Reglerteknik Ö3

Köp övningshäfte på kårbokhandeln

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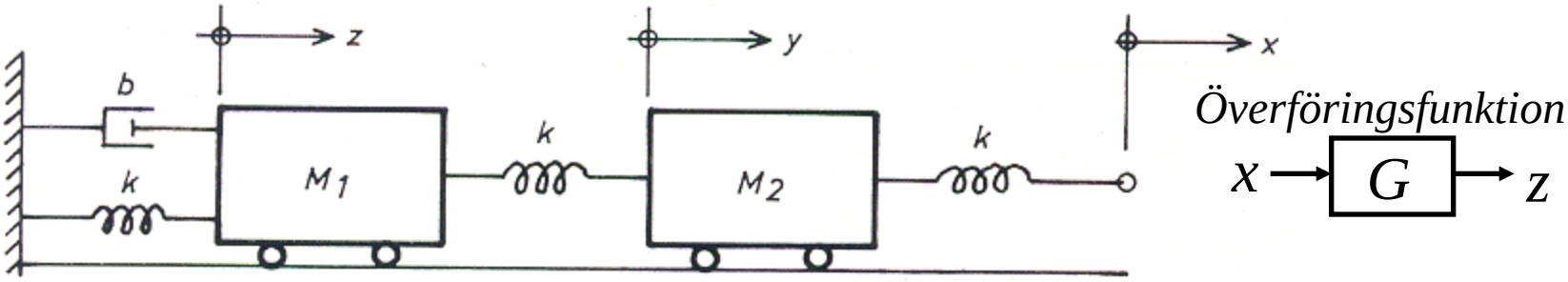
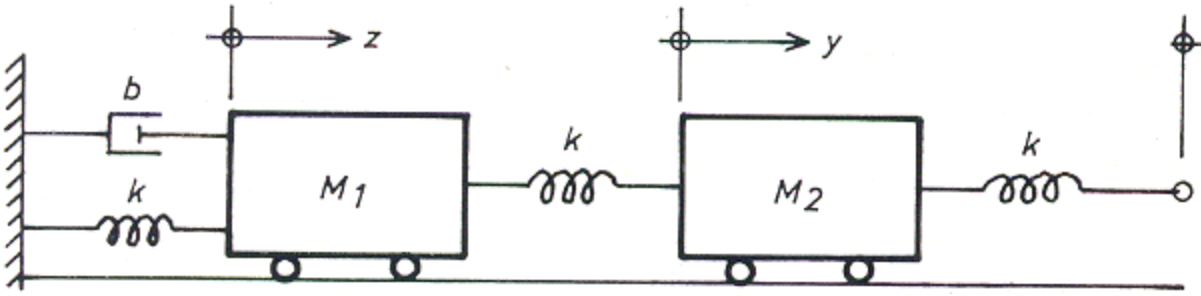
7.2 Mekanisk modell



Ställ upp differentialekvation och överföringsfunktion från *x* till *z*.

7.2 lösning Mekanisk modell



Överföringsfunktion

$x \rightarrow \boxed{G} \rightarrow z$

$x \rightarrow \boxed{G} \rightarrow z \quad \equiv \quad x \rightarrow \boxed{G_2} \xrightarrow{y} \boxed{G_1} \rightarrow z$

Laplace-transform direkt!

$L:\ M_1: \quad k(Y-Z)-bsZ-kZ = M_1 s^2 Z$

$\Rightarrow \quad Z(M_1 s^2 + bs + 2k) = kY$

$L:\ M_2: \quad k(X-Y)-k(Y-Z) = M_2 s^2 Z$

$\Rightarrow \quad Y = \dfrac{kX + kZ}{M_2 s^2 + 2k}$ *eliminera Y !*

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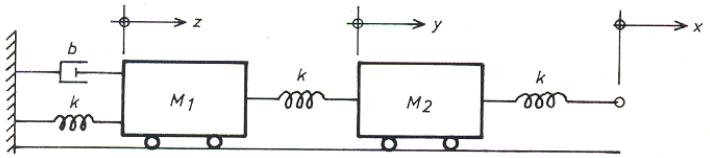
7.2 lösning Mekanisk modell



$$Z(M_1 s^2 + bs + 2k) = k\,\frac{kX + kZ}{M_2 s^2 + 2k} \qquad \textit{gör liknämnigt !}$$

$$\frac{Z(M_1 s^2 + bs + 2k)\left(M_2 s^2 + 2k\right)}{M_2 s^2 + 2k} = \frac{k^2 X + k^2 Z}{M_2 s^2 + 2k}$$

$$Z(M_1 s^2 + bs + 2k)(M_2 s^2 + 2k) - k^2 Z = k^2 X$$

$$\Rightarrow \quad G(s) = \frac{Z}{X} = \frac{k^2}{M_1 M_2 s^4 + M_2 b s^3 + 2k(M_1 + M_2)s^2 + 2bks + 3k^2}$$

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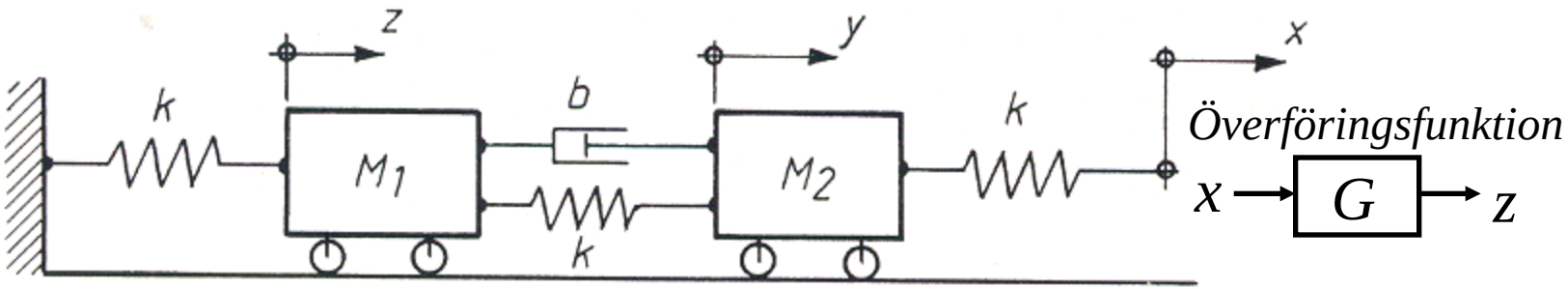
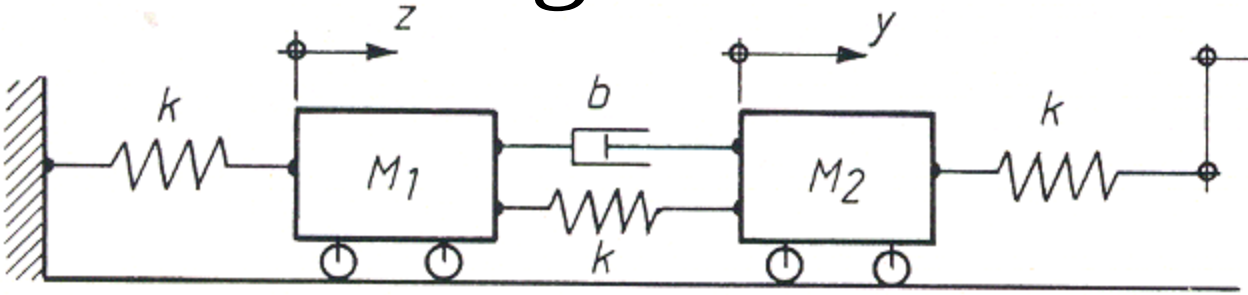
7.3 Mekanisk modell



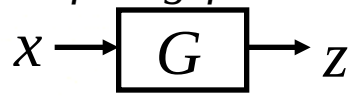
Ställ upp differentialekvation och överföringsfunktion från x till z.

