

Reglerteknik 4

Kapitel 8



Köp bok och övningshäfte på kårbokhandeln

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Lektion 4 kap 8

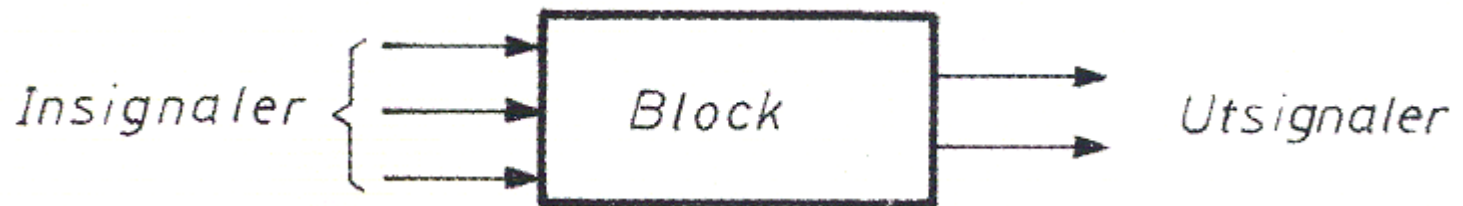
- Blockschemareduktion

- Förenkla komplicerade blockschemor – jämför med ellärans **ersättningsresistans**.
- Reducera blockschemat till enklaste form – jämför med ellärans **tvåpolssats**.

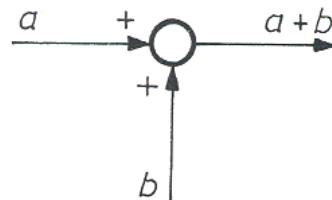
Blockschema

Blockschemat är ett schematiskt och tydligt sätt att beskriva ett regelsystems funktion. Man använder tre symboler:

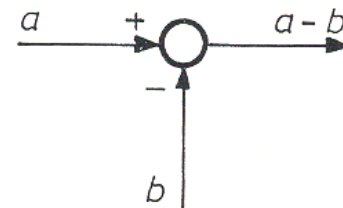
- Signaler
- Block
- Signaler



- Summerings/Differens punkter



Summeringspunkt



Differenspunkt

Överföringsfunktioner ”baklänges”

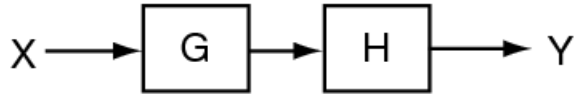
Ofta är man också intresserad av överföringsfunktioner mellan andra punkter i blockschemat än den egentliga instorheten och utstorheten.

Det kan gälla hur en **störning** fortplantar sig till utgången, eller hur stor en **inre styrsignal** till ett ställdon blir.

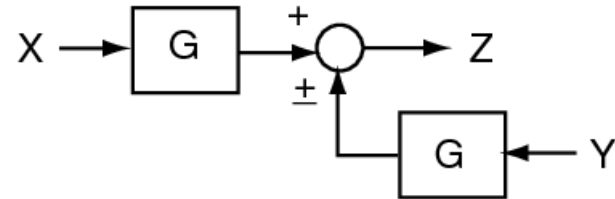
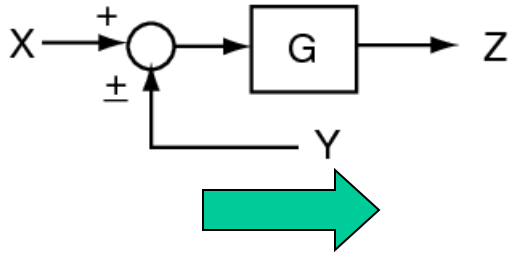
Man ritar då om blockschemat så att man får nya blockschemor för dessa andra överföringsfunktioner.

Block schemareduktionsregler

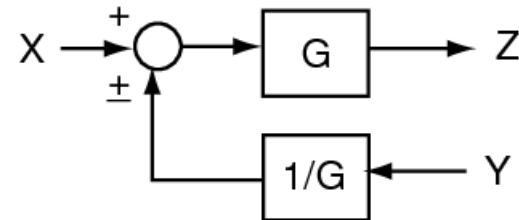
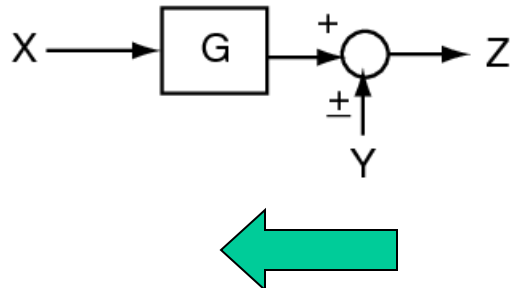
Cascaded blocks



Moving a summer behind a block

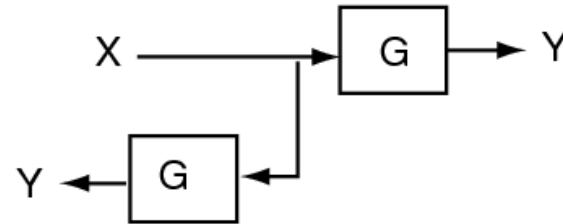
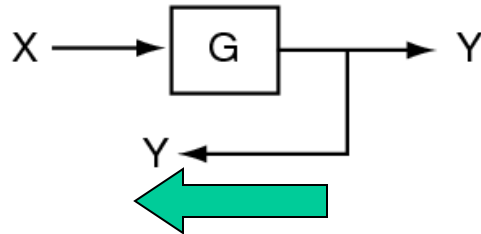


Moving a summer ahead of a block

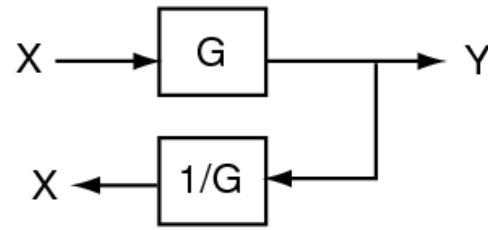
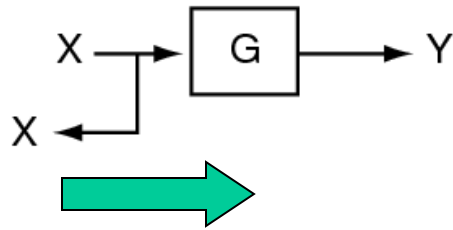


Blockschemareduktionsregler

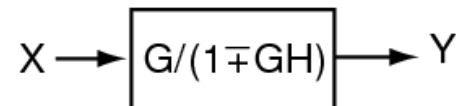
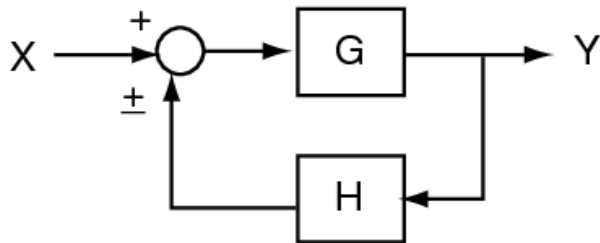
Moving a pickoff ahead of a block



