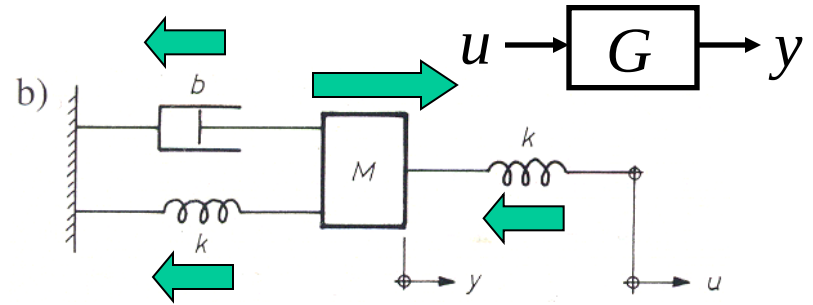
Fjäder 2:

$$F = k(\vec{u} - \vec{y})$$

Dämpare:

$$F = -b\dot{\vec{y}}$$

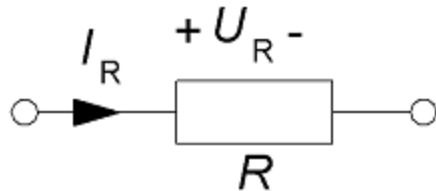
Kraftekvationen: $\sum F = M \cdot \ddot{\vec{y}}$



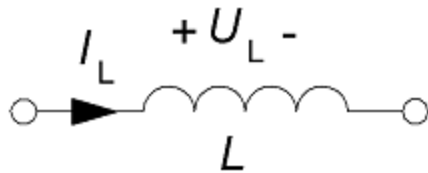
$$M \cdot \ddot{y} = k(u - y) - ky - b\dot{y} \quad \xLeftrightarrow{L:} \quad Ms^2Y = kU - 2kY - bsY$$

$$G(s) = \frac{Y}{U} = \frac{k}{Ms^2 + bs + 2k}$$

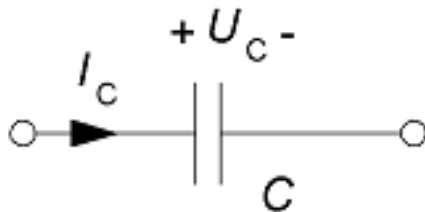
Elektriska system, grundelement



$$u_R = i_R \cdot R \quad U_R = I_R \cdot R$$



$$u_L = L \cdot \frac{d}{dt} i_L \quad U_L = Ls \cdot I_L$$

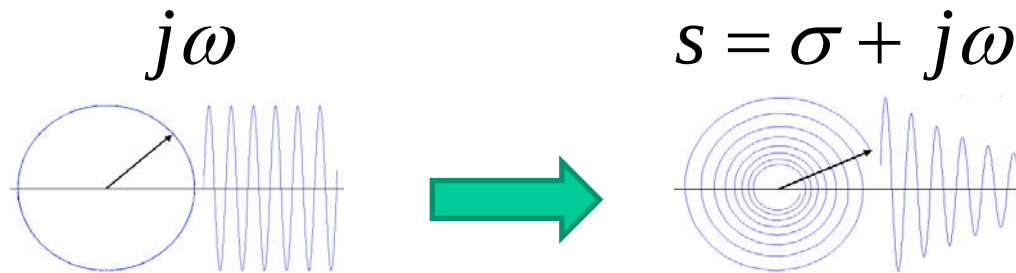


$$u_C = \frac{Q_C}{C} = \frac{\int i_C dt}{C} \quad U_C = \frac{1}{C \cdot s} \cdot I_C$$

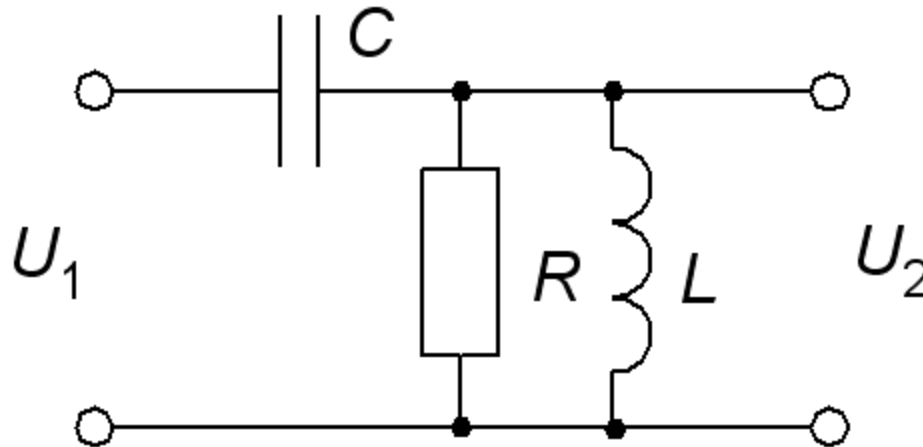
Elektriska nät, $j\omega$ -metoden

$$s = \sigma + j\omega \quad \text{im}[s] = \omega$$

Du som redan kan $j\omega$ -metoden ställer upp de elektriska överföringsfunktionerna direkt, och **byter** sedan $j\omega$ mot s .



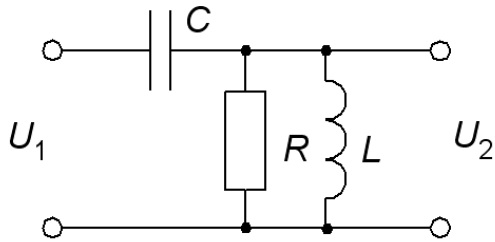
Sammansett elektriskt system



$$R \parallel L = \frac{j\omega L \cdot R}{R + j\omega L}$$

$$\frac{\underline{U}_2}{\underline{U}_1} = \frac{\frac{j\omega L \cdot R}{R + j\omega L}}{\frac{1}{j\omega C} + \frac{j\omega L \cdot R}{R + j\omega L}} \Rightarrow G(s) = \frac{\frac{sL \cdot R}{R + sL}}{\frac{1}{sC} + \frac{sL \cdot R}{R + sL}}$$

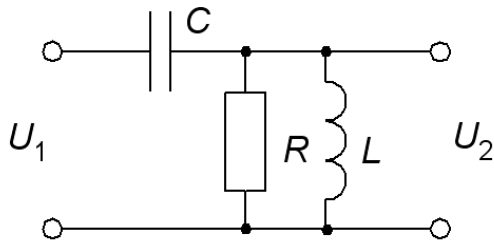
Sammansett elektriskt system



$$G(s) = \frac{\frac{sL \cdot R}{R + sL}}{\frac{1}{sC} + \frac{sL \cdot R}{R + sL}}$$

$$G(s) = \frac{\frac{sL \cdot R}{R + sL}}{\frac{1}{sC} + \frac{sL \cdot R}{R + sL}} = \frac{LRs \cdot sC}{(R + Ls) + LRs \cdot sC} = \boxed{\frac{LRCs^2}{LRCs^2 + Ls + R}}$$

Sammanfattning elektriskt system MATLAB

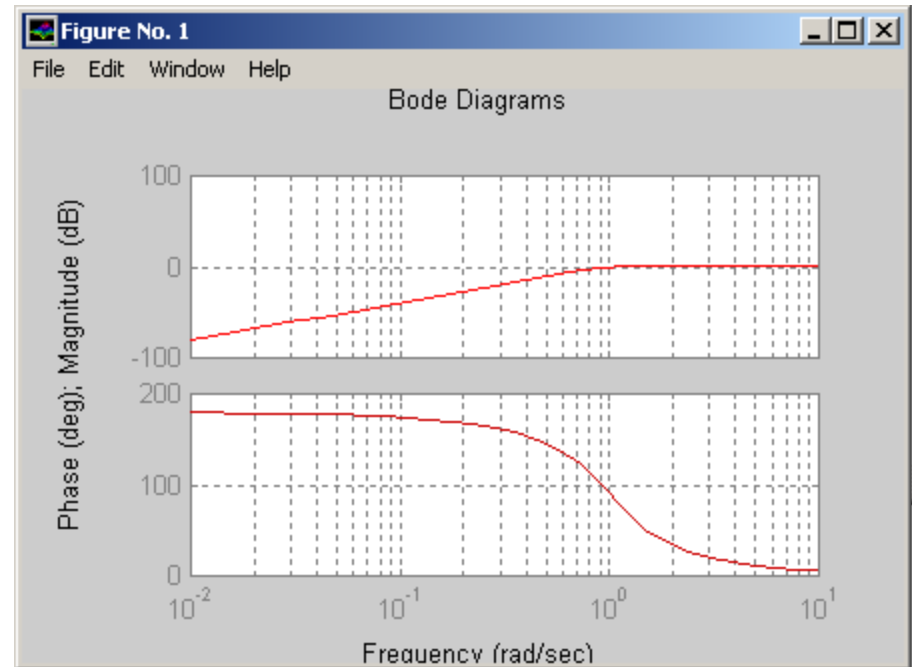


$$G(s) = \frac{LRCs^2}{LRCs^2 + Ls + R}$$

För enkelhets skull $L = R = C = 1$

```
G=tf([1,0,0],[1,1,1])  
bode(G)
```

Från ellärans laborationer är
Du van vid **Bode**-diagrammet.

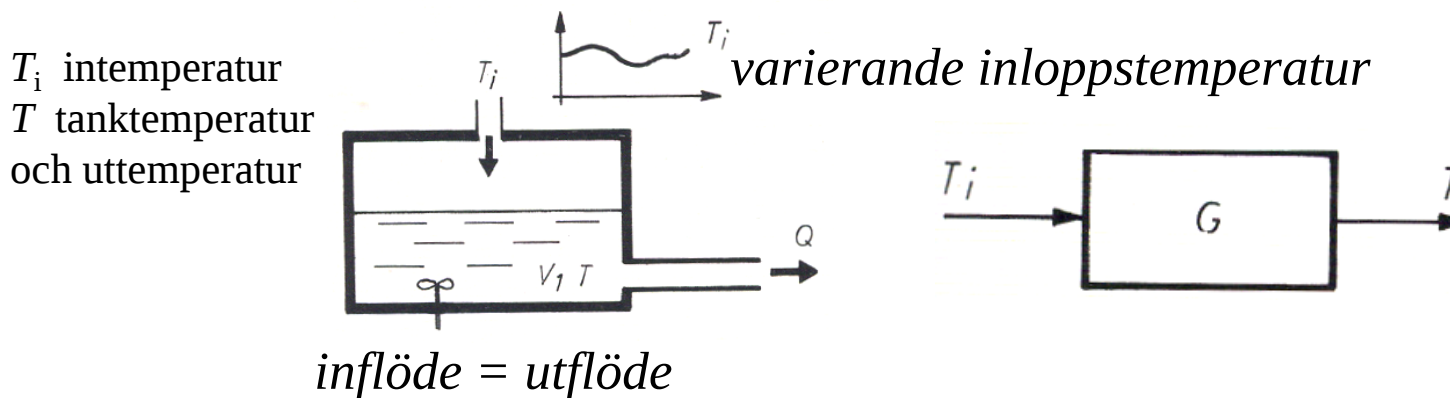


Temperatursamband

Energibalans: $\Delta P = P_{in} - P_{out} \quad \frac{dE}{dt} = P_{in} - P_{out}$

Termisk energi: $E = T \cdot V \cdot c \cdot \rho$ T temperatur, V volym 4 m³,
 c värmekapacitet, ρ densitet

Utflöde Q_{out} och inflöde Q_{in} är **samma** $Q = 0,1$ m³/s. Sökt, överföringsfunktion mellan T uttemperatur och T_i intemperatur.