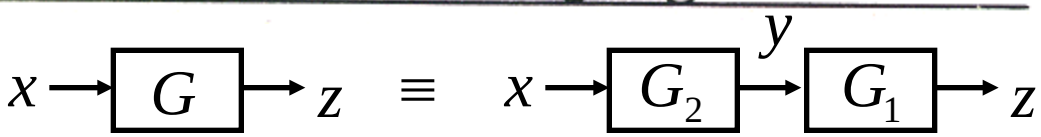
$$x \rightarrow \boxed{G} \rightarrow z \quad \equiv \quad x \rightarrow \boxed{G_2} \xrightarrow{y} \boxed{G_1} \rightarrow z$$

7.3 lösning Mekanisk modell



Överföringsfunktion





Laplace-transform direkt!

$L:\ M_1:\quad k(Y-Z)+bs(Y-Z)-kZ=M_1s^2Z$

$\Rightarrow\quad Z(M_1s^2+bs+2k)=(bs+k)Y$

$L:\ M_2:\quad k(X-Y)-k(Y-Z)-bs(Y-Z)=M_2s^2Y$

$\Rightarrow\quad Y(M_2s^2+bs+2k)=kX+kZ+bsZ$

$$Y=\frac{kX+kZ+bsZ}{M_2s^2+bs+2k}$$

eliminera Y !

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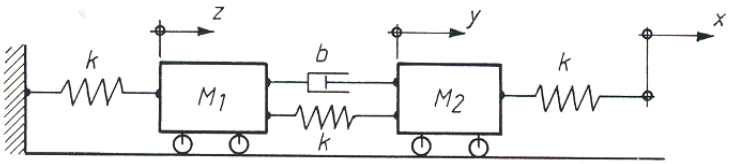
7.3 lösning Mekanisk modell



$$Z(M_1 s^2 + bs + 2k) = (bs + k)\frac{kX + kZ + bsZ}{M_2 s^2 + bs + 2k} \qquad \textit{gör liknämnigt !}$$

$$Z(M_1 s^2 + bs + 2k)(M_2 s^2 + bs + 2k) = (bs + k)(kX + kZ + bsZ)$$

$$Z\left(M_1 M_2 s^4 + b(M_1 + M_2)s^3 + (2k(M_1 + M_2) + b^2)s^2 + 4kbs + 4k^2\right) =$$

$$= X(bks + k^2) + Z(b^2 s^2 + 2bks + k^2)$$

$$\Rightarrow \quad G(s) = \frac{Z}{X} = \frac{bks + k^2}{M_1 M_2 s^4 + b(M_1 + M_2)s^3 + (2k(M_1 + M_2) + b^2)s^2 + 2bks + 3k^2}$$

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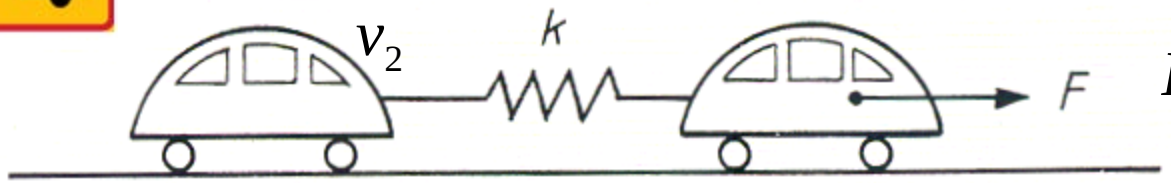
7.4 Mekanisk modell

Ex. Bogsering.



Överföringsfunktion

Ställ upp differentialekvation och överföringsfunktion från F till v_2.

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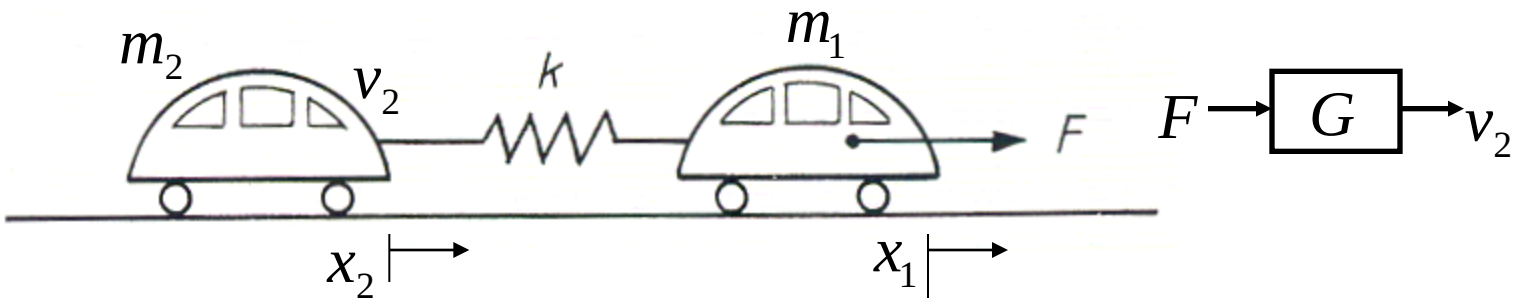
7.4 lösning Mekanisk modell



Laplace-transform direkt!