Moving a pickoff behind a block

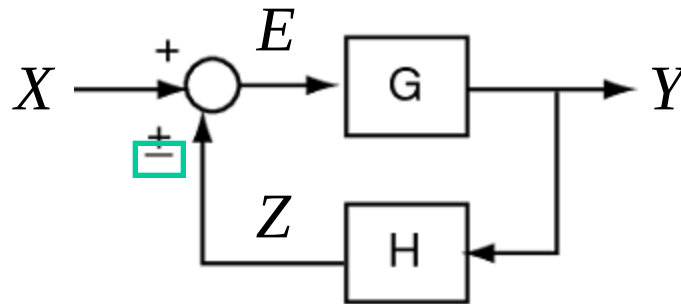


Eliminating a feedback loop



Blockschemareduktionsregler - feedback

Feedback
- negative
+ positive

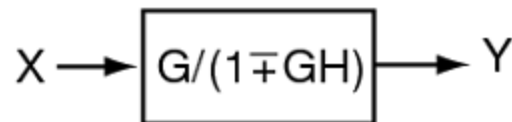


$$Y = E \cdot G \quad E = X \pm Z \quad Z = Y \cdot H$$

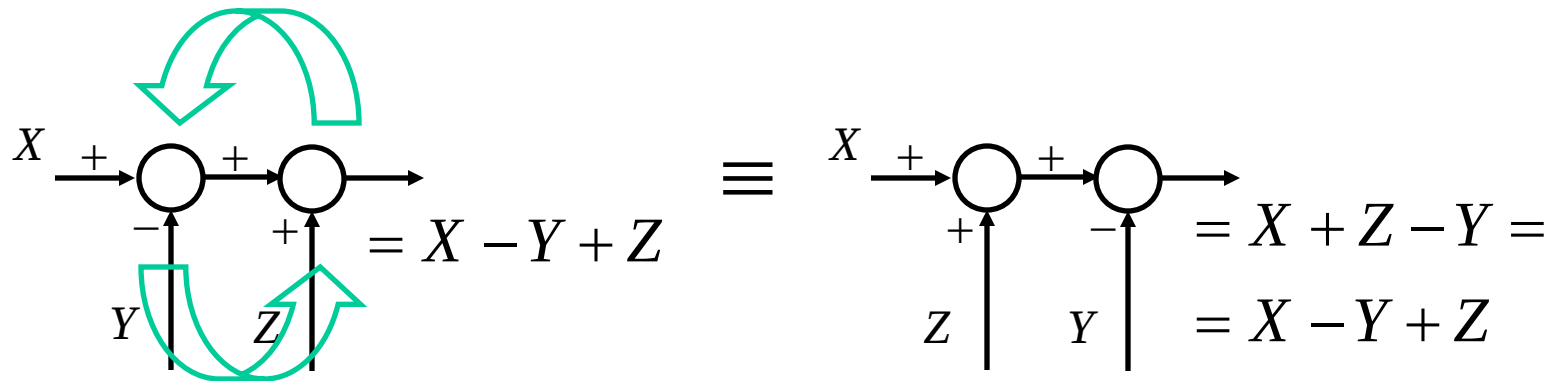
$$Y = EG = (X \pm Z) \cdot G = (X \pm Y \cdot H) \cdot G$$

$$Y = X \cdot G \pm Y \cdot H \cdot G \Leftrightarrow Y \mp Y \cdot H \cdot G = X \cdot G$$

$$G_{closedloop} = \frac{Y}{X} = \frac{G}{1 \mp GH}$$

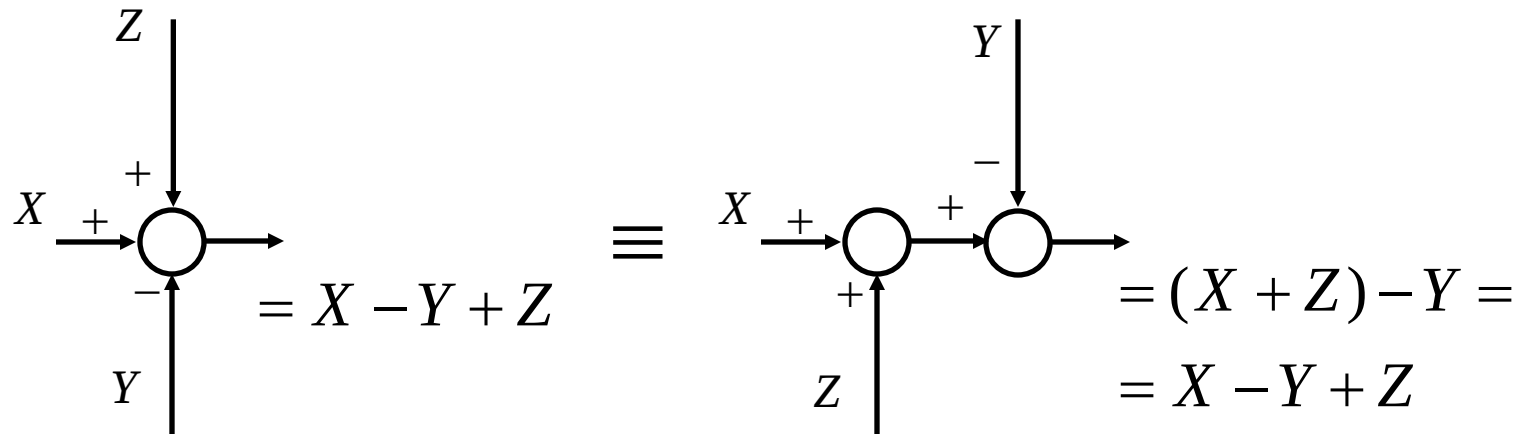


$$X - y + z = X + z - y$$



- Man får flytta summations och differenspunkter längs signalvägen.

$$x - y + z = (x + z) - y$$

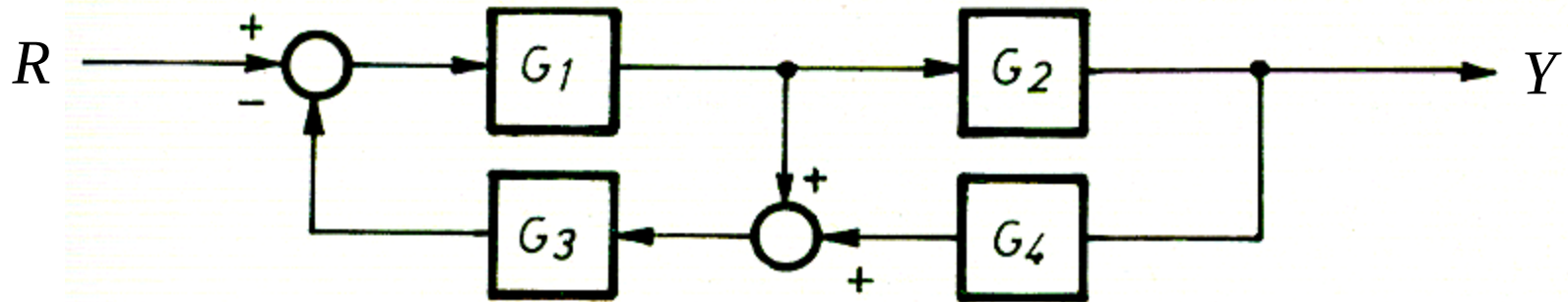


- Man kan dela upp, eller slå ihop, summationspunkter.

Ekvationsmetoden

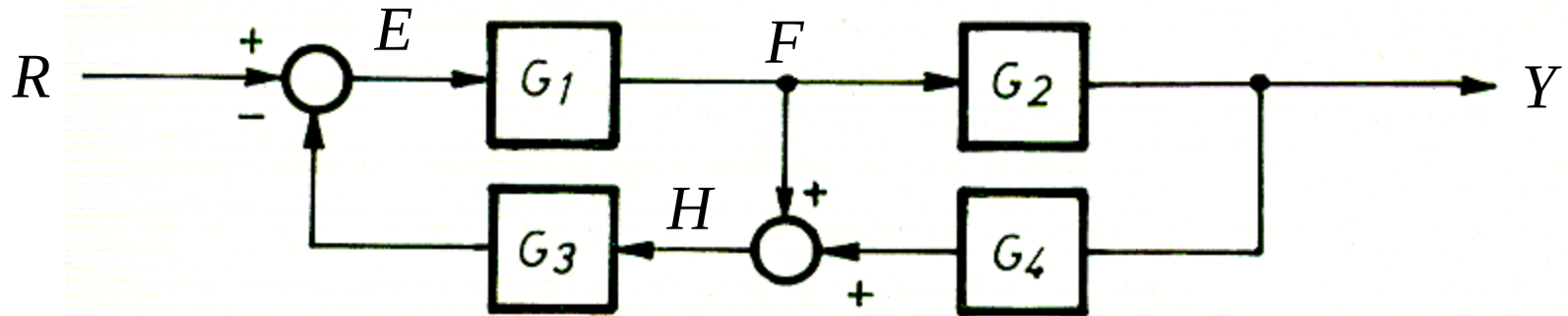
Det matematiska alternativet för blockschemareduktion är att införa mellanvariabler, och ställa upp och lösa ett ekvationssystem för blockschemat.

Ex. Ekvationsmetoden



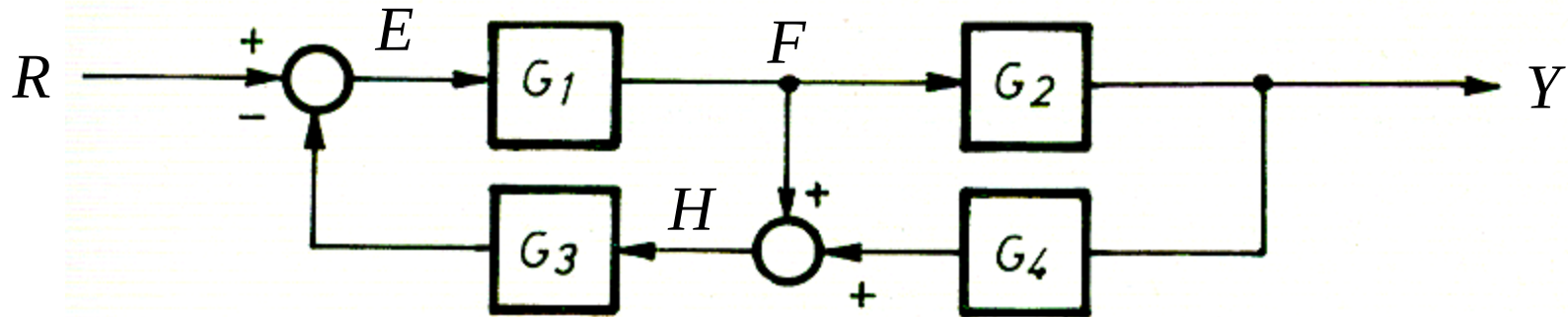
- Inför alla mellanvariabler, $E F H$.