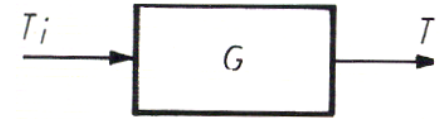


Temperatursamband

$$E = T \cdot V \cdot c \cdot \rho \quad \text{Värmeenergi i tanken}$$

$$P_{in} = T_i \cdot Q \cdot c \cdot \rho \quad \text{Tillförd effekt}$$

$$P_{out} = T \cdot Q \cdot c \cdot \rho \quad \text{Bortförd effekt}$$



Energibalans:

$$\frac{dE}{dt} = P_{in} - P_{out}$$

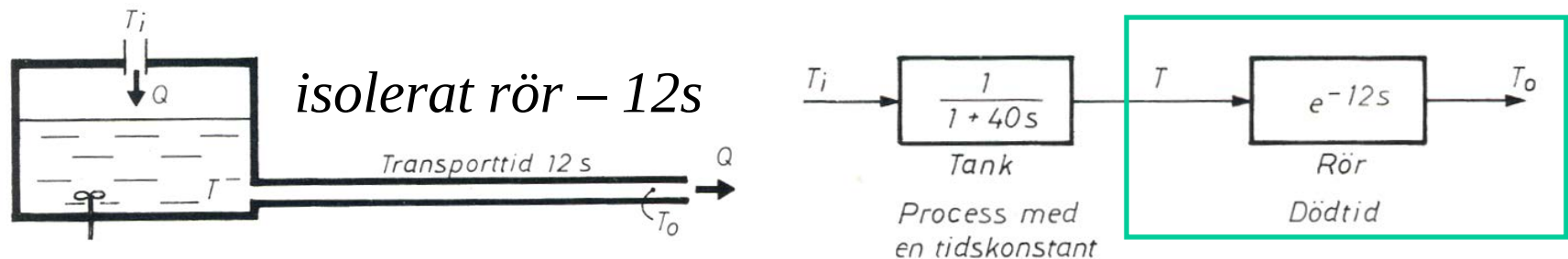
$$\frac{d}{dt}(TVc\rho) = T_i Q c \rho - T Q c \rho \Rightarrow V \cancel{c \rho} \frac{dT}{dt} = T_i Q \cancel{c \rho} - T Q \cancel{c \rho} \Rightarrow$$

$$V \frac{dT}{dt} + QT = QT_i \quad \xLeftrightarrow{L:} \quad V \cdot T(s) \cdot s + Q \cdot T(s) = Q \cdot T_i(s)$$

$$G(s) = \frac{Q}{Vs + Q} = \frac{1}{1 + \frac{V}{Q}s}$$

$$G = \frac{1}{1 + 40s}$$

Temperatursamband med dödtid



Isolerat rör. Överföringsfunktion mellan T_o och T .

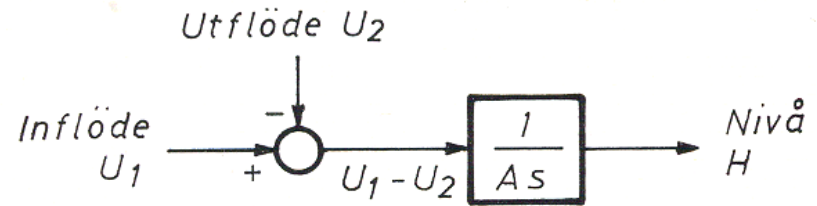
$$T_o(t) = T(t-12) \quad G(s) = e^{-12s} \quad G = G_1 \cdot G_2 = \frac{e^{-12s}}{1+40s}$$

`G=tf([1],[40 1],'InputDelay',12)`

Transfer function:

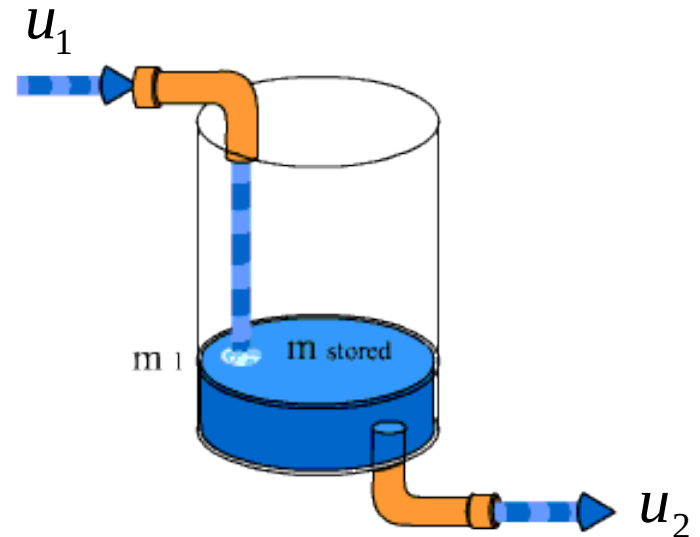
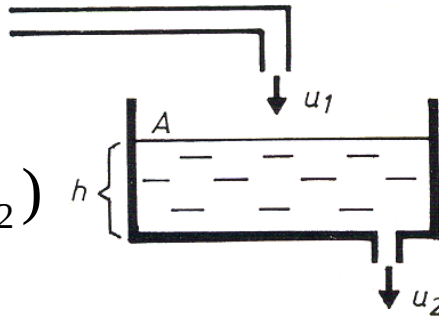
$$\exp(-12*s) * \frac{1}{40 s + 1}$$

Nivå samband



Balansekvation:

$$\frac{dV}{dt} = A \frac{dh}{dt} = (u_1 - u_2)$$



u_1 u_2 In- och utflöde

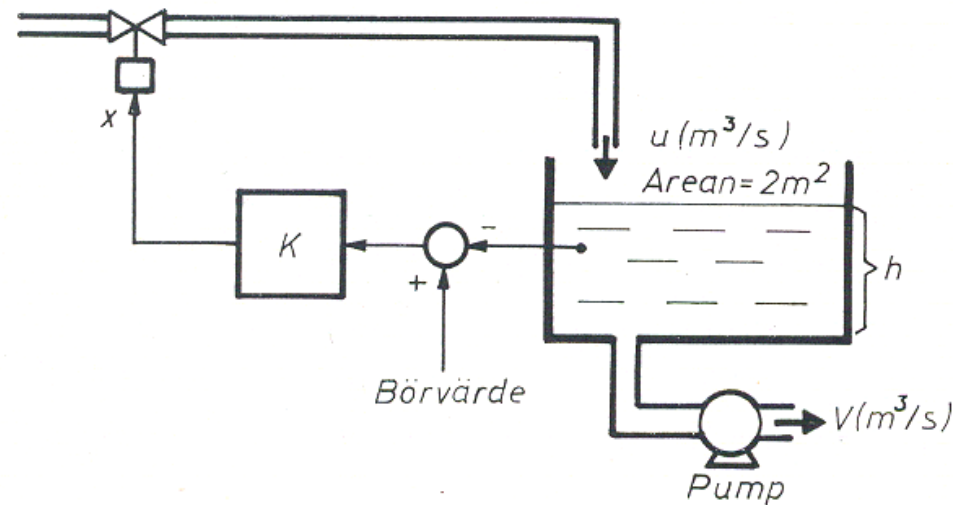
A h Area och nivå

$V = A \cdot h$ Vätskevolym i tank

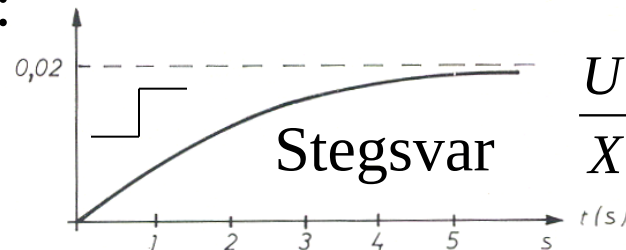
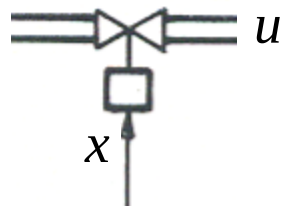
$$\frac{dV}{dt} = A \frac{dh}{dt} = (u_1 - u_2) \quad \Leftrightarrow \quad AHs = (U_1 - U_2) \Rightarrow$$

$$G(s) = \frac{H}{(U_1 - U_2)} = \frac{1}{As}$$

7.11 Nivåregulator för tank

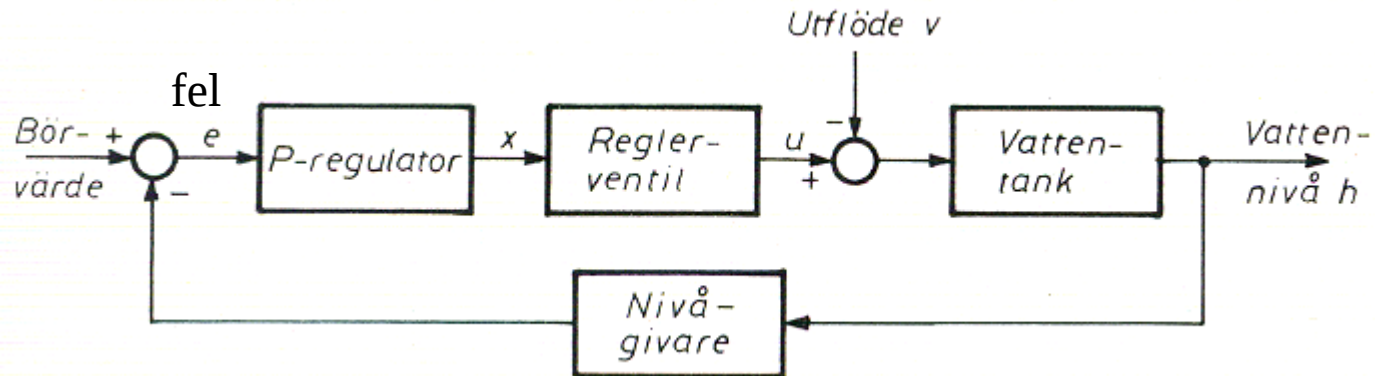
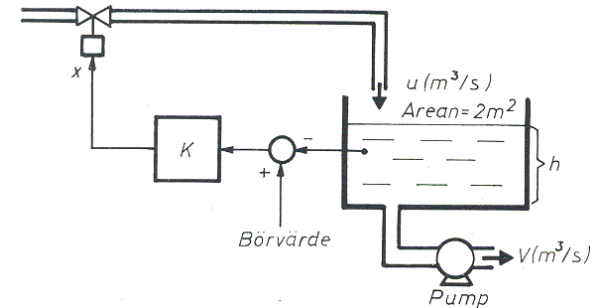