$L:$ $m_2:$ $\quad k(X_1 - X_2) - bsX_2 = m_2 s^2 X_2$

$\Rightarrow \quad X_1 \cdot k = X_2(m_2 s^2 + bs + k)$

$L:$ $m_1:$ $\quad F - k(X_1 - X_2) - bsX_1 = m_1 s^2 X_2$

$\Rightarrow \quad X_1 = \dfrac{F + kX_2}{m_1 s^2 + bs + k}$

eliminera X_1

$$v_2 = \frac{dx_2}{dt}$$

$L:$ $V_2 = sX_2$

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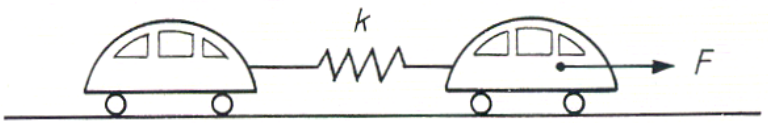
7.4 lösning Mekanisk modell



$$k \cdot \frac{F + kX_2}{m_1 s^2 + bs + k} = X_2(m_2 s^2 + bs + k)$$

$$kF + k^2 X_2 = X_2(m_2 s^2 + bs + k)(m_1 s^2 + bs + k)$$

$$kF = X_2\left(m_1 m_2 s^4 + b(m_1 + m_2)s^3 + (\,k(m_1 + m_2) + b^2)s^2 + (2kbs + k^2 - k^2)\right)$$

$$V_2 = sX_2 \quad \Rightarrow$$

$$G(s) = \frac{V_2}{F} = \frac{sX_2}{F} = \frac{k}{m_1 m_2 s^3 + b(m_1 + m_2)s^2 + (\,k(m_1 + m_2) + b^2)s + 2kb}$$

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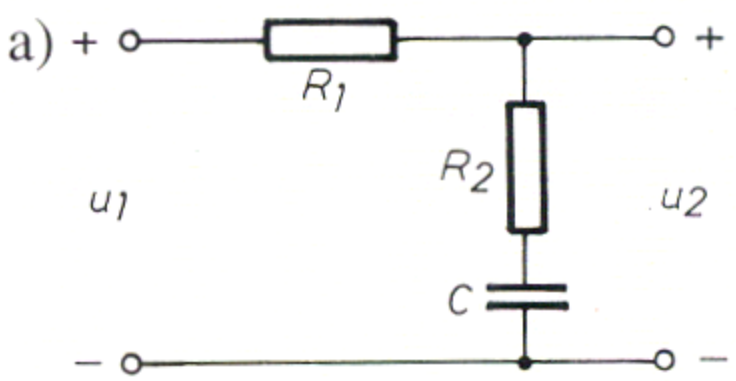
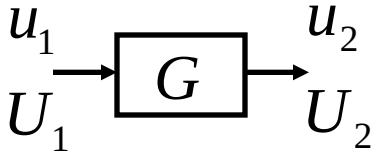
7.5 **a** Elektrisk modell



Överföringsfunktion



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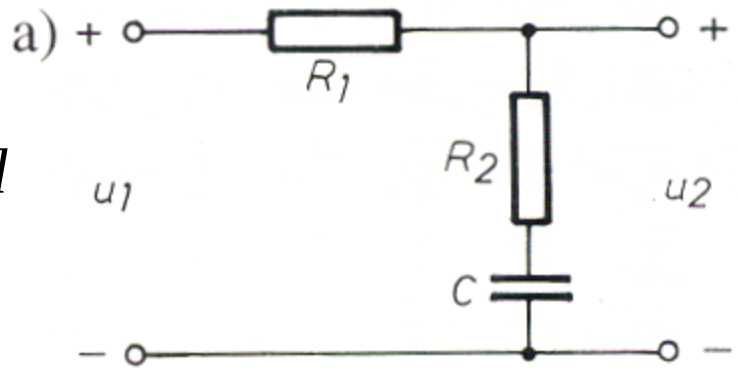
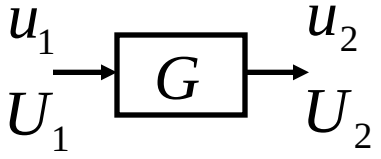
7.5 **a** lösning Elektrisk modell

Enklast med jω-metoden



Överföringsfunktion



Spänningsdelning

$$U_2 = U_1 \frac{R_2 + \frac{1}{j\omega C}}{R_1 + R_2 + \frac{1}{j\omega C}} \cdot \frac{j\omega C}{j\omega C} = U_1 \frac{R_2 j\omega C + 1}{(R_1 + R_2) j\omega C + 1}$$

$$\frac{U_2}{U_1} = \frac{R_2 j\omega C + 1}{(R_1 + R_2) j\omega C + 1} \quad \{s \to j\omega\} \quad \boxed{\frac{U_2}{U_1} = \frac{R_2 Cs + 1}{(R_1 + R_2) Cs + 1}}$$

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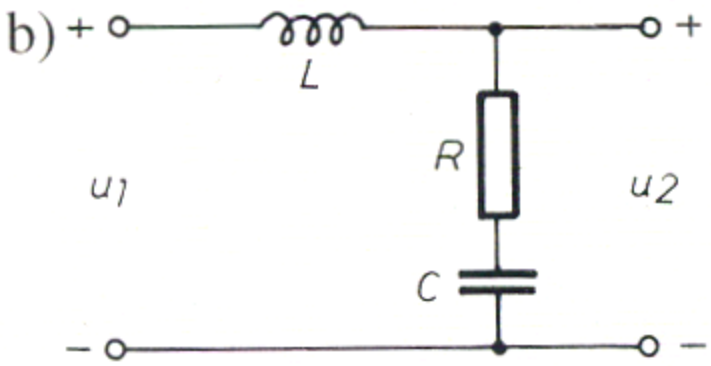
7.5 **b** Elektrisk modell



Överföringsfunktion

u_1 → G → u_2

U_1 U_2

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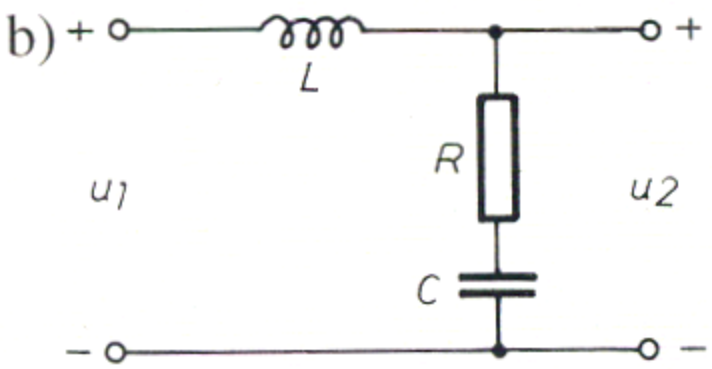
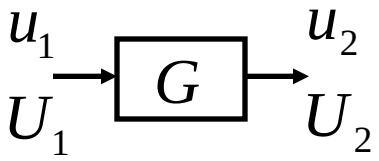
7.5 **b** lösning Elektrisk modell