Ex. Ekvationsmetoden



- Ställ upp ekvationssystemet.

Ex. Ekvationsmetoden



$$E = R - G_3 H \quad (1)$$

$$F = E G_1 \quad (2) \quad (2) \rightarrow (3)$$

$$Y = F G_2 \quad (3) \quad (2) \rightarrow (4)$$

$$H = F + G_4 Y \quad (4)$$

$$E = R - G_3 H \quad (1)$$

$$Y = E G_1 G_2 \quad (3')$$

$$H = E G_1 + G_4 Y \quad (4')$$

Ex. Ekvationsmetoden

$$E = R - G_3 H \quad (1)$$

$$Y = EG_1 G_2 \quad (3') \quad (4') \rightarrow (1)$$

$$H = EG_1 + G_4 Y \quad (4')$$

$$E = R - G_3 (EG_1 + G_4 Y_3) \quad (1')$$

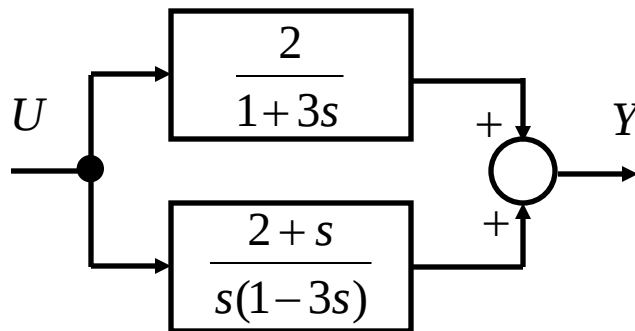
$$Y = EG_1 G_2 \quad (3')$$

$$Y = EG_1 G_2 \Rightarrow \frac{Y}{G_1 G_2} = E \quad (3') \rightarrow (1')$$

$$\frac{Y}{G_1 G_2} = R - G_3 \left(\frac{Y}{G_2} + G_4 Y \right) \Rightarrow Y = RG_1 G_2 - Y(G_1 G_3 + G_1 G_2 G_3 G_4)$$

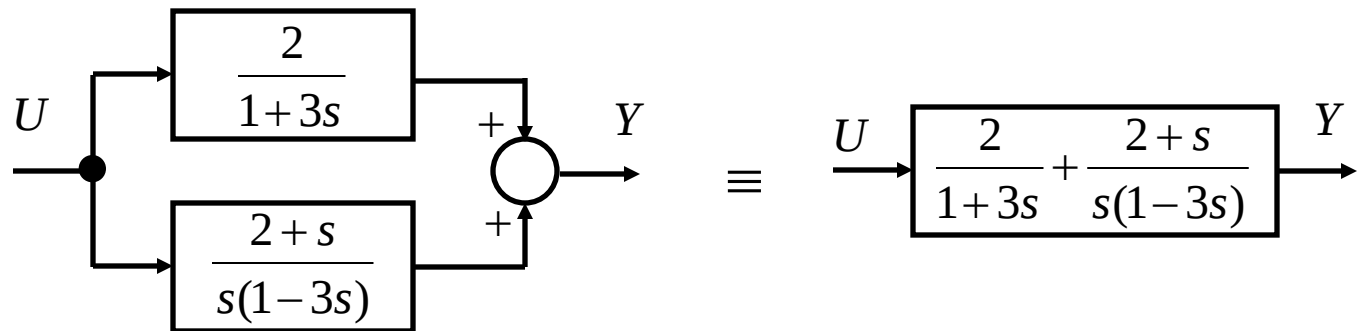
$$\Rightarrow G\left(\frac{Y}{R}\right) = \frac{G_1 G_2}{1 + G_1 G_3 + G_1 G_2 G_3 G_4}$$

8.1 Blockschemareduktion



$$G = \frac{Y}{U} = ?$$

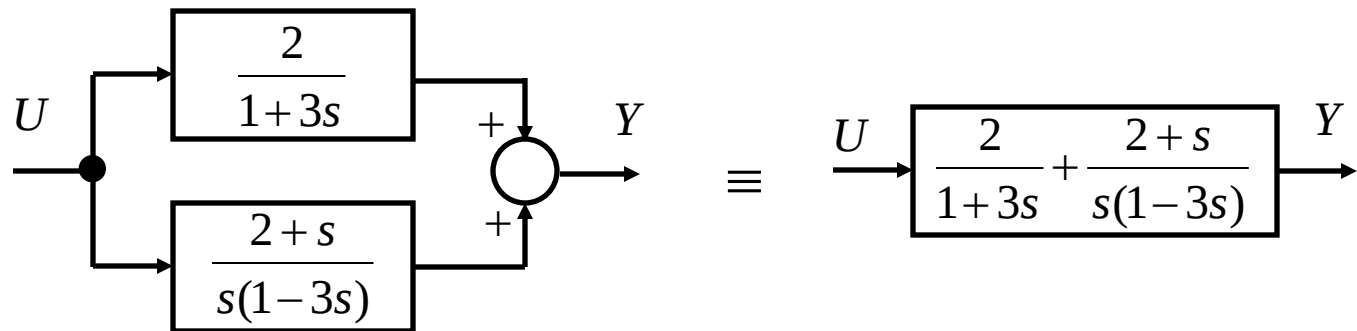
8.1 lösn. Blockschemareduktion



$$\frac{2}{1+3s} + \frac{2+s}{s(1-3s)} \Rightarrow \frac{2s(1-3s) + (2+s)(1+3s)}{s(1+3s)(1-3s)}$$

$$\Rightarrow G = \frac{Y}{U} = \frac{9s - 3s^2 + 2}{s(1-9s^2)}$$

8.1 med Matlab



`G1=tf([2],[3,1])`

Transfer function:

2

3 s + 1

`G2=tf([1,2],[-3,1,0])`

Transfer function:

-s - 2

3 s^2 - s

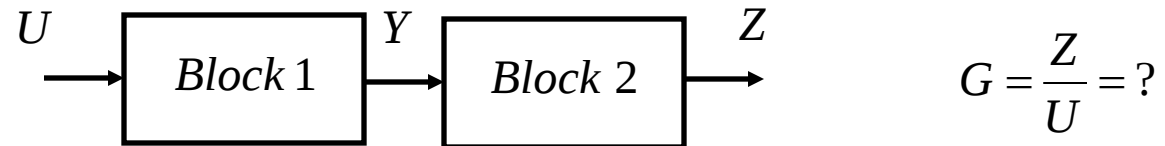
`G=G1+G2`

Transfer function:

3 s^2 - 9 s - 2

9 s^3 - s

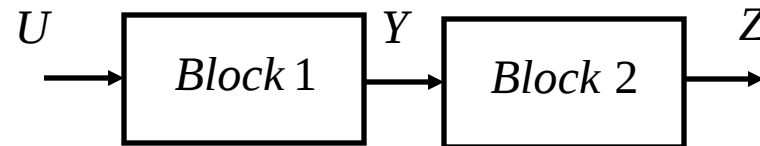
8.2 Blockschemareduktion



$$\text{Block 1: } \ddot{y} + 2\dot{y} + 3y = 5\ddot{u} + u$$

$$\text{Block 2: } 3\dot{z} + 2z = 6y$$

8.2 lösn. Blockschemareduktion



$$G = \frac{Z}{U} = ?$$

$$\ddot{y} + 2\dot{y} + 3y = 5\ddot{u} + u$$

$$\{L:\} \quad Y(s^2 + 2s + 3) = U(5s^2 + 1) \quad \Rightarrow \quad G_1\left(\frac{Y}{U}\right) = \frac{5s^2 + 1}{s^2 + 2s + 3}$$

$$3\dot{z} + 2z = 6y$$

$$\{L:\} \quad Z(3s + 2) = Y(6) \quad \Rightarrow \quad G_2\left(\frac{Z}{Y}\right) = \frac{6}{3s + 2}$$