Reglerventil:

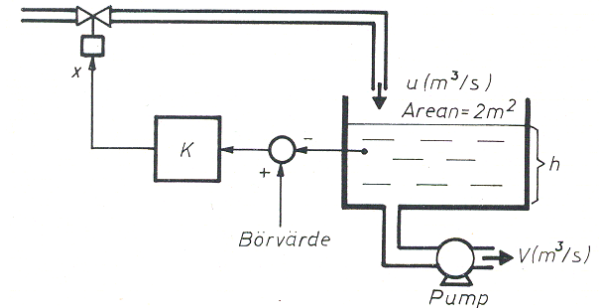


Vi har mätt upp
reglerventilens
stegsvar

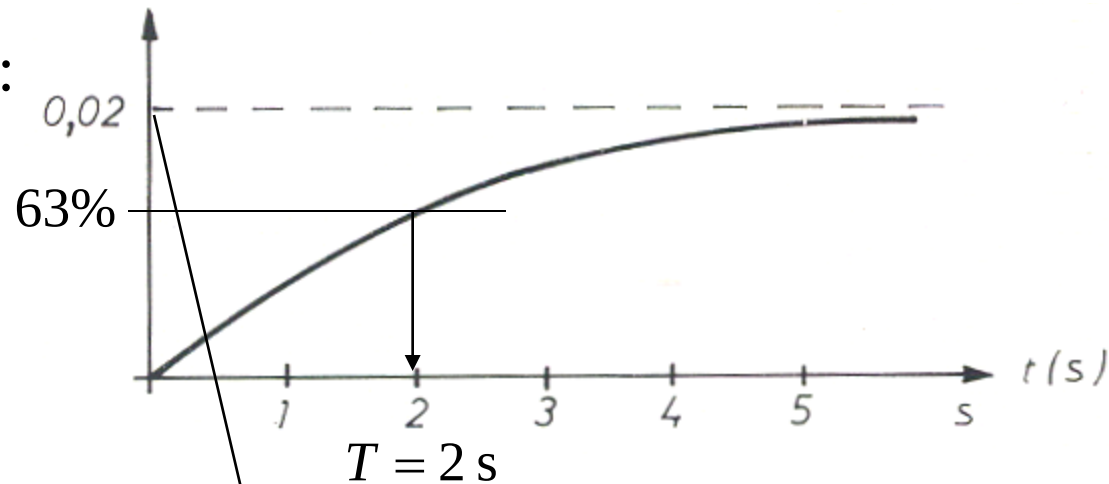
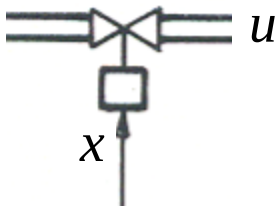
7.11 a Nivåregulator blockschema



7.11 b Nivåregulator reglerventil



Reglerventil:

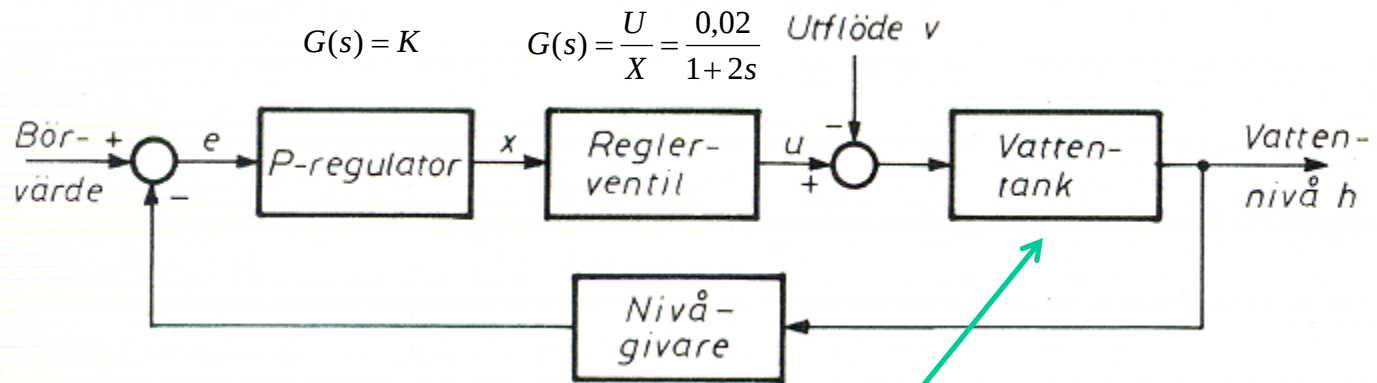
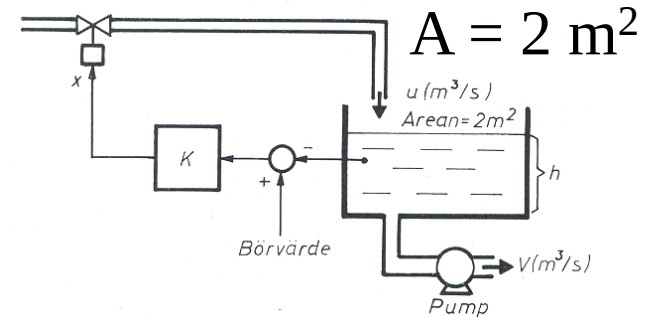


$$G(s) = \frac{U}{X} = \frac{0,02}{1 + 2s}$$

Överföringsfunktion



7.11 c Nivåregulator vattentank



Tank:

$$\frac{dV}{dt} = A \frac{dh}{dt} = (u - v) \quad \{L:\} \quad AsH = (U - V)$$

$$G(s) = \frac{H}{(U - V)} = \frac{1}{As} = \frac{1}{2s}$$

Överföringsfunktion

$$(U - V) \rightarrow \boxed{G} \rightarrow H$$

Har ni sett vattentankarna på labbet?

Uppgift 7.12 kommer ni säkert studera noggrant!

(7.12 Kopplade tankar)

U_0 = inflöde tank 1

U_{10} , U_{11} = utflöden tank 1

U_2 = utflöde tank 2

A_1 = area tank 1

A_2 = area tank 2

h_1 = nivå tank 1

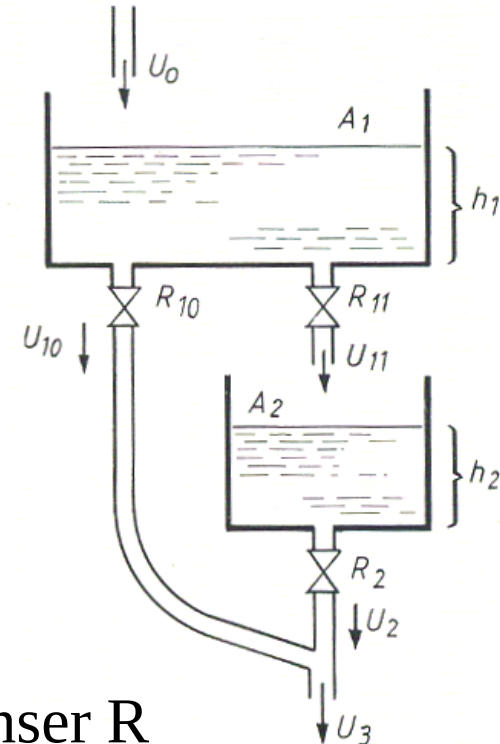
h_2 = nivå tank 2

Sökt överföringsfunktion

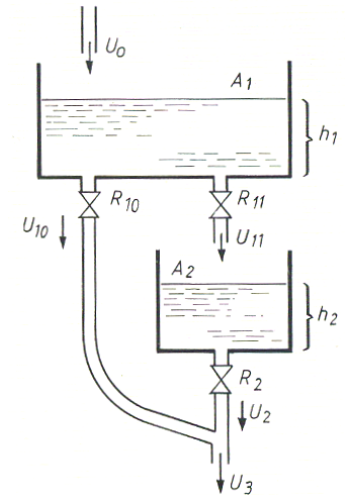


Ventilresistanser R

$$U_{10} = \frac{h_1}{R_{10}} \quad U_{11} = \frac{h_1}{R_{11}} \quad U_2 = \frac{h_2}{R_2}$$



(7.12 lösning Kopplade tankar)



Tank 1:
$$A_1 \frac{dh_1}{dt} = u_0 - u_{10} - u_{11} = u_0 - \frac{h_1}{R_{10}} - \frac{h_1}{R_{11}} \quad \{L:\}$$

$$\left(A_1 s + \frac{1}{R_{10}} + \frac{1}{R_{11}} \right) H_1 = U_0 \quad (1)$$

Tank 2:
$$A_2 \frac{dh_2}{dt} = u_{11} - u_2 = u_{11} - \frac{h_2}{R_{11}} - \frac{h_2}{R_2} \quad \{L:\}$$

$$\left(A_2 s + \frac{1}{R_2} \right) H_2 = \frac{1}{R_{11}} H_1 \quad (2)$$

Summa:
$$\{L:\} \quad U_3 = U_{10} + U_2 = \frac{1}{R_{10}} H_1 + \frac{1}{R_2} H_2 \quad (3)$$