Överföringsfunktion



Spänningsdelning

$$U_2 = U_1 \frac{R + \frac{1}{j\omega C}}{j\omega L + R + \frac{1}{j\omega C}} \quad \{s \to j\omega\} \quad U_1 \frac{R + \frac{1}{sC}}{sL + R + \frac{1}{sC}}$$

$$\frac{U_2}{U_1} = \frac{RCs + 1}{LCs^2 + RCs + 1}$$

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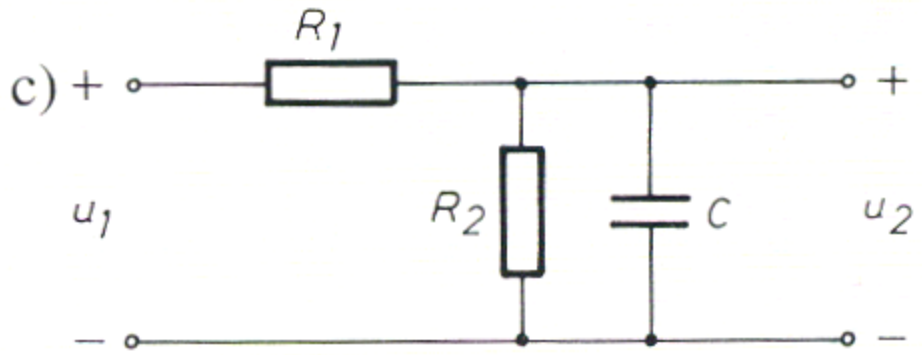
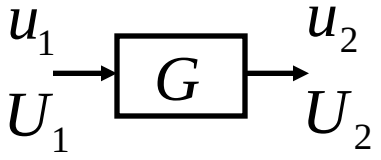
7.5 **c** Elektrisk modell



Överföringsfunktion



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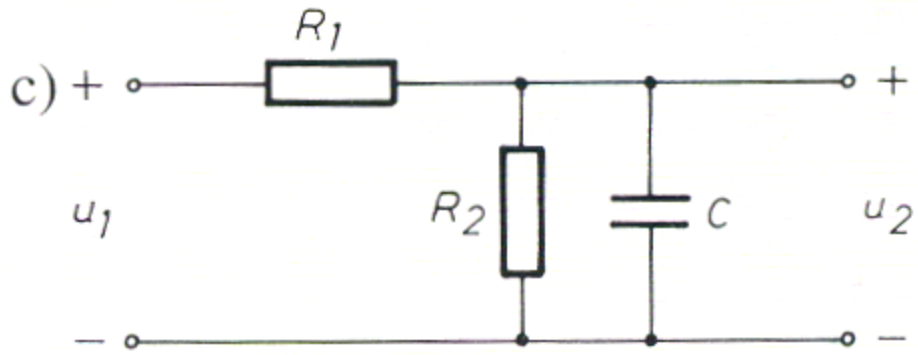
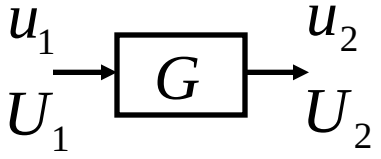
7.5 **c** lösning Elektrisk modell



Överföringsfunktion



$$R_2 \,||\, C = \frac{R_2 \cdot \dfrac{1}{j\omega C}}{R_2 + \dfrac{1}{j\omega C}} = \{s \to j\omega\} = \frac{R_2}{R_2 Cs + 1}$$

Spänningsdelning

$$\frac{U_2}{U_1} = \frac{\dfrac{R_2}{R_2 Cs + 1}}{R_1 + \dfrac{R_2}{R_2 Cs + 1}} = \boxed{\frac{R_2}{R_1 R_2 Cs + (R_1 + R_2)}}$$

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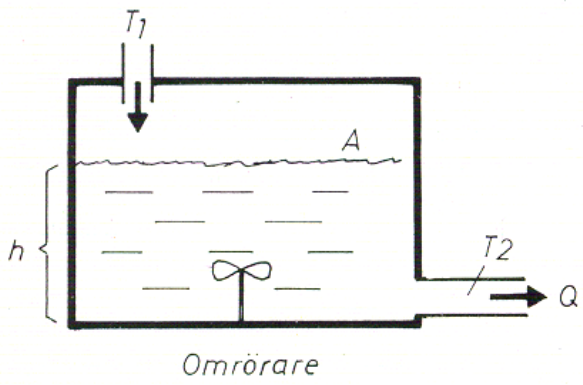
7.6 Termisk modell



T_1 inloppstemperatur [°]
T_2 utloppstemperatur [°]
Q genomflöde 0,1 [m^3/s]
A, h area och höjd 3 [m^3] 0,5 [m]

Överföringsfunktion

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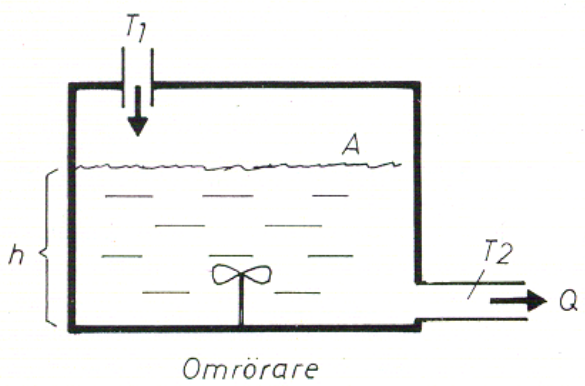
7.6 lösning Termisk modell



T_1 inloppstemperatur [°]
T_2 utloppstemperatur [°]
Q genomflöde 0,1 [m^3/s]
A, h area och höjd 3 [m^3] 0,5 [m]

Överföringsfunktion

tankens energi — *utrunnen värmeeffekt*

$$\frac{dE}{dt} = P_{in} - P_{ut}$$

tillförd värmeeffekt

temperatur — *vätskans värme-kapacitivitet*

$$E = T \cdot m \cdot c$$

massa

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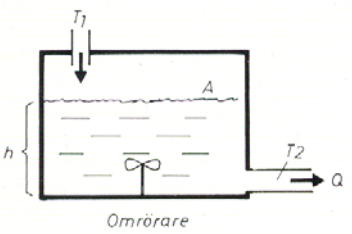
7.6 lösning Termisk modell

$$E = T \cdot c \cdot m = T \cdot c \cdot V\rho = T \cdot c \cdot Ah\rho$$

densitet — *volym = area·höjd* — *volym*



Överföringsfunktion

$$Q = \frac{V}{t}$$

$$P = \frac{T \cdot m \cdot c}{t} = \frac{T \cdot V \cdot \rho \cdot c}{t} = T \cdot c \cdot \rho \cdot Q$$

$$\frac{dE}{dt} = P_{in} - P_{ut} \quad \Leftrightarrow \quad Ah\rho c\frac{dT}{dt} = P_{in} - P_{ut}$$

$$Ah\cancel{\rho c}\frac{dT_2}{dt} = T_1 \cdot \cancel{\rho \cdot c} \cdot Q - T_2 \cdot \cancel{\rho \cdot c} \cdot Q \quad \{L:\}$$

$$AhsT_2 = T_1Q - T_2Q \quad G(s) = \frac{T_2}{T_1} = \frac{Q}{Ahs + Q} = \frac{1}{1 + \frac{Ah}{Q}s} = \frac{1}{1+15s}$$