8.2 lösn. Blockschemareduktion

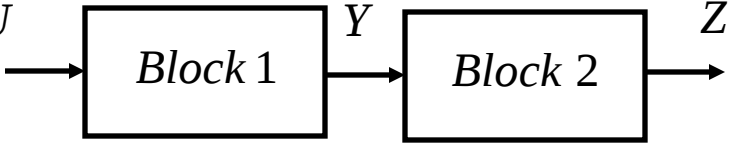


$$G_1\left(\frac{Y}{U}\right) = \frac{5s^2 + 1}{s^2 + 2s + 3} \quad G_2\left(\frac{Z}{Y}\right) = \frac{6}{3s + 2}$$

$$G\left(\frac{Z}{U} = \frac{Y}{U} \cdot \frac{Z}{Y}\right) = G_1 \cdot G_2 = \frac{6(5s^2 + 1)}{(3s + 2)(s^2 + 2s + 3)} = \frac{30s + 6}{3s^3 + 8s^2 + 13s + 6}$$

8.2 med Matlab

$$G = \frac{Z}{U} = ?$$

$$G_1\left(\frac{Y}{U}\right) = \frac{5s^2 + 1}{s^2 + 2s + 3} \quad G_2\left(\frac{Z}{Y}\right) = \frac{6}{3s + 2}$$


```

graph LR
    U --> B1[Block 1]
    B1 -- Y --> B2[Block 2]
    B2 -- Z --> Z
  
```

```
G1=tf([5,0,1],[1,2,3])
```

Transfer function:

$$5 s^2 + 1$$

$$s^2 + 2 s + 3$$

```
G2=tf([6],[3,2])
```

Transfer function:

$$6$$

$$3 s + 2$$

$$G=G1*G2$$

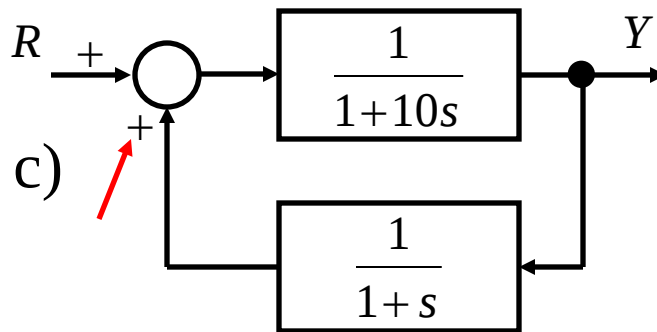
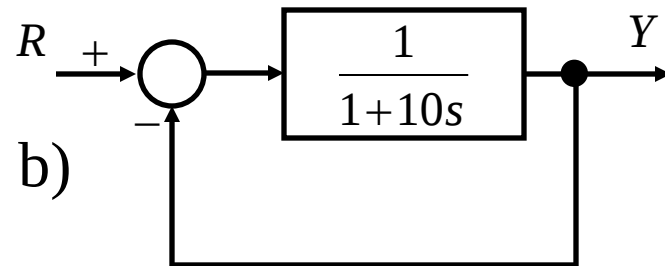
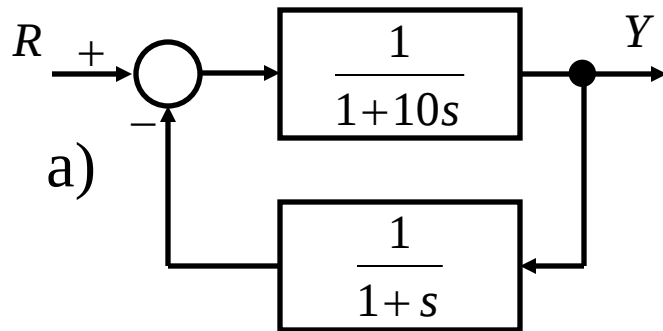
Transfer function:

$$30 s^2 + 6$$

$$3 s^3 + 8 s^2 + 13 s + 6$$

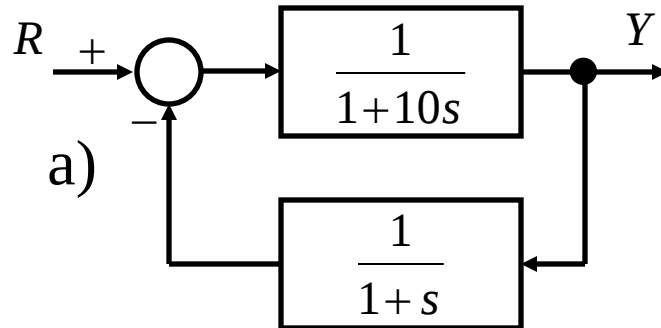
8.3 Feedback

$$X \longrightarrow \boxed{G/(1 \mp GH)} \longrightarrow Y$$



8.3 a lösn. Feedback

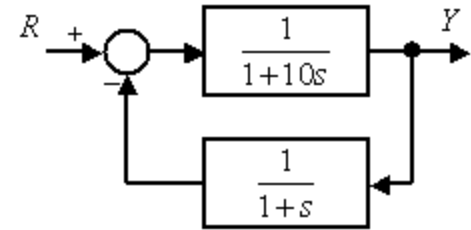
$$G_{closedloop}\left(\frac{Y}{X}\right) = \frac{G}{1 \mp GH}$$



$$G_1 = \frac{1}{10s+1} \quad G_2 = \frac{1}{s+1}$$

$$G_{closedloop}\left(\frac{Y}{R}\right) = \frac{G_1}{1 + G_1 \cdot G_2} = \frac{\frac{1}{10s+1}}{1 + \frac{1}{10s+1} \cdot \frac{1}{s+1}}$$

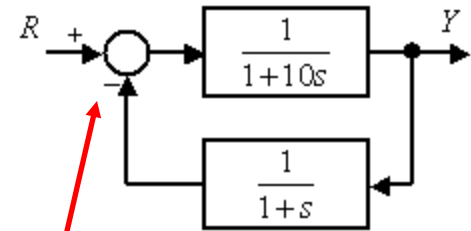
8.3 a lösn. Feedback



$$\begin{aligned}\frac{G_1}{1+G_1 \cdot G_2} &= \frac{\frac{1}{10s+1}}{1+\frac{1}{10s+1} \cdot \frac{1}{s+1}} = \frac{s+1}{(10s+1)(s+1)+1} = \\ &= \frac{s+1}{10s^2+11s+2}\end{aligned}$$

8.3 a med Matlab

$$G_1 = \frac{1}{10s+1} \quad G_2 = \frac{1}{s+1}$$



```
G1=tf([1],[10,1])
Transfer function:
```

```
      1
-----
10 s + 1
```

```
G2=tf([1],[1,1])
Transfer function:
```

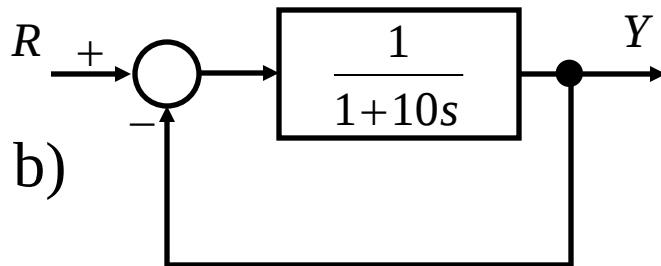
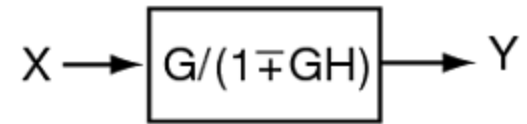
```
      1
-----
1 s + 1
```

```
G=feedback(G1,G2,-1)
```

```
Transfer function:
```

```
          s + 1
-----
10 s^2 + 11 s + 2
```


8.3 b lösn. Feedback

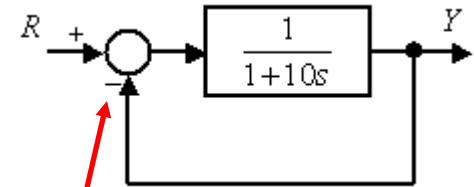


$$G_1 = \frac{1}{10s+1} \quad G_2 = 1$$

$$G_{closedloop} \left(\frac{Y}{R} \right) = \frac{G_1}{1 + G_1 \cdot G_2} = \frac{\frac{1}{10s+1}}{1 + \frac{1}{10s+1} \cdot 1} = \frac{1}{10s+2}$$

8.3 b med Matlab

$$G_1 = \frac{1}{10s+1} \quad G_2 = 1$$



```
G1=tf([1],[10,1])  
Transfer function:
```

```
      1  
-----  
10 s + 1
```

```
G2=tf(1)
```

```
Transfer function:
```