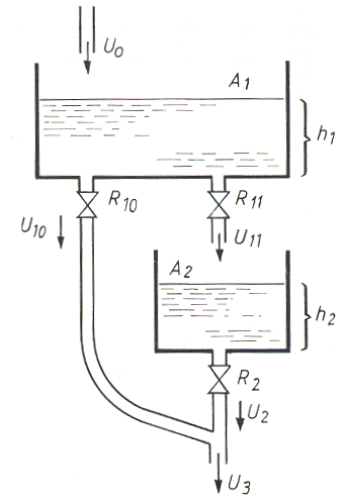
(7.12 lösning Kopplade tankar)

(1), (2) & (3)

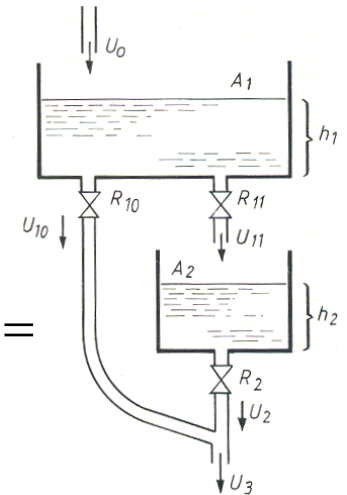
$$U_3 = \frac{1}{R_{10}} \cdot \frac{U_0}{A_1 s + \frac{1}{R_{10}} + \frac{1}{R_{11}}} + \frac{1}{R_2} \cdot \frac{\frac{1}{R_{11}} \cdot H_1}{A_2 s + \frac{1}{R_2}} \quad (4)$$

(1) & (4)

$$U_3 = \frac{U_0}{A_1 R_{10} s + 1 + \frac{R_{10}}{R_{11}}} + \frac{\frac{1}{R_{11}} \cdot H_1}{A_2 R_2 s + 1} \cdot \frac{U_0}{A_1 s + \frac{1}{R_{10}} + \frac{1}{R_{11}}} \Rightarrow$$



(7.12 lösning Kopplade tankar)



$$\begin{aligned} \Rightarrow \frac{U_3}{U_0} &= \frac{1}{A_1 R_{10} + 1 + \frac{R_{10}}{R_{11}}} + \frac{1}{(A_2 R_2 s + 1) \left(A_1 R_{11} s + \frac{R_{11}}{R_{10}} + 1 \right)} = \\ &= \frac{R_{11}}{A_1 R_{10} R_{11} s + R_{11} + R_{10}} + \frac{R_{10}}{(A_2 R_2 s + 1) (A_1 R_{10} R_{11} s + R_{11} + R_{10})} = \\ &= \frac{A_2 R_2 R_{11} s + R_{11} + R_{10}}{(A_2 R_2 s + 1) (A_1 R_{10} R_{11} s + R_{11} + R_{10})} = \end{aligned}$$

$$= \frac{A_2 R_2 R_{11} s + (R_{10} + R_{11})}{A_1 A_2 R_{10} R_{11} R_2 s^2 + (A_1 R_{10} R_{11} + A_2 R_2 (R_{10} + R_{11})) s + (R_{10} + R_{11})}$$

Experimentella analyser

Man går in med ”testsignaler” på processen och analyserar svaret på utgången.

- **Stegsvarsanalys**

Stegsvarstabell i Formelsamlingen

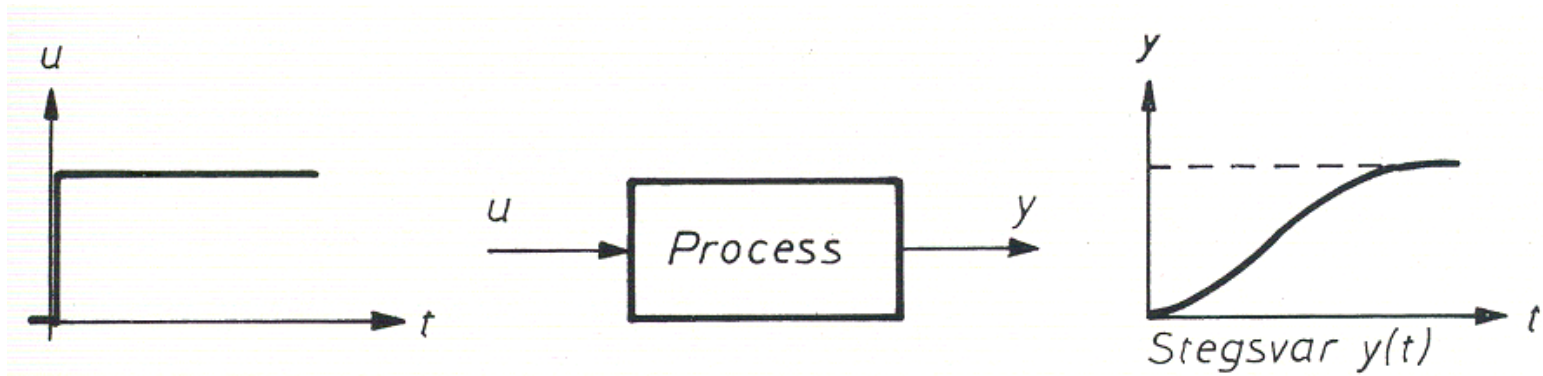
- **Frekvensanalys**

Testa med sinus-generator och rita Bodediagram.

- **Parametrisk identifiering**

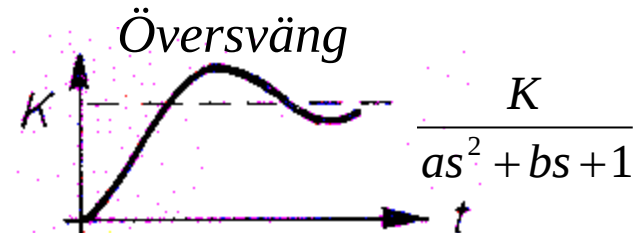
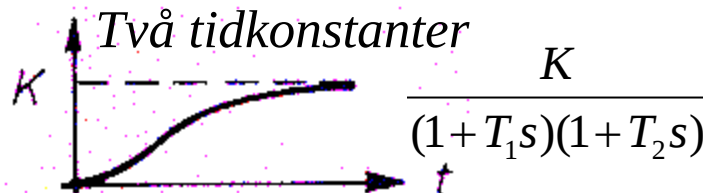
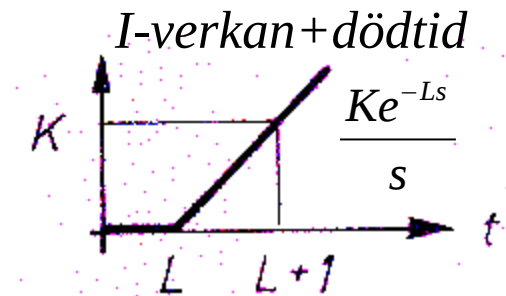
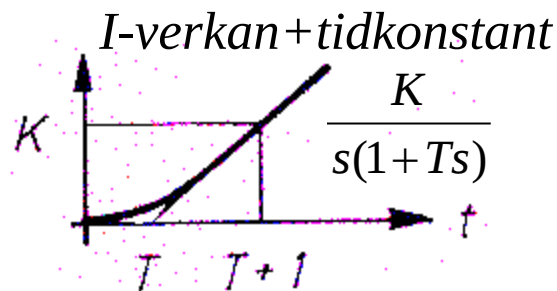
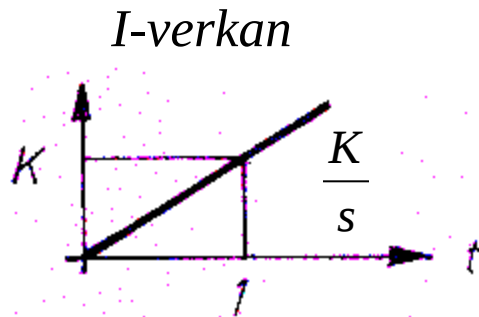
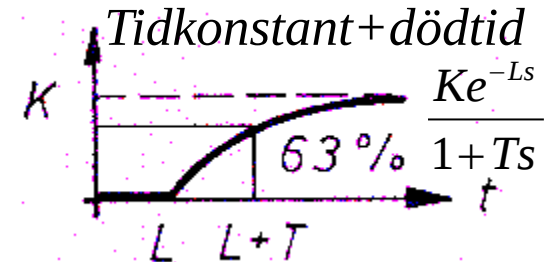
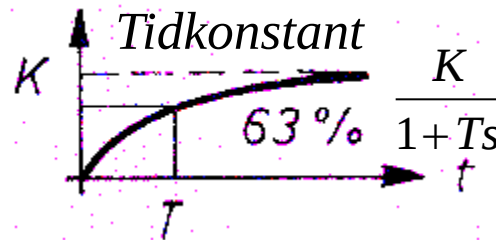
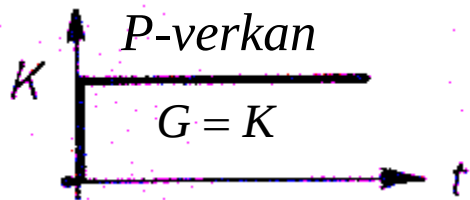
Identifiera parametrar med minsta kvadratmetoden.