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Laplacetransformtabell

$L[f(t)]$

$L^{-1}[F(s)]$

Laplacetransform $F(s)$	Tidsfunktion $f(t)$ för $t > 0$
1	Impulsfunktion $\delta(t)$
$\frac{1}{s}$	Stegfunktion $\sigma(t)$
$\frac{1}{s^2}$	Rampfunktion t
$\frac{1}{s^3}$	$\frac{t^2}{2}$
$\frac{1}{s+a}$	e^{-at}
$\frac{1}{(s+a)(s+b)}$	$\frac{e^{-at}-e^{-bt}}{b-a}$
$\frac{1}{s(1+as)}$	$1-e^{-\frac{t}{a}}$
$\frac{1}{s(1+as)(1+bs)}$	$1-\frac{a\cdot e^{-\frac{t}{a}}}{a-b}-\frac{b\cdot e^{-\frac{t}{b}}}{b-a}$
$\frac{1}{(s+a)(s+b)(s+c)}$	$\frac{e^{-at}}{(b-a)(c-a)}+\frac{e^{-bt}}{(c-b)(a-b)}+\frac{e^{-ct}}{(a-c)(b-c)}$
$\frac{s+a}{(s+b)(s+c)}$	$\frac{(a-b)e^{-bt}-(a-c)e^{-ct}}{c-b}$
$\frac{1}{(s+a)^2}$	$t\cdot e^{-at}$
$\frac{1}{s^2+a^2}$	$\frac{1}{a}\cdot \sin at$
$\frac{s}{s^2+a^2}$	$\cos at$
$\frac{1}{s(s^2+a^2)}$	$\frac{1}{a^2}[1-\cos at]$
$\frac{1}{s^2(s^2+a^2)}$	$\frac{1}{a^3}[at-\sin at]$
$\frac{s+a}{(s+a)^2+b^2}$	$e^{-at}\cdot \cos bt$
$\frac{b}{(s+a)^2+b^2}$	$e^{-at}\cdot \sin bt$

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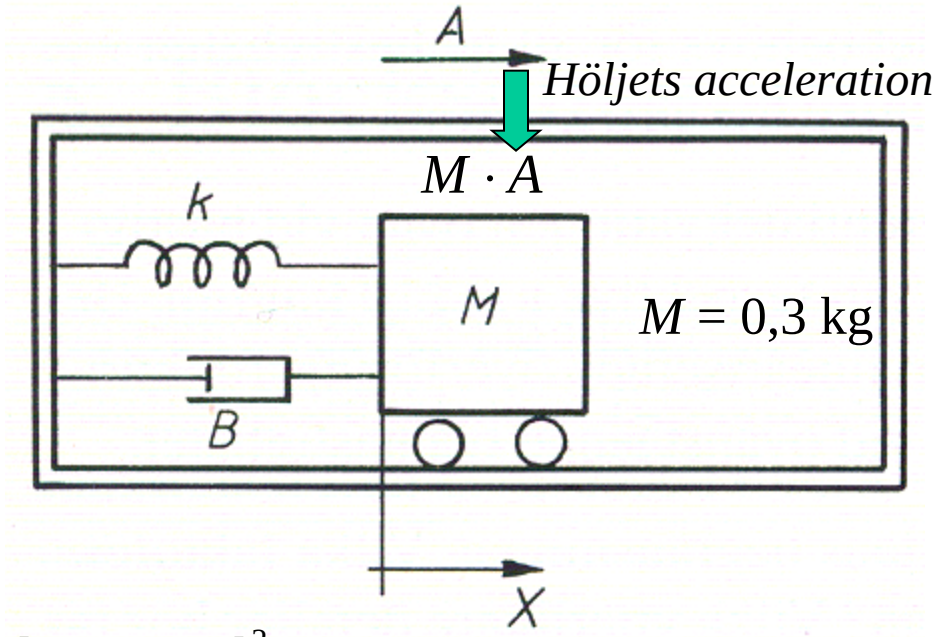
7.15 Accelerometer

Newtons lag

$$\sum F = M\frac{d^2x}{dt^2}$$



$$M \cdot A + k \cdot x + B\frac{dx}{dt} = -M\frac{d^2x}{dt^2} \quad \{L:\} \quad MA + kX + BXs = -MXs^2$$

$$\Rightarrow \quad G(s) = \frac{X}{A} = -\frac{M}{Ms^2 + Bs + k}$$

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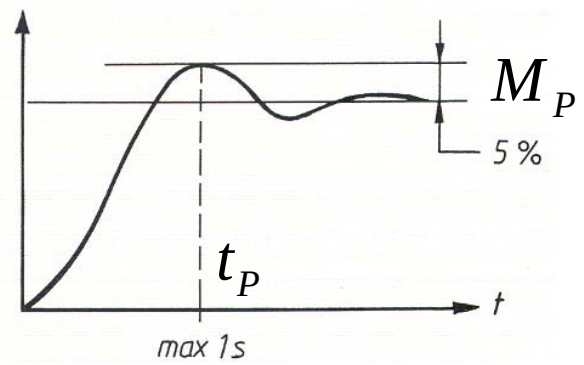
Formelsamling komplexa rötter

Processens stegsvar. Relativa dämpningen ζ. Odämpad egen-svängning ω_0. Översväng M_p, tid för översväng t_p.



Formelsamling (2:a ordningens system med komplexa rötter):

$$G(s) = \frac{K\omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2} \qquad M_P = \exp\left[\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}\right] \qquad t_P = \frac{\pi}{\omega_0\sqrt{1-\zeta^2}}$$

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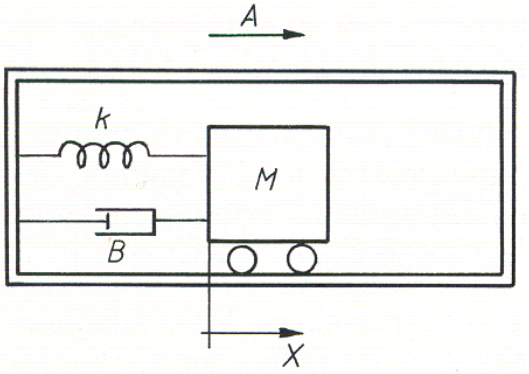
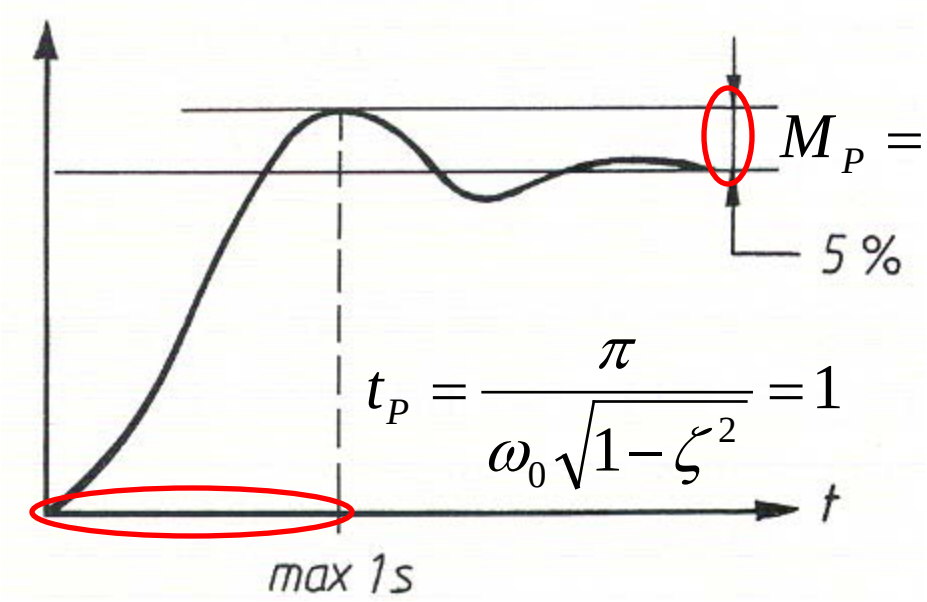
7.15 **b** Accelerometer parametrar