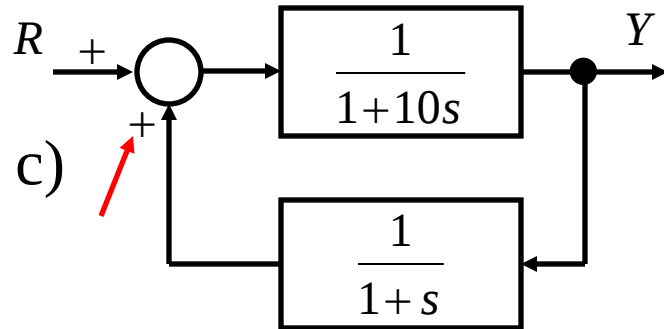
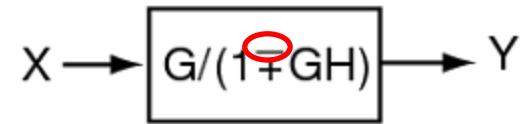
```
1  
Static gain.
```

```
G=feedback(G1,G2,-1)
```

```
Transfer function:
```

```
      1  
-----  
10 s + 2
```


8.3 c lös. Feedback

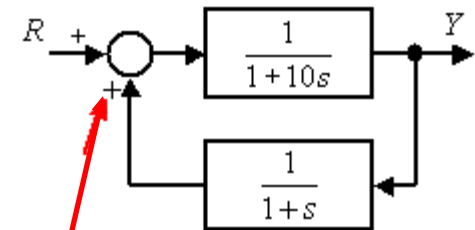


$$G_1 = \frac{1}{10s+1} \quad G_2 = \frac{1}{s+1}$$

$$\begin{aligned} \frac{G_1}{1 - G_1 \cdot G_2} &= \frac{\frac{1}{10s+1}}{1 - \frac{1}{10s+1} \cdot \frac{1}{s+1}} = \frac{s+1}{(10s+1)(s+1) - 1} = \\ &= \frac{s+1}{10s^2 + 11s} \end{aligned}$$

8.3 c med Matlab

$$G_1 = \frac{1}{10s+1} \quad G_2 = \frac{1}{s+1}$$



```
G1=tf([1],[10,1])
Transfer function:
```

```
      1
-----
10 s + 1
```

```
G2=tf([1],[1,1])
Transfer function:
```

```
      1
-----
1 s + 1
```

```
G=feedback(G1,G2,+1)
```

```
Transfer function:
```

```
      s + 1
-----
10 s^2 + 11 s
```


(Begynnelse) och Slutvärde

Kommer Du ihåg? $L : _ \Rightarrow \boxed{\frac{1}{s}}$

$$\lim_{t \rightarrow 0+} f(t) = \lim_{s \rightarrow \infty} s \cdot F(s)$$

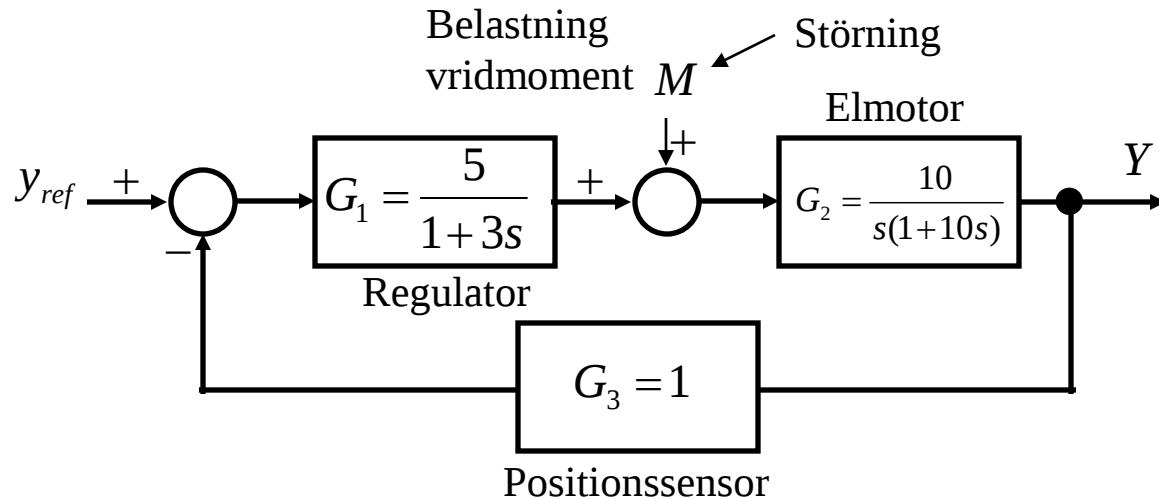
$$\lim_{t \rightarrow \infty} f(t) = \lim_{s \rightarrow 0} \boxed{s} \cdot F(s) \quad \bullet \text{ slutvärde}$$

*Vad som händer efter lång tid avgörs av
laplacetransformens lågfrekvenssegenskaper.
Slutvärdet efter ett steg om man låter $s \rightarrow 0$.*

$$\boxed{\frac{1}{s} \times s = 1}$$

8.4 Positionsreglering

$$X \rightarrow \boxed{G/(1 \mp GH)} \rightarrow Y$$



a) $G_{y_{ref} \rightarrow Y} = ?$

För reglerstorheten.

b) $G_{M \rightarrow Y} = ?$

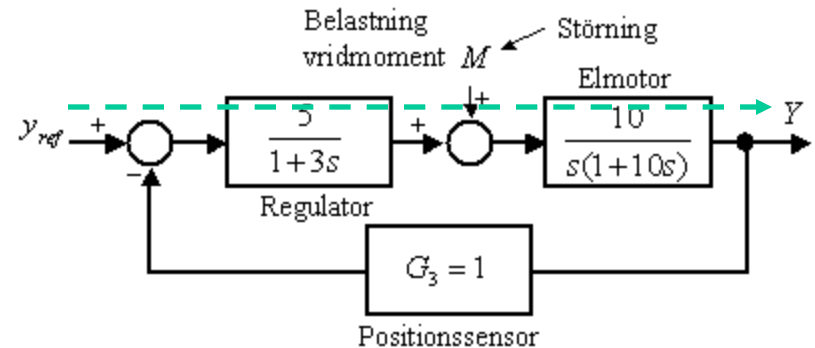
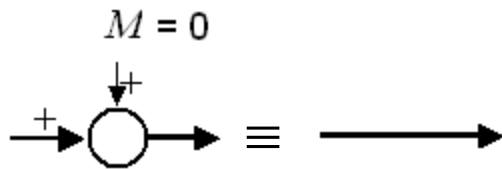
För störstorheten.

Den storhet som *inte* ingår i överföringsfunktionen kan sättas = 0. För linjära system gäller ju superposition.

8.4 a lösn. Positionsreglering

$$G_1 = \frac{5}{3s+1} \quad G_2 = \frac{10}{s(s+1)} \quad G_3 = 1$$

Antag $M = 0$



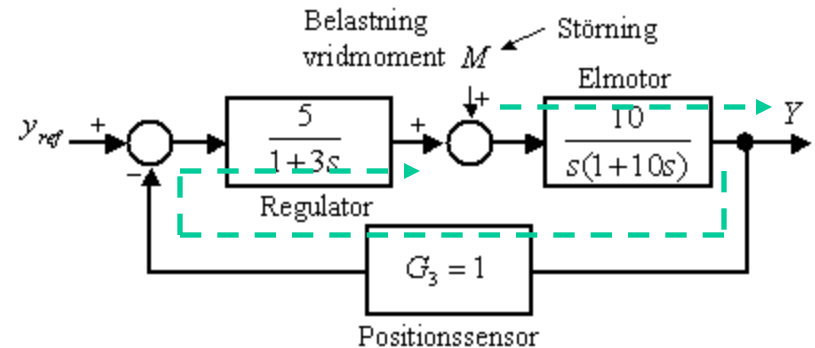
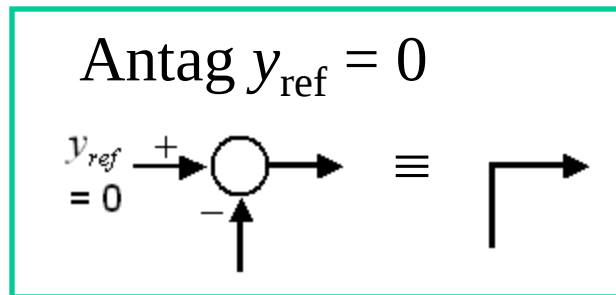
$$G\left(\frac{Y}{Y_{REF}}\right) = \frac{G_1 \cdot G_2}{1 + G_1 \cdot G_2 \cdot G_3} = \frac{\frac{5}{3s+1} \cdot \frac{10}{s(s+1)}}{1 + \frac{5}{3s+1} \cdot \frac{10}{s(s+1)} \cdot 1} = \frac{50}{s(3s+1)(s+1) + 50} =$$

$$= \frac{50}{30s^3 + 13s^2 + s + 50}$$

($s = 0 \rightarrow G = 1$, en *stegändring* hos styrstorheten slår igenom med 100% på utstorhetens *slutvärde*. $s \cdot 1/s = 1$)