Stegsvar

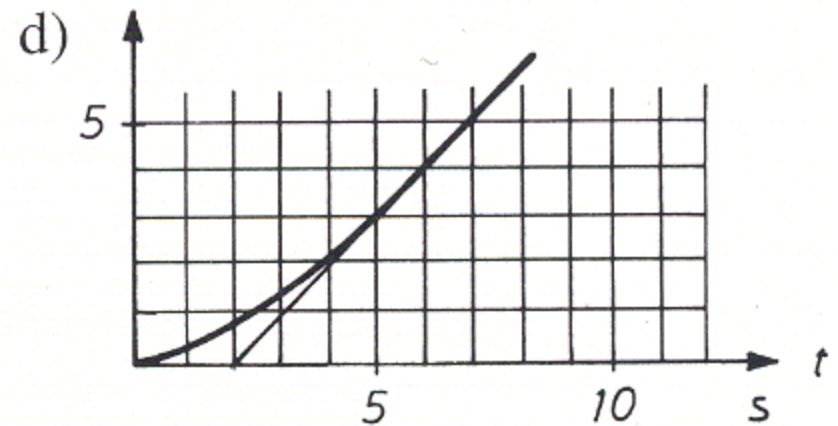
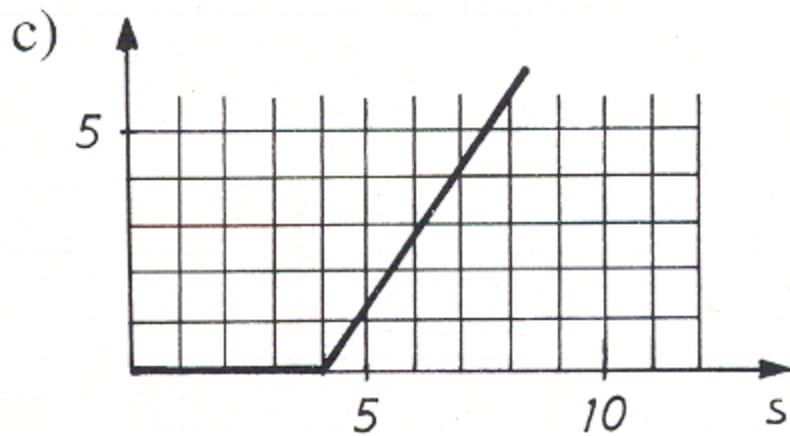
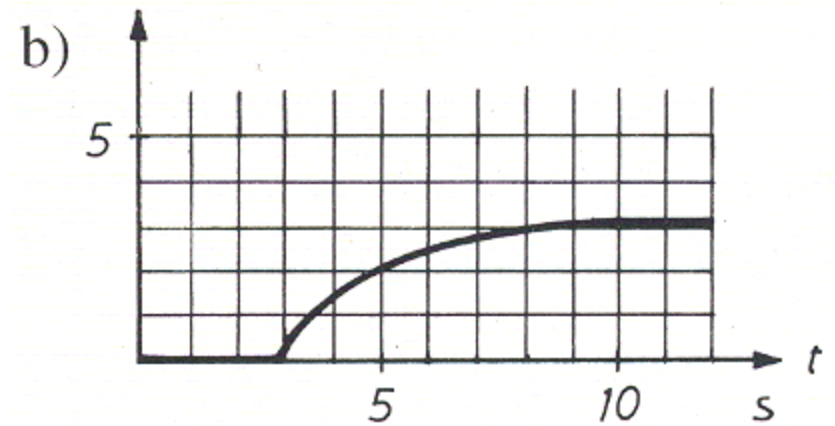
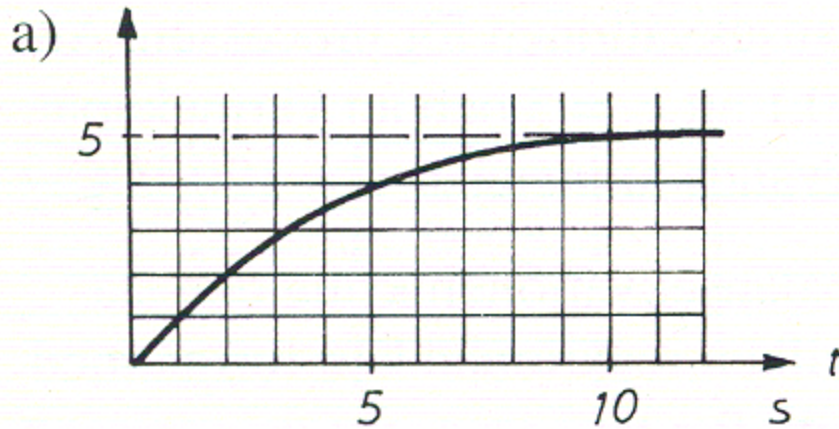


- Enhetssteget är den vanligaste "testsignalen".

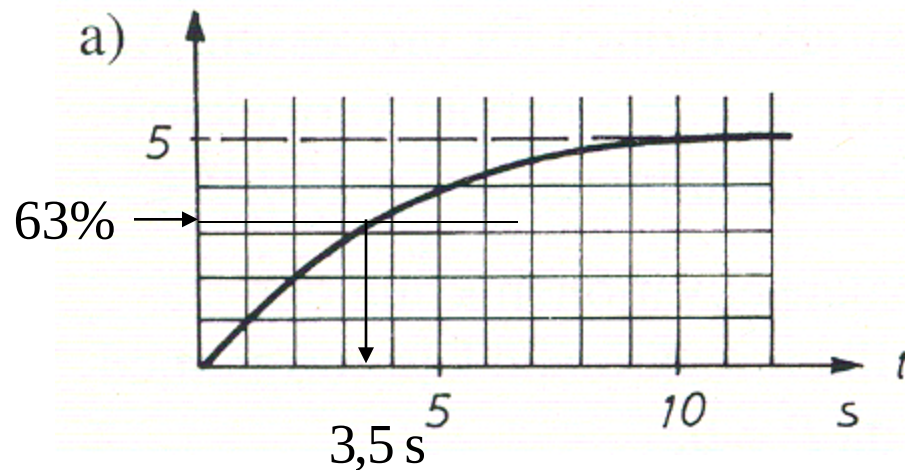
Stegsvarstabel



7.16 Stegvarsanalys



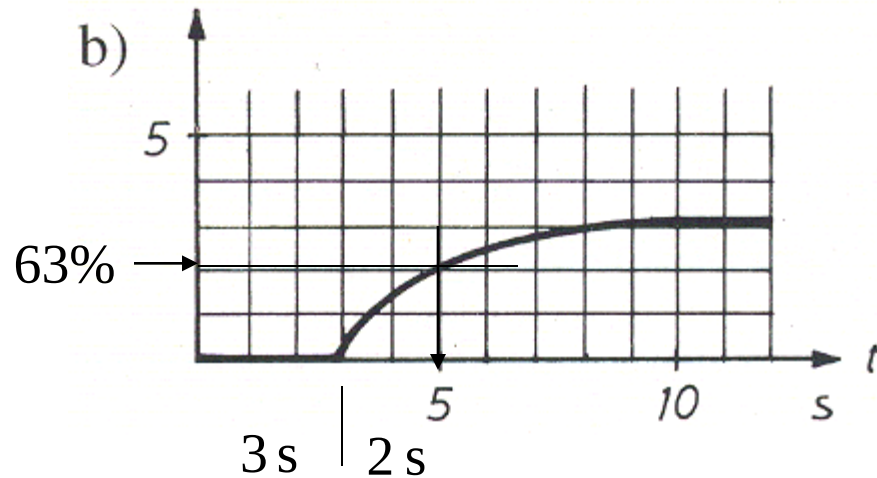
7.16 a lösning Stegvarsanalys



$$\frac{K}{1+Ts} \Rightarrow \frac{5}{1+3,5s}$$

- Enkel tidkonstant

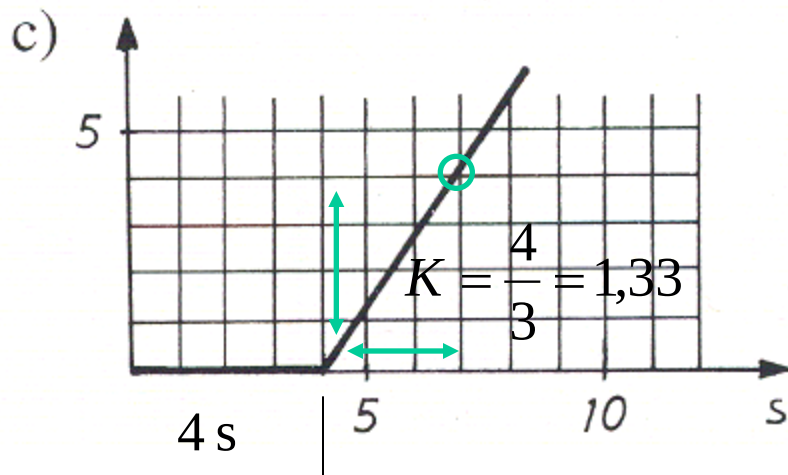
7.16 b lösning Stegvarsanalys



$$\frac{Ke^{-Ls}}{1+Ts} \Rightarrow \frac{3 \cdot e^{-3s}}{1+2s}$$

- Dödtid
- Enkel tidkonstant

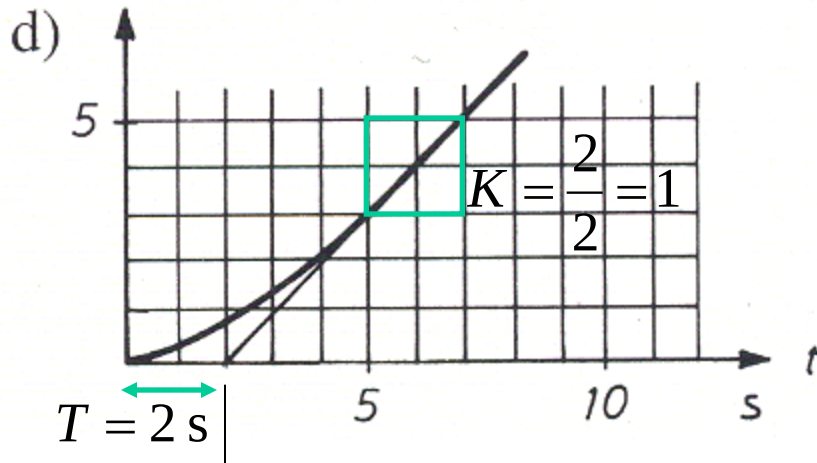
7.16 c lösning Stegvarsanalys



$$\frac{Ke^{-Ls}}{s} \Rightarrow \frac{1,33 \cdot e^{-4s}}{s}$$

- Dödtid
- Integrering

7.16 d lösning Stegvarsanalys

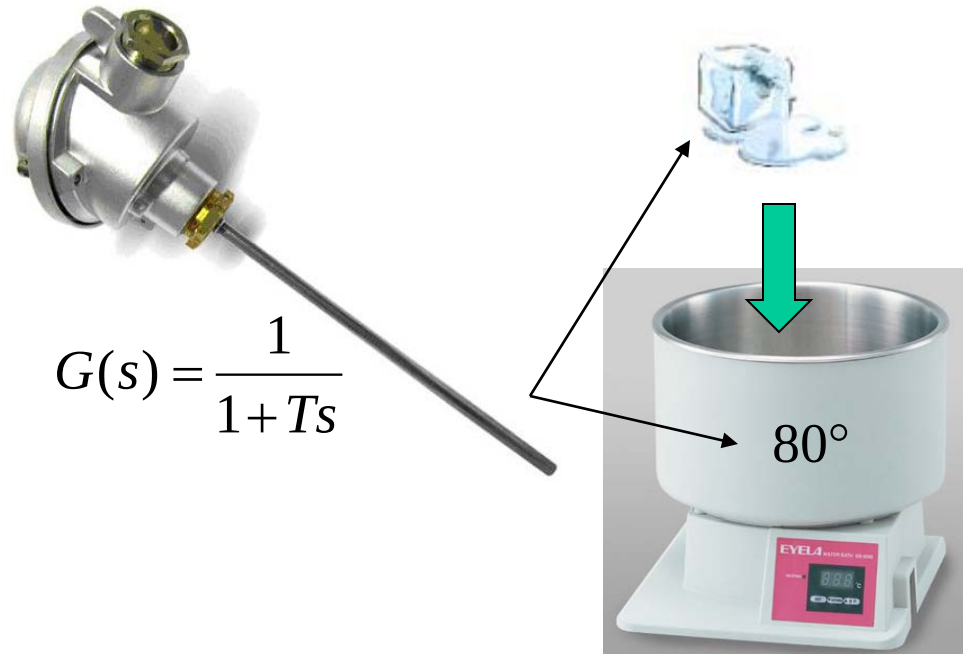


$$\frac{K}{s(1+Ts)} \Rightarrow \frac{1}{s(1+2s)}$$

- Integrering
- Tidkonstant

7.9 Temperaturgivare, tidkonstant

Från 0° till 80°
efter 20 s visar
givaren 65°
Bestäm
tidkonstanten T.