$$G(s) = \frac{K\omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2}$$



$$M_P = \exp\left[\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}\right] = 0{,}05$$

$$t_P = \frac{\pi}{\omega_0\sqrt{1-\zeta^2}} = 1$$

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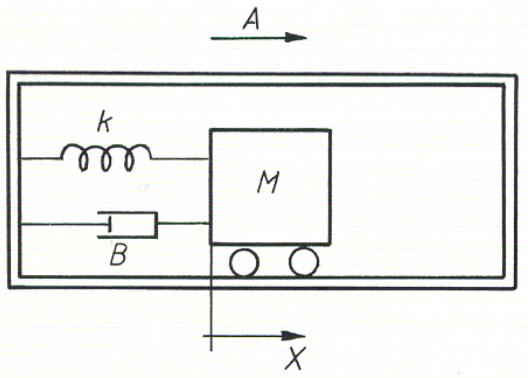
7.15 **b** Accelerometer parametrar



$$M_P = \exp\left[\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}\right] = 0{,}05 \quad \Rightarrow \quad \frac{-\zeta\pi}{\sqrt{1-\zeta^2}} = \ln 0{,}05 = -3$$

$$\Rightarrow \quad \zeta\pi = 3\cdot\sqrt{1-\zeta^2} \quad \Rightarrow \quad \pi^2\zeta^2 = 3^2(1-\zeta^2) \quad \Rightarrow \quad \zeta^2 = \frac{9}{\pi^2+9}$$

$$\Rightarrow \quad \boxed{\zeta = 0{,}69}$$

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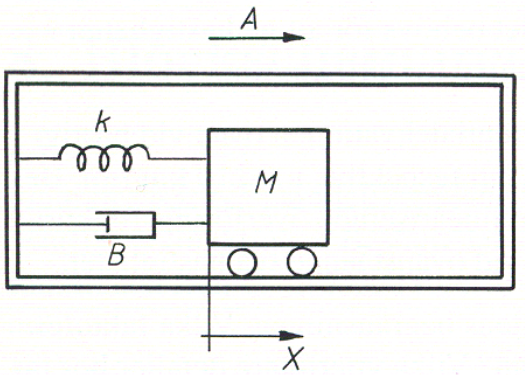
7.15 **b** Accelerometer parametrar



$$G(s) = \frac{K\omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2}$$ ← *gain*

$\zeta = 0{,}69 \quad M = 0{,}3$ kg

annan gain påverkar inte parametrarna

$$G(s) = -\frac{M}{Ms^2 + Bs + k} = -\frac{1}{s^2 + \frac{B}{M}s + \frac{k}{M}} \quad \Rightarrow \quad \omega_0 = \sqrt{\frac{k}{M}}$$

$$\frac{B}{M} = 2\zeta\omega_0 \quad \Rightarrow \quad B = 0{,}3 \cdot 2 \cdot 0{,}69 \cdot \sqrt{\frac{k}{0{,}3}}$$

$$\Rightarrow \quad B = \sqrt{0{,}3} \cdot 2 \cdot 0{,}69 \cdot \sqrt{k} = 0{,}76 \cdot \sqrt{k}$$

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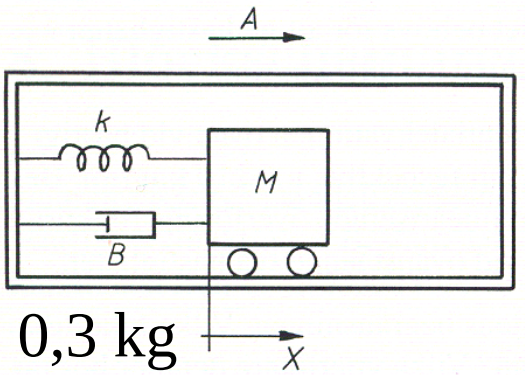
7.15 **b** Accelerometer parametrar



$M = 0{,}3 \text{ kg}$

$\zeta = 0{,}69$

$$\omega_0 = \sqrt{\frac{k}{M}}$$

$$t_P = \frac{\pi}{\omega_0\sqrt{1-\zeta^2}} = 1 \quad \Rightarrow \quad \frac{\sqrt{M}\cdot\pi}{\sqrt{k}\cdot\sqrt{1-\zeta^2}} = 1$$

$$\Rightarrow \quad \sqrt{k} = \frac{\sqrt{M}\cdot\pi}{\sqrt{1-\zeta^2}} = \frac{\sqrt{0{,}3}\cdot\pi}{\sqrt{1-0{,}69^2}} = 2{,}38$$

$$\Rightarrow \quad \boxed{k = 5{,}65}$$

$$\boxed{B} = 0{,}76\cdot\sqrt{k} = 0{,}76\cdot 2{,}38 = \boxed{1{,}8}$$

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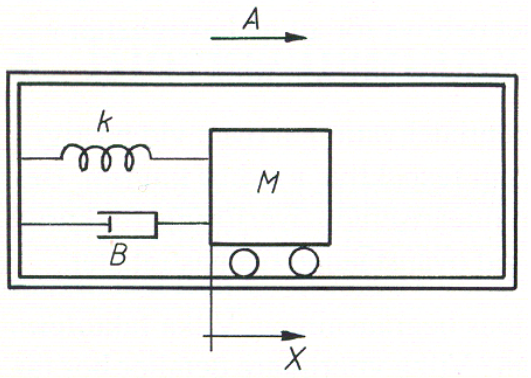
7.15 **c** Accelerometer stegsvar 1 m/s^2



$$G(s) = -\frac{M}{Ms^2 + Bs + k} = \frac{0{,}3}{0{,}3s^2 + 1{,}8s + 5{,}65}$$

Slutvärdet vid stegändring A = 1 m/s^2

$$\lim_{s \to 0}\left[s \cdot \frac{1}{s} \cdot G(s) \right] = \lim_{s \to 0}\left[-1 \cdot \frac{0{,}3}{0{,}3s^2 + 1{,}8s + 5{,}65} \right] = -\frac{0{,}3}{5{,}65} = -0{,}053$$

Vid accelerationen A 1 m/s^2 ”backar” massan M sträckan 5,3 cm.