8.4 b lösn. Positionsreglering

$$G_1 = \frac{5}{3s+1} \quad G_2 = \frac{10}{s(s+1)} \quad G_3 = 1$$

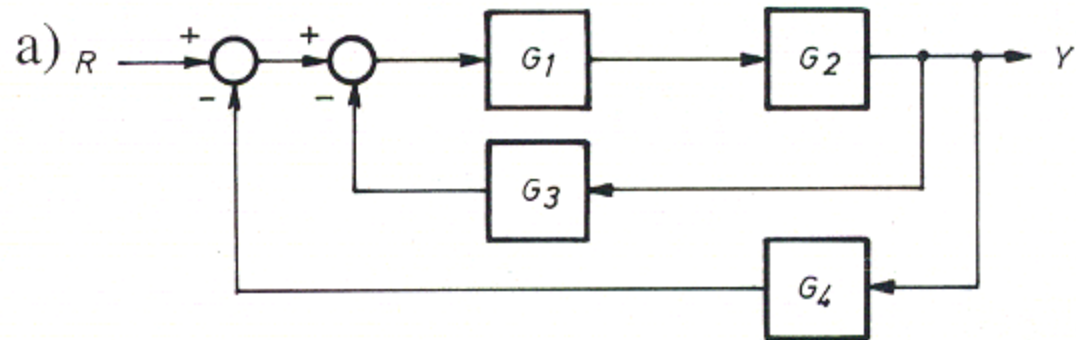


$$G\left(\frac{Y}{M}\right) = \frac{G_2}{1 + G_1 \cdot G_3} = \frac{\frac{10}{s(s+1)}}{1 + \frac{5}{3s+1} \cdot 1} = \frac{10(3s+1)}{s(s+1)(3s+1) + 5s(s+1)} =$$

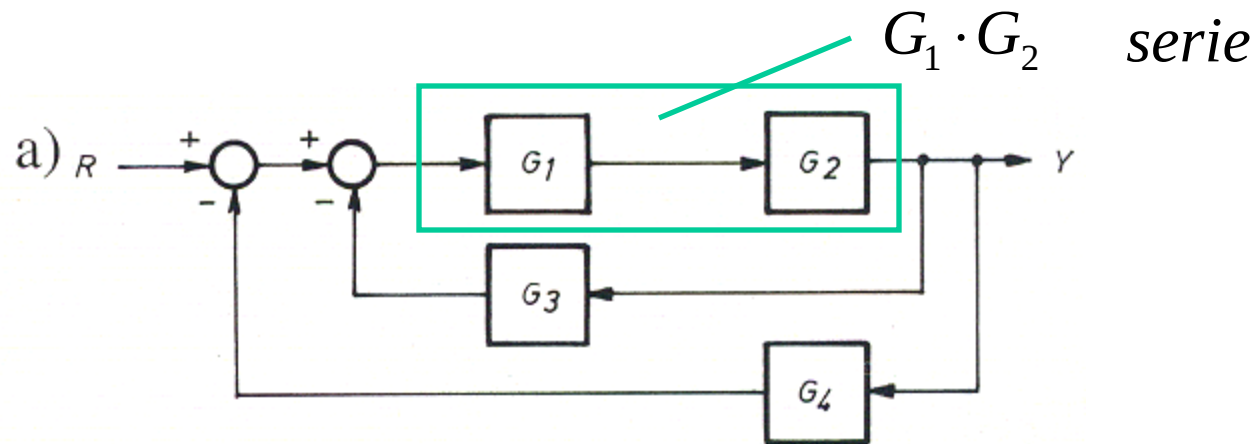
$$= \frac{30s+10}{30s^3 + 13s^2 + s + 50}$$

($s = 0 \rightarrow G = 1/5$, en *stegändring* hos störningen påverkar *slutvärdet* med 20%. $s \cdot 1/s = 1$)