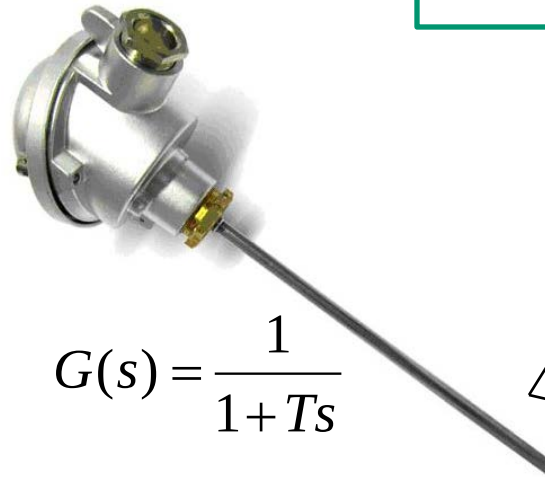


$$G(s) = \frac{1}{1 + Ts}$$

7.9 lösning, tidkonstant

$$\frac{1}{s(1+Ts)} \quad \{L^{-1}\} \quad 1 - e^{-\frac{t}{T}}$$

Från 0° till 80°
efter 20 s visar
givaren 65°
Bestäm
tidkonstanten T.



$$G(s) = \frac{1}{1+Ts}$$

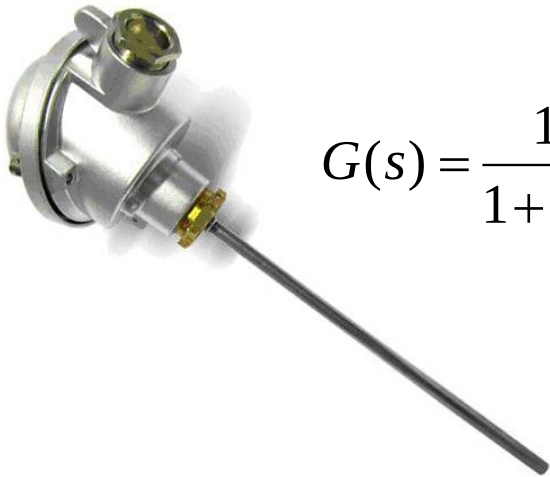


Temperatursteg: $\frac{80}{s} \Rightarrow \frac{80}{s} \cdot \frac{1}{1+Ts}$

$$\{L^{-1}:\} \quad x(t) = 80(1 - e^{-\frac{t}{T}}) \Rightarrow 65 = 80(1 - e^{-\frac{20}{T}})$$

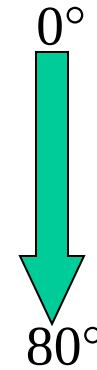
$$-\frac{20}{T} = \ln\left(\frac{80-65}{80}\right) \Rightarrow T = -\frac{20}{\ln\left(\frac{80-65}{80}\right)} \approx 12 \text{ s}$$

7.9 lösning, ”snabbformeln”



$$G(s) = \frac{1}{1 + Ts}$$

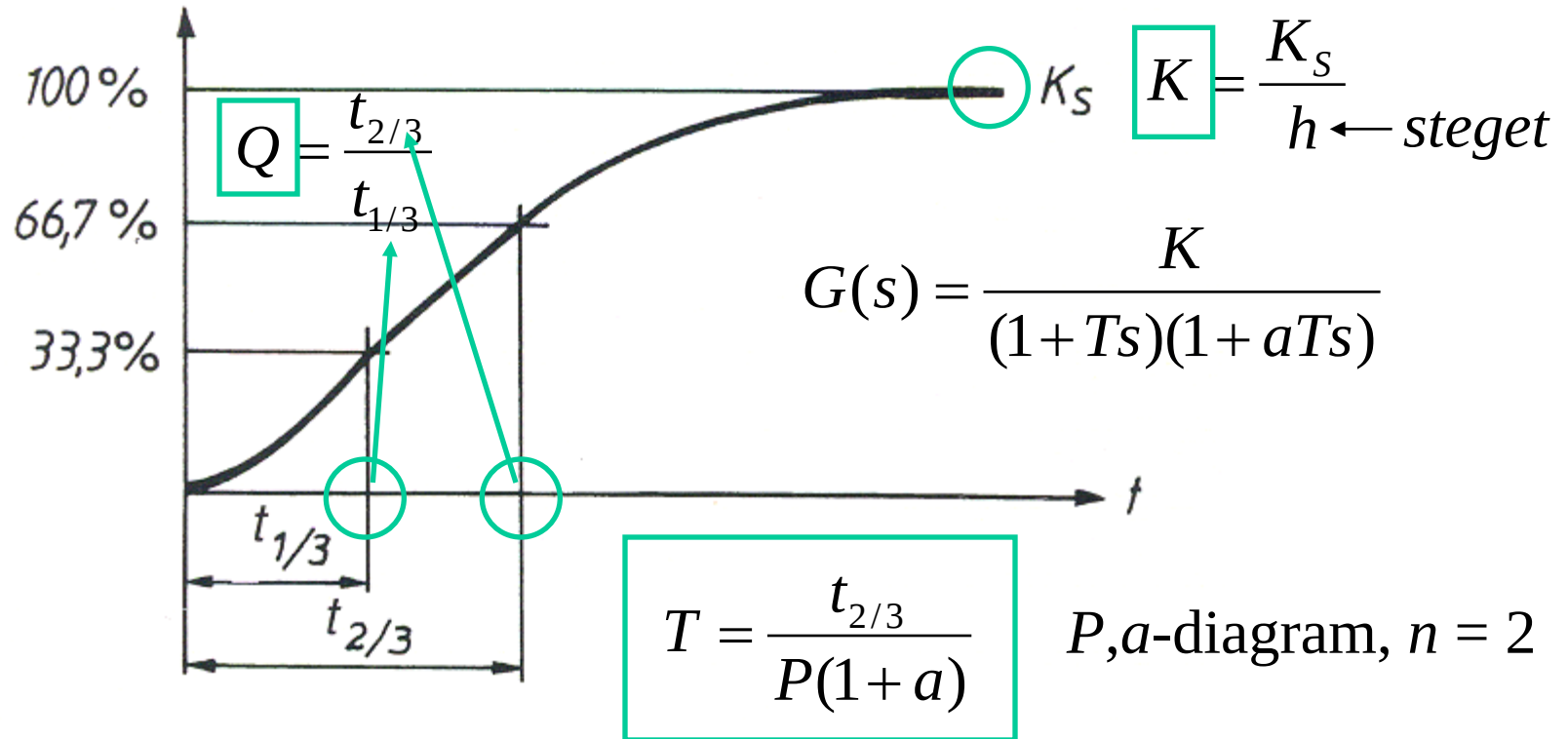
Från 0° till 80°
efter 20 s visar
givaren 65°
Bestäm
tidkonstanten T.



$$t = T \ln\left(\frac{\text{hela}}{\text{resten}}\right) \Rightarrow 20 = T \ln\left(\frac{80 - 0}{80 - 65}\right)$$