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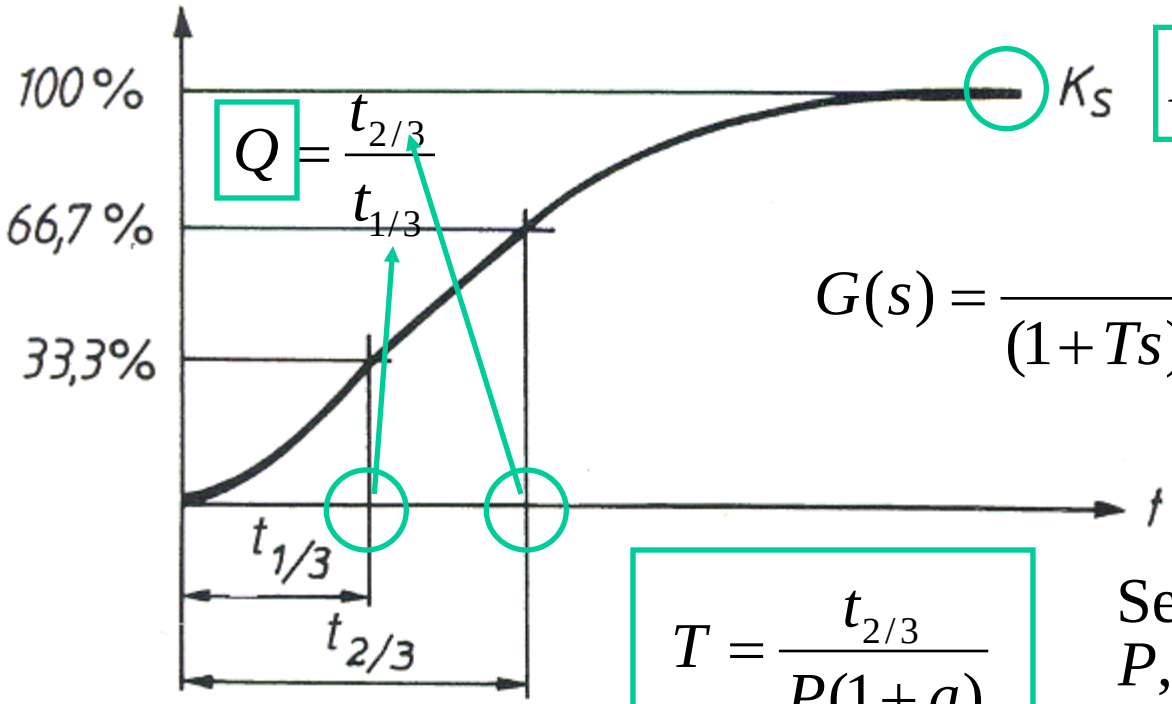
Processer med **två** tidkonstanter



$$K = \frac{K_S}{h} \leftarrow \textit{steget}$$

$$G(s) = \frac{K}{(1+Ts)(1+aTs)}$$

$$T = \frac{t_{2/3}}{P(1+a)}$$

Se
P,*a*-diagram, *n* = 2

$1{,}92 < Q < 2{,}71$ *Ok med två tidkonstanter*

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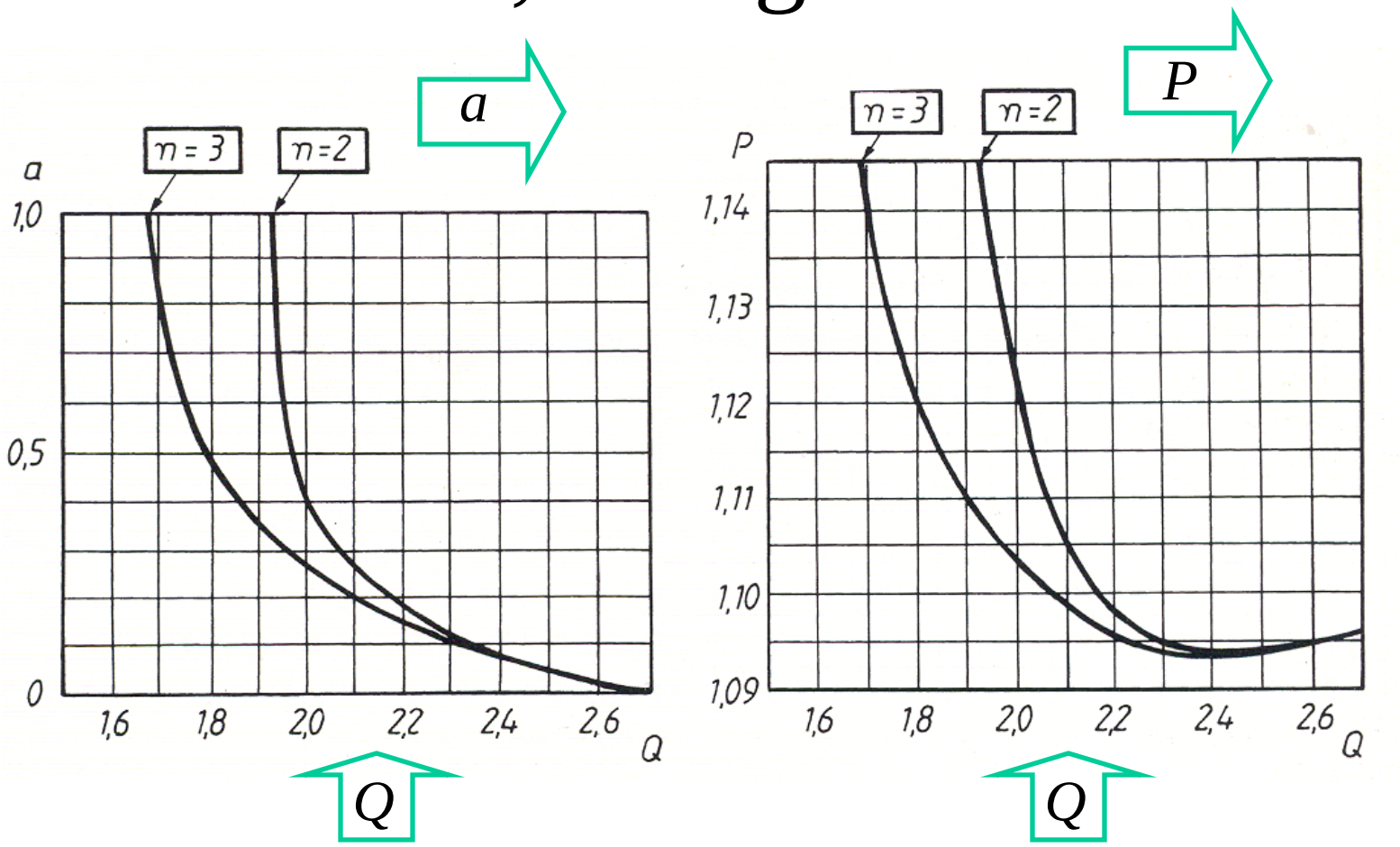
P,*a*-diagram