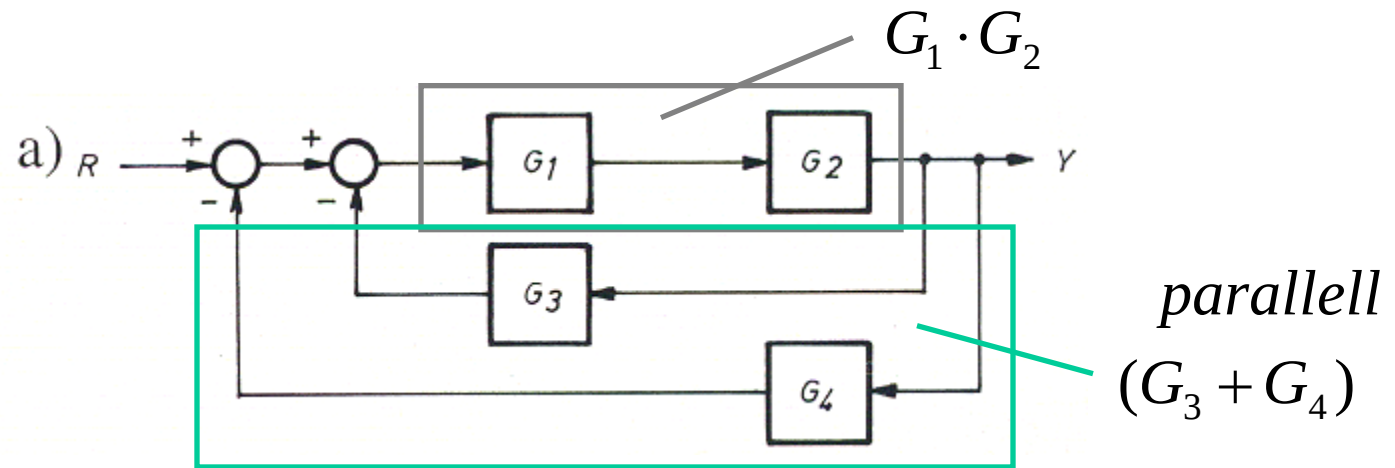
8.5 a Blockschemareduktion



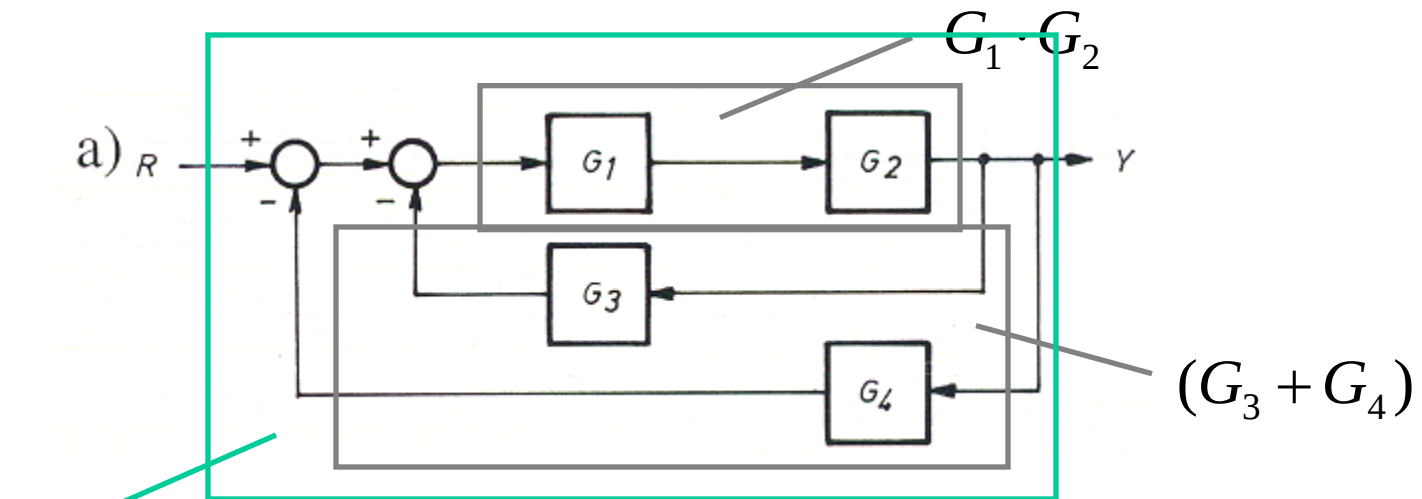
8.5 a lösn. Blockschemareduktion



8.5 a lösn. Blockschemareduktion



8.5 a lösn. Blockschemareduktion

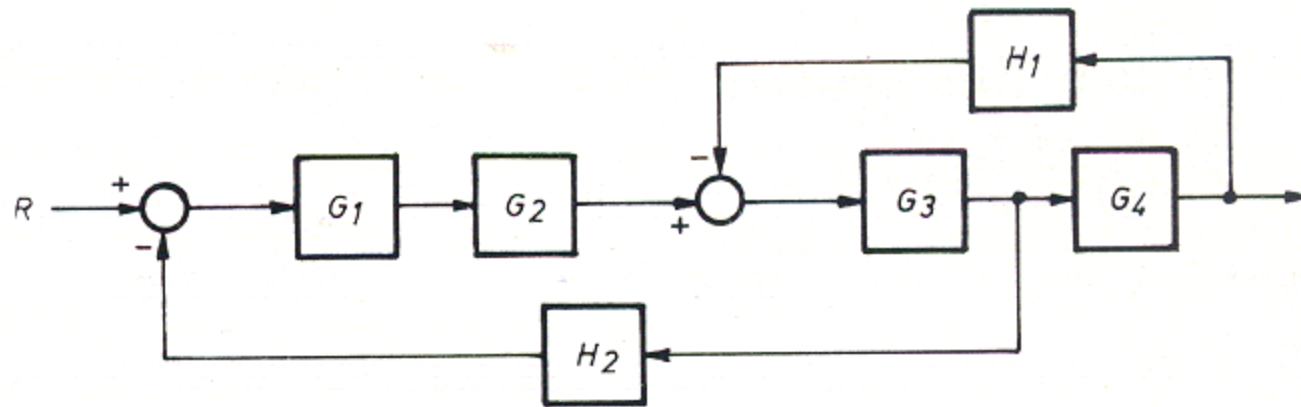


$$\frac{G_1 G_2}{1 + G_1 G_2 (G_3 + G_4)} \quad \text{negative feedback}$$

$$G\left(\frac{Y}{R}\right) = \frac{G_1 G_2}{1 + G_1 G_2 G_3 + G_1 G_2 G_4}$$

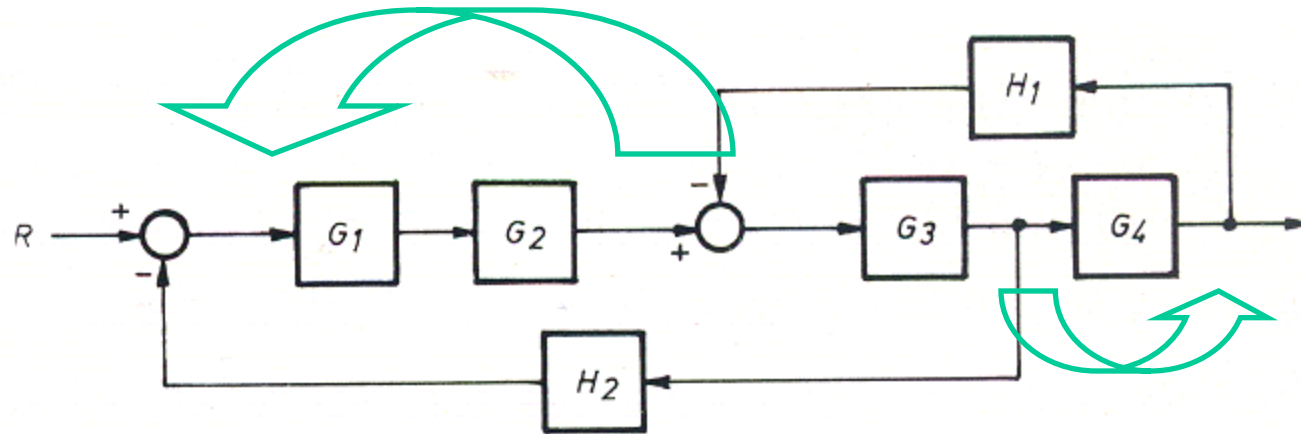
8.5 c Blockschemareduktion

c)

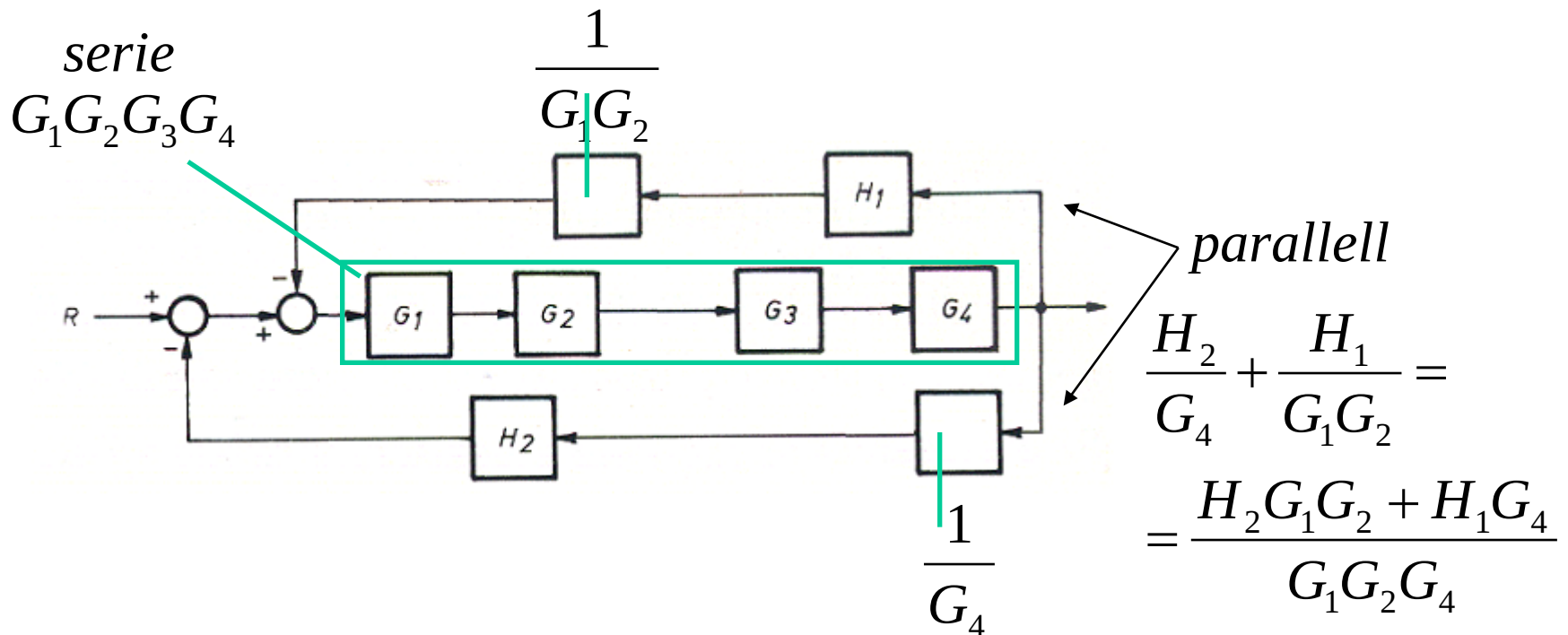


8.5 c Blockschemareduktion

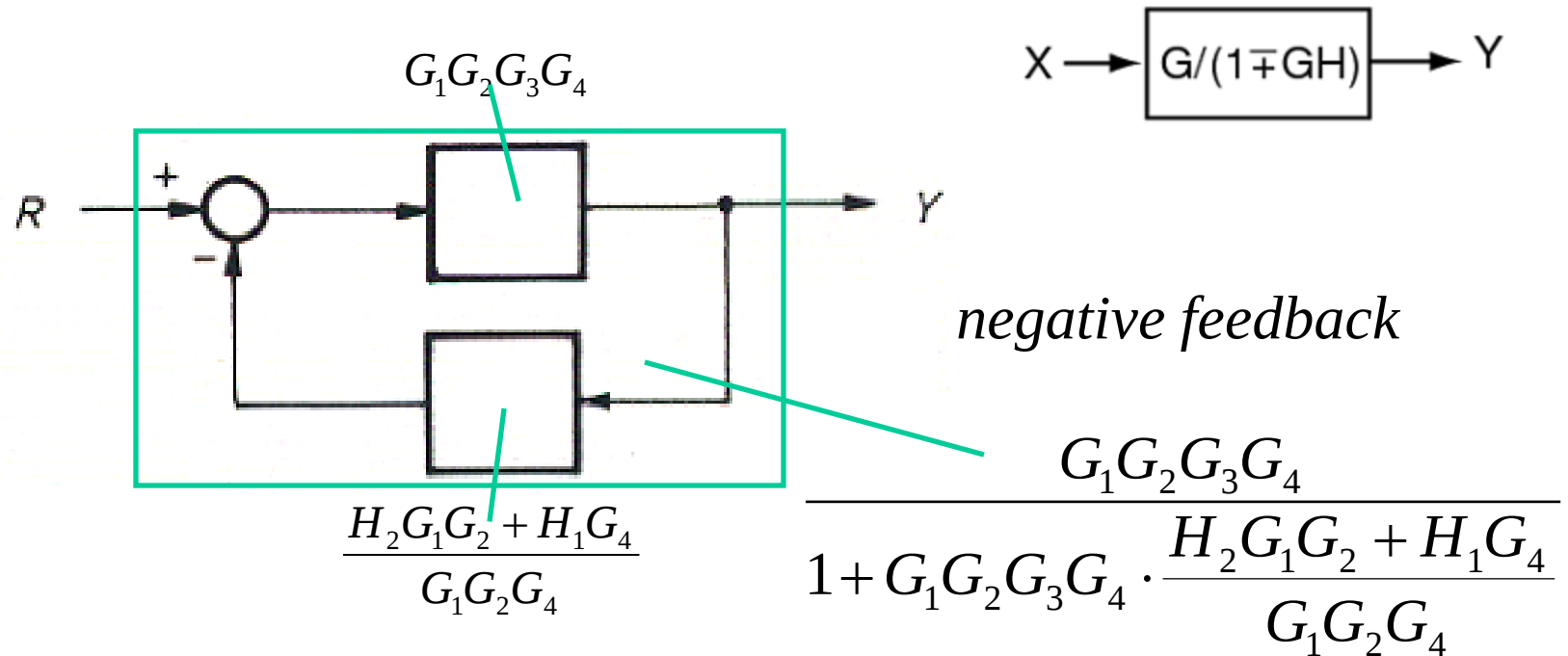
c)