$$T = \frac{20}{\ln\left(\frac{80}{80 - 65}\right)} \approx 12 \text{ s}$$

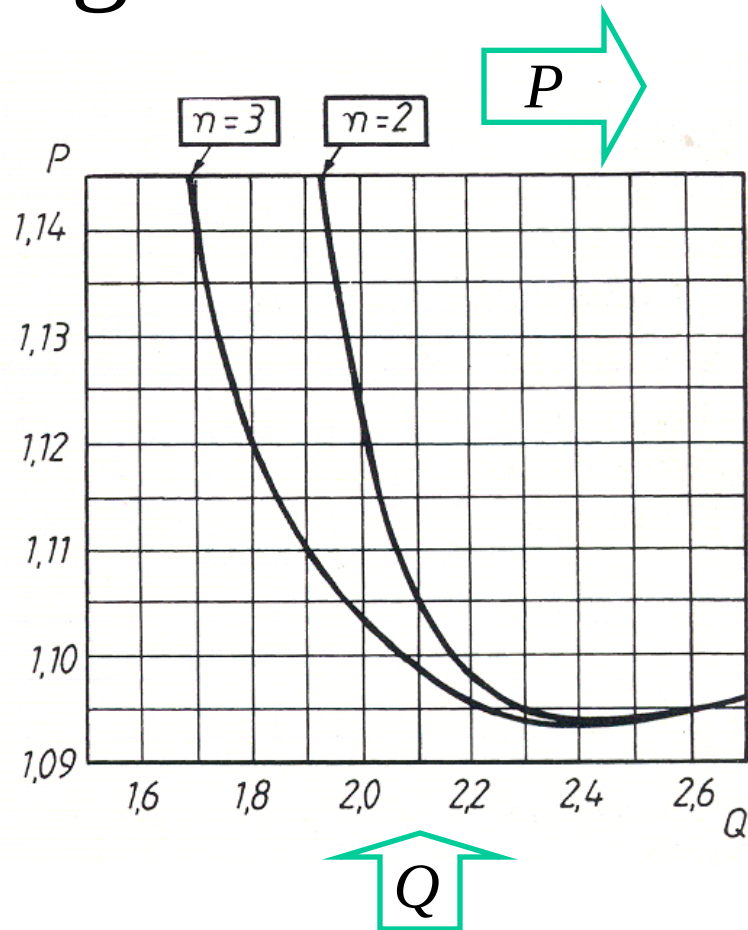
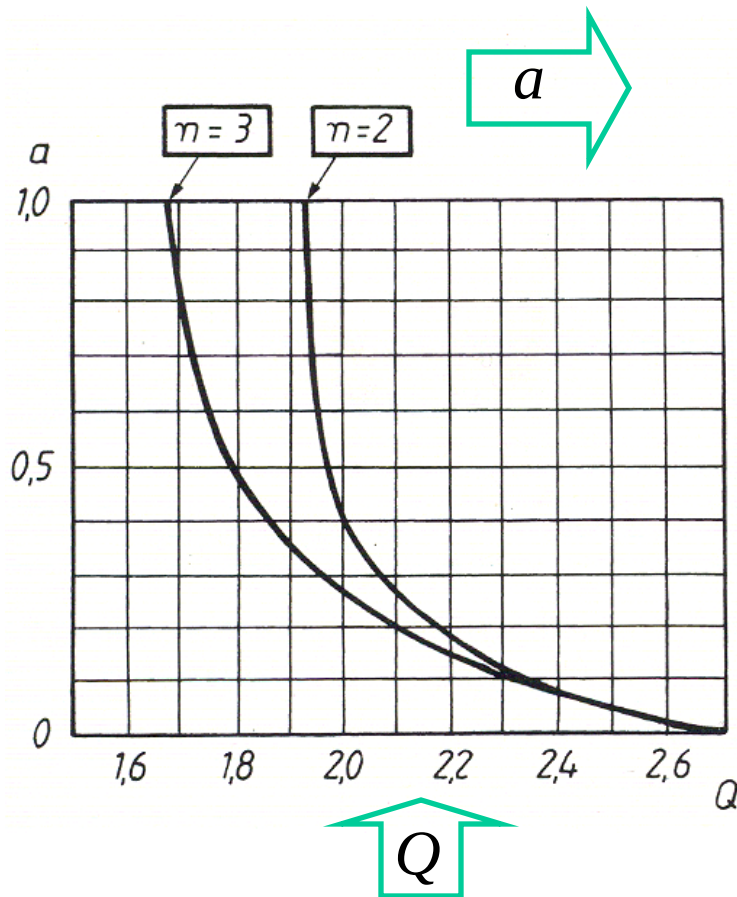
Processer med två tidkonstanter



$$1,92 < Q < 2,71$$

P, a -diagram

$$Q = \frac{t_{2/3}}{t_{1/3}}$$



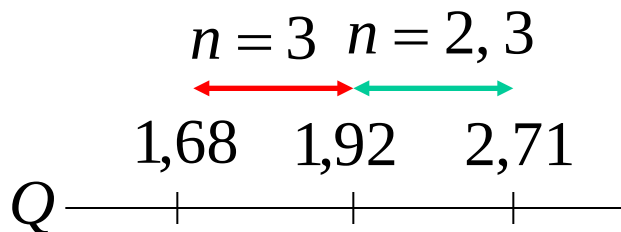
Processer med tre tidkonstanter

$$G(s) = \frac{K}{(1+Ts)(1+aTs)(1+a^2Ts)}$$

$$1,68 < Q < 2,71$$

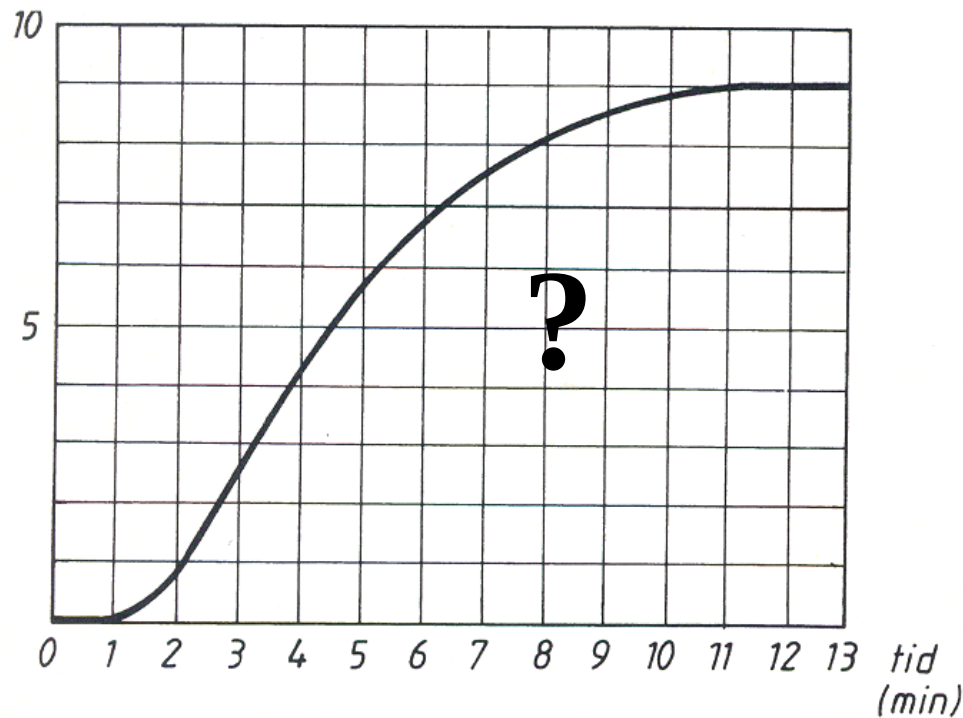
$$T = \frac{t_{2/3}}{P(1+a+a^2)}$$

P, a -diagram, $n = 3$

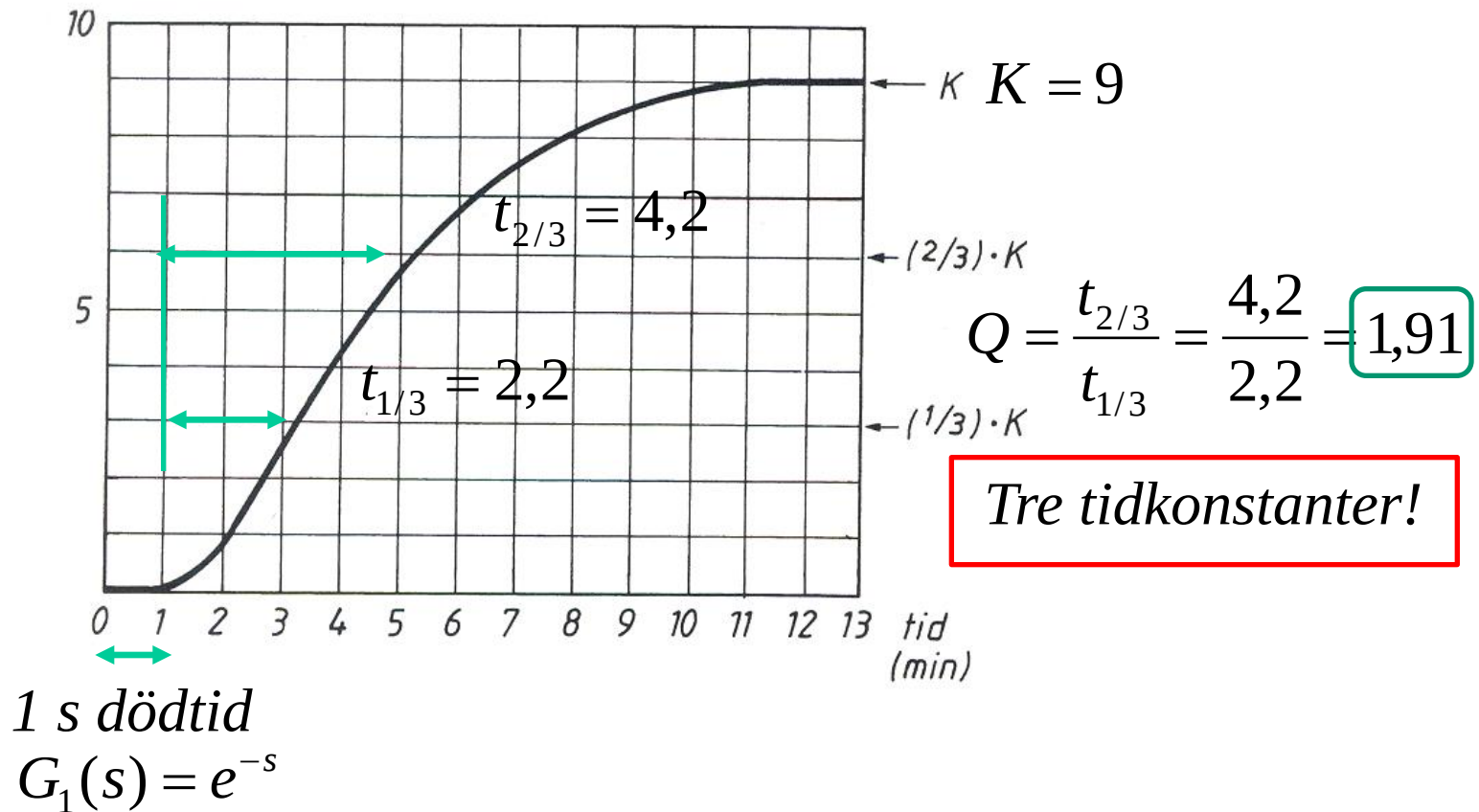


- Om $1,68 < Q < 1,92$ vet man att det är tre tidkonstanter.
- Om $1,92 < Q < 2,71$ kan man välja mellan 2 eller 3 tidkonstanter i modellen.

Ex. Identifiera Process?

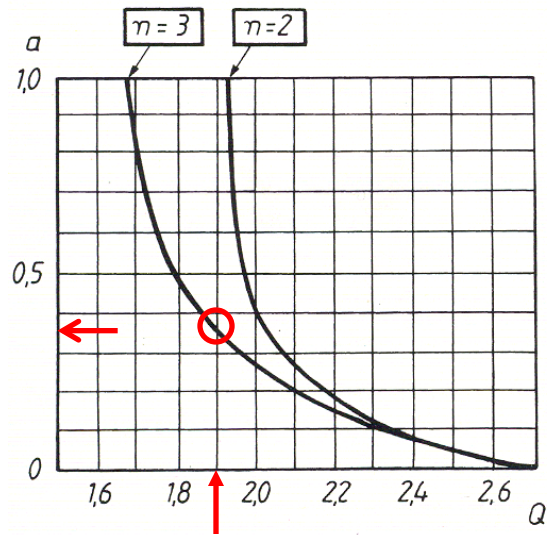


Ex. Identifiera Process?