$$Q = \frac{t_{2/3}}{t_{1/3}}$$



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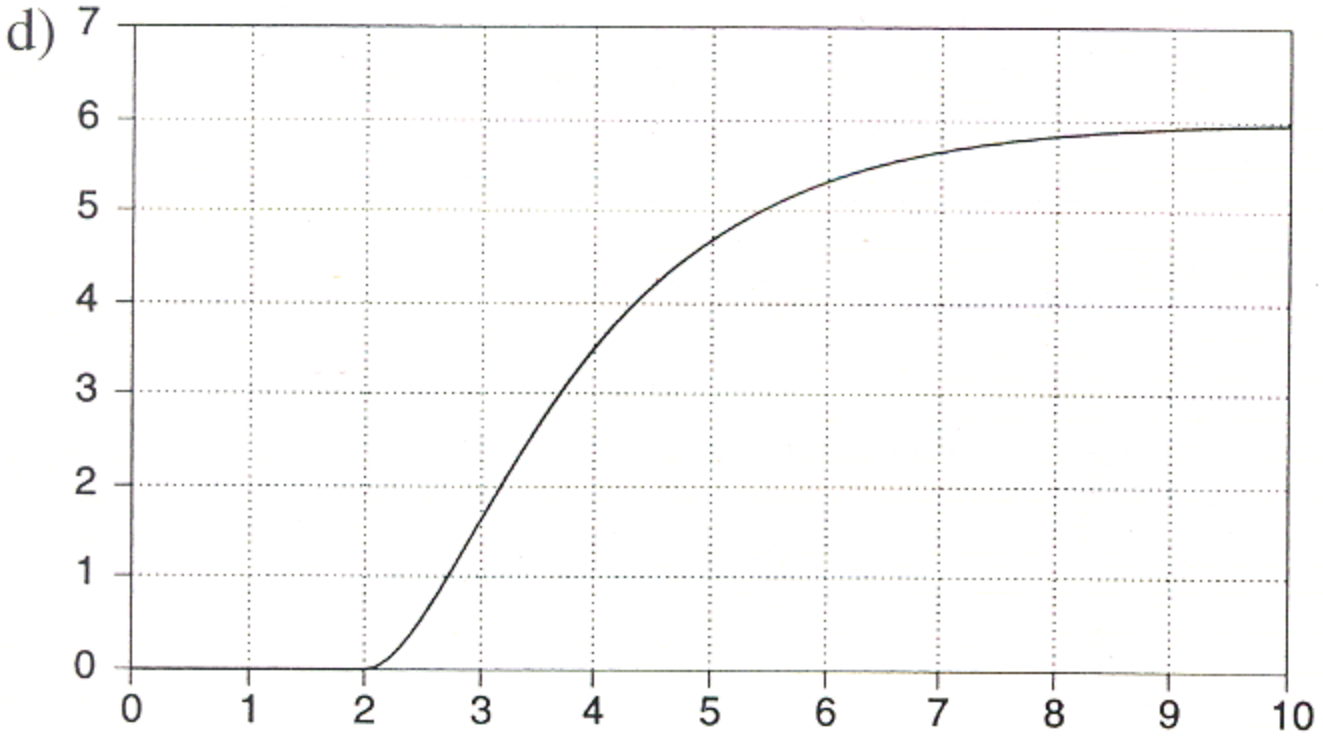
7.17 d Stegsvarsanalys



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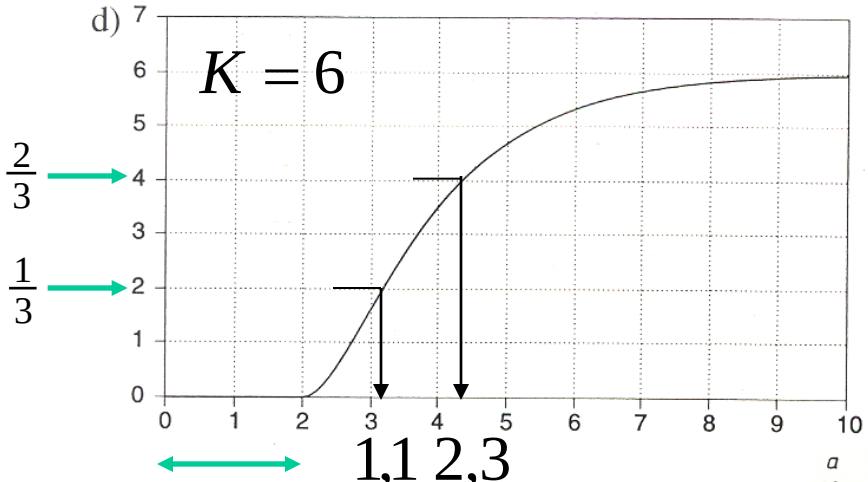
7.17 **d** lösning Stegsvarsanalys

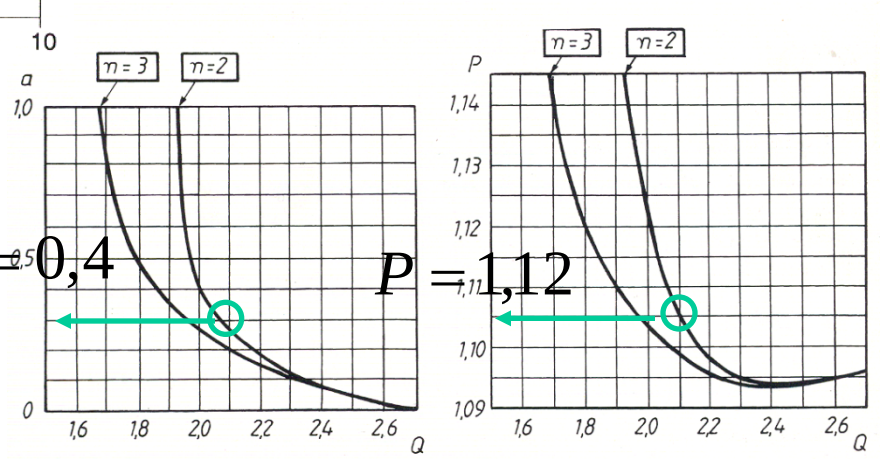


$$L = 2$$

dödtid

$$Q = \frac{t_{2/3}}{t_{1/3}} = \frac{2,3}{1,1} = 2,09$$

Modell med två tidkonstanter



$a = 0,4$ $\quad P = 1,12$

$$T = \frac{t_{2/3}}{P(1+a)} =$$

$$= \frac{2,3}{1,12(1+0,4)} = 1,47$$

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7.17 **d** lösning Stegsvarsanalys

$$G(s) = \frac{K \cdot e^{-2s}}{(1+Ts)(1+aTs)} = \frac{6 \cdot e^{-2s}}{(1+1{,}47s)(1+0{,}4 \cdot 1{,}47s)}$$

$$G(s) = \frac{6 \cdot e^{-2s}}{(1+1{,}47s)(1+0{,}59s)}$$

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