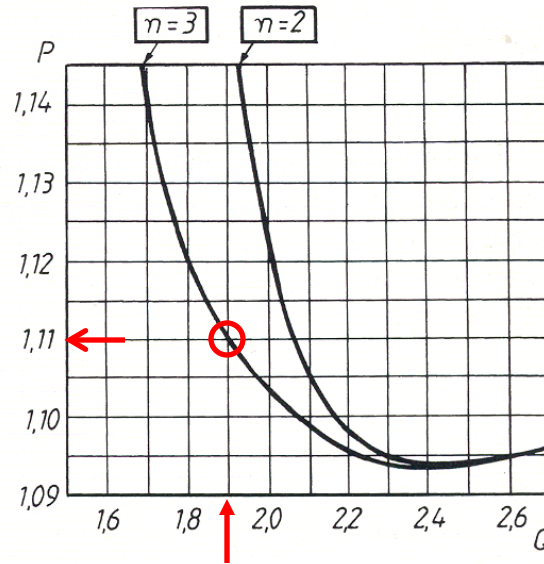


P, a -diagrammet

$$Q = 1,91 \Rightarrow$$



$$\Rightarrow a = 0,33$$



$$\Rightarrow P = 1,11$$

$$T = \frac{t_{2/3}}{P(1 + a + a^2)}$$

$$T = \frac{4,2}{1,11(1 + 0,33 + 0,33^2)} = 2,63$$

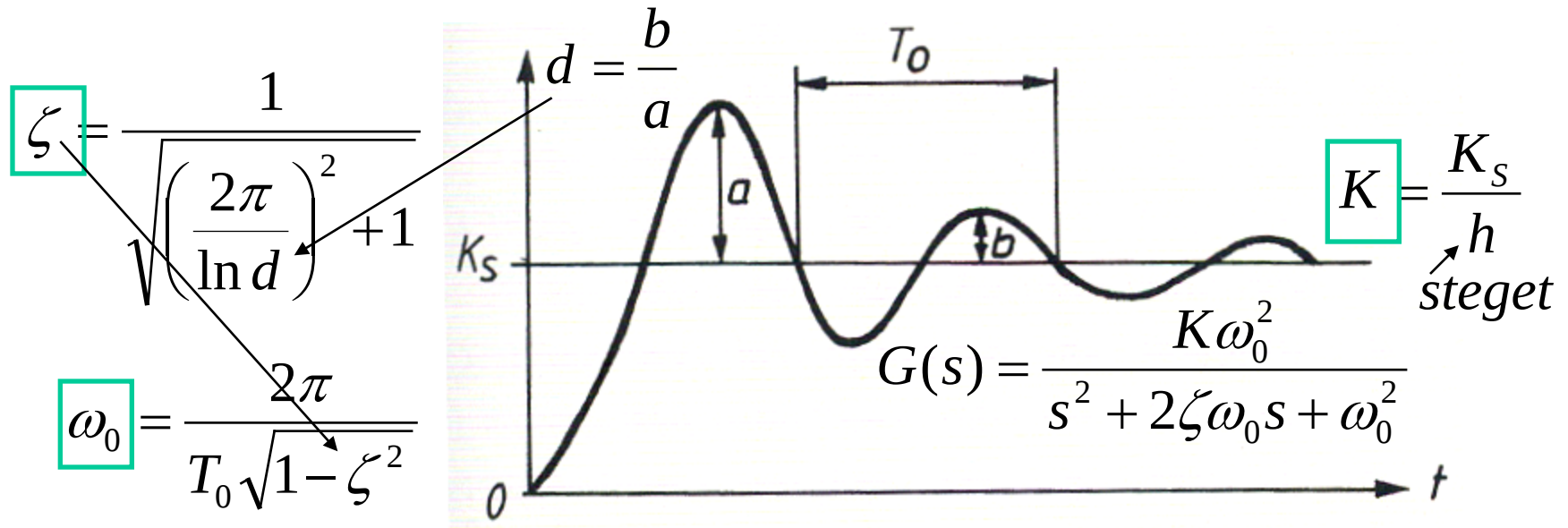
$$aT = 0,33 \cdot 2,63 = 0,87$$

$$a^2T = 0,33^2 \cdot 2,63 = 0,29$$

Approximativ överföringsfunktion

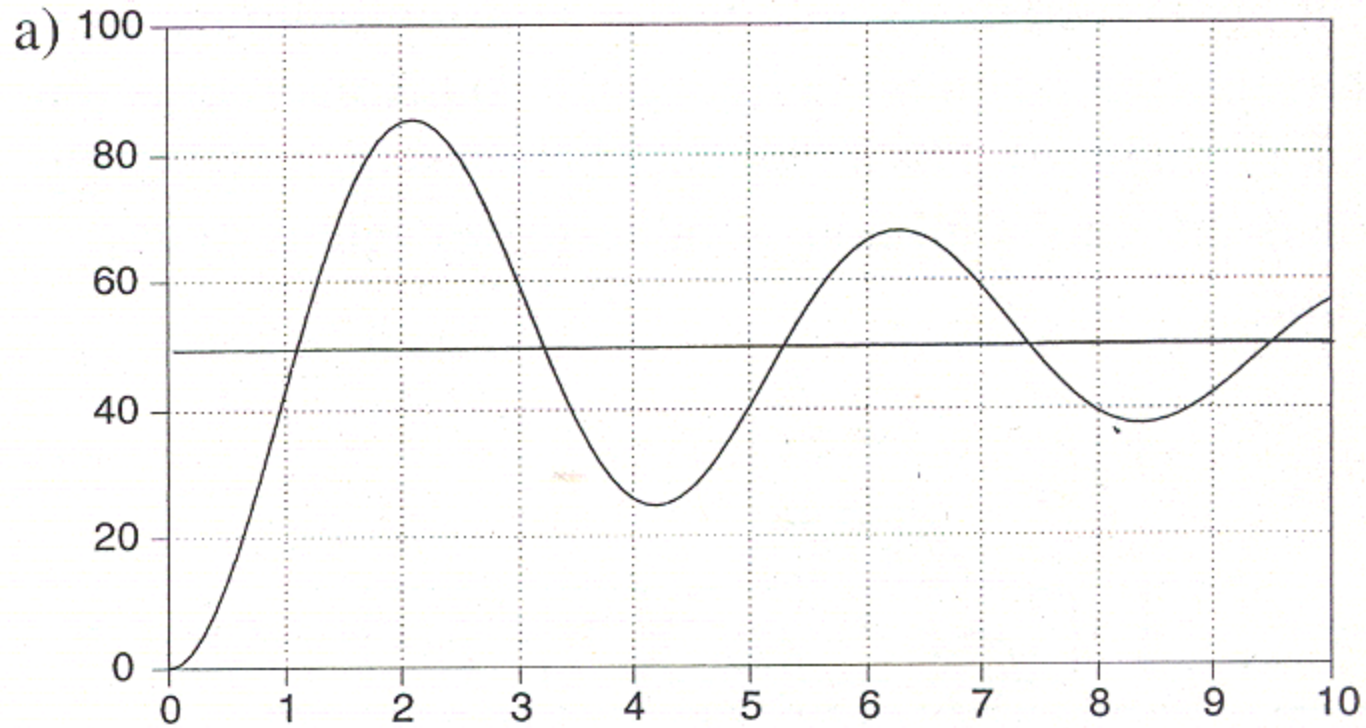
$$\begin{aligned} G(s) &= e^{-s} \cdot \frac{K}{(1+Ts)(1+aTs)(1+a^2Ts)} = \\ &= \frac{9 \cdot e^{-s}}{(1+2,63s)(1+0,87s)(1+0,29s)} \end{aligned}$$

Processer med översväng

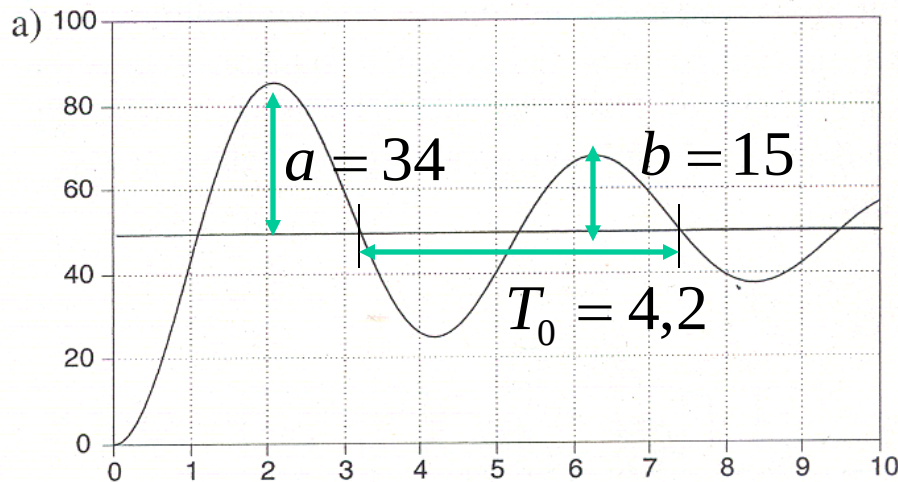


- Parametrar som kan identifieras efter ett stegsvarsexperiment.

7.17 a Stegvarsanalys



7.17 a lösning Stegvarsanalys



$$d = \frac{b}{a} = \frac{15}{34} = 0,44$$

$$K = \frac{K_s}{h} = \frac{50}{1} = 50$$

$$\omega_0 = \frac{2\pi}{T_0 \sqrt{1-\zeta^2}} =$$

$$= \frac{2\pi}{4,2 \sqrt{1-0,13^2}} \approx 1,51$$

$$\zeta = \frac{1}{\sqrt{\left(\frac{2\pi}{\ln d}\right)^2 + 1}} = \frac{1}{\sqrt{\left(\frac{2\pi}{\ln 0,44}\right)^2 + 1}} \approx 0,13$$

$$G(s) = \frac{K\omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2} = \frac{50 \cdot 1,51^2}{s^2 + 2 \cdot 0,13 \cdot 1,51 s + 1,51^2} = \frac{9,58}{s^2 + 0,39 s + 2,28}$$