8.5 c Blockschemareduktion



8.5 c Blockschemareduktion

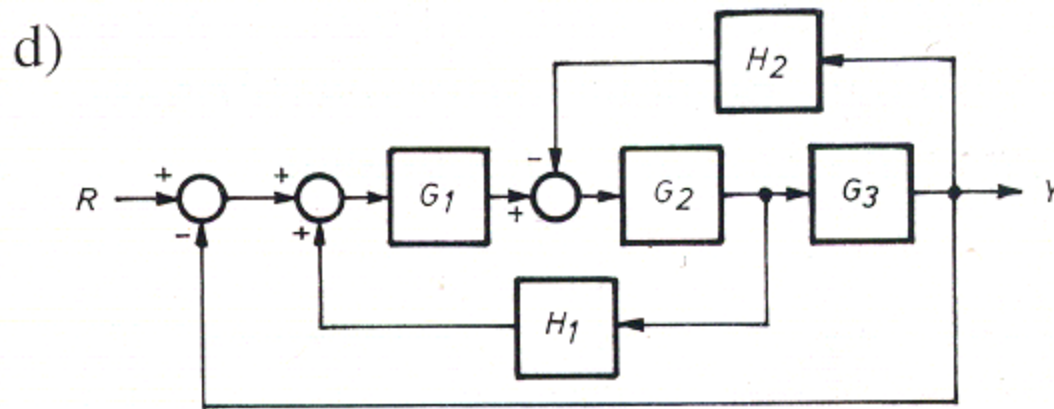


8.5 c Blockschemareduktion

$$\frac{G_1 G_2 G_3 G_4}{1 + \cancel{G_1} \cancel{G_2} G_3 \cancel{G_4} \cdot \frac{H_2 G_1 G_2 + H_1 G_4}{\cancel{G_1} \cancel{G_2} \cancel{G_4}}} = \frac{G_1 G_2 G_3 G_4}{1 + G_3 (H_2 G_1 G_2 + H_1 G_4)} =$$

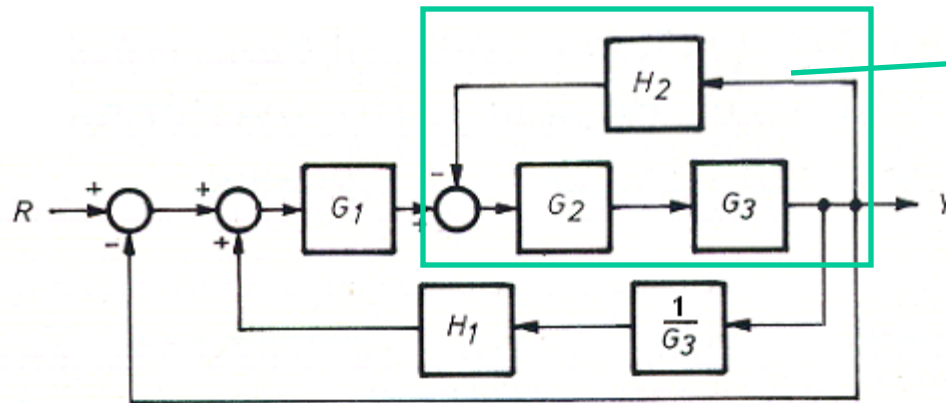
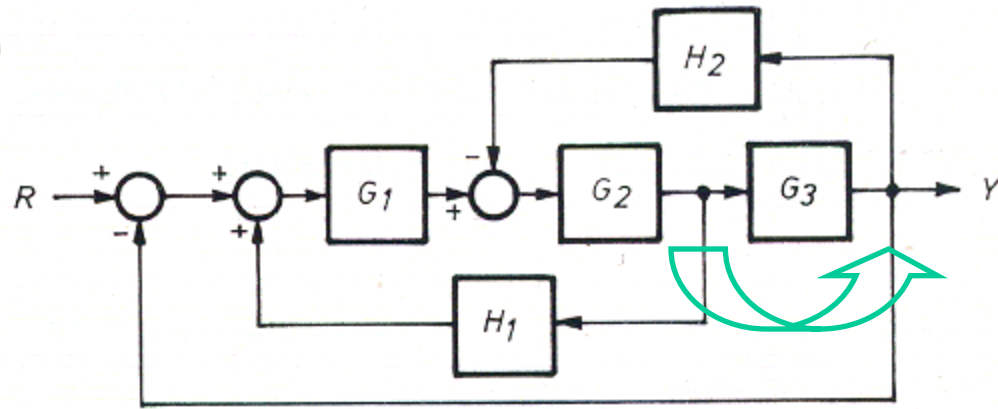
$$= \frac{G_1 G_2 G_3 G_4}{1 + H_2 G_1 G_2 G_3 + H_1 G_3 G_4}$$

8.5 d Blockschemareduktion



8.5 d Blockschemareduktion

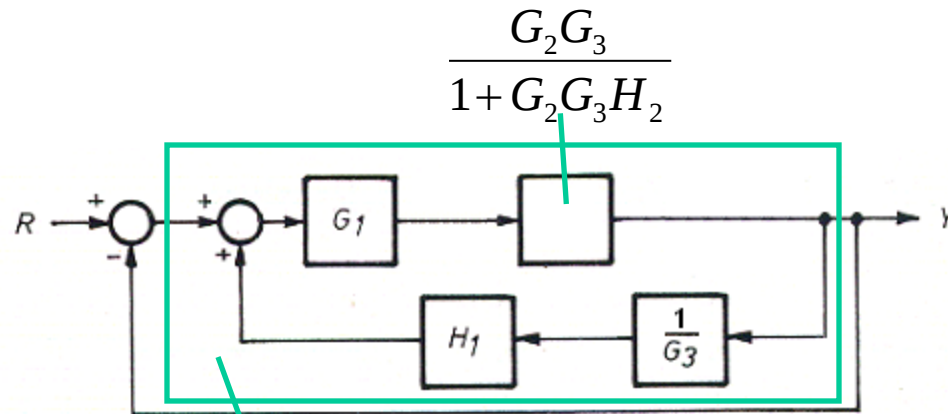
d)



negative feedback

$$\frac{G_2 G_3}{1 + G_2 G_3 H_2}$$

8.5 d Blockschemareduktion



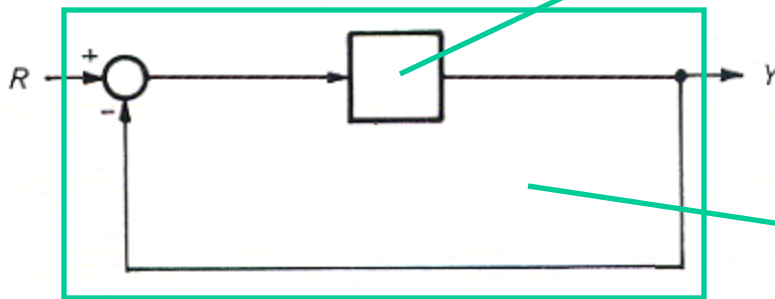
*positive
feedback*

$$\frac{G_1 G_2 G_3}{1 + G_2 G_3 H_2}$$

$$1 - \frac{H_1}{G_3} \cdot \frac{G_1 G_2 G_3}{1 + G_2 G_3 H_2}$$

8.5 d Blockschemareduktion

$$\frac{\frac{G_1 G_2 G_3}{1 + G_2 G_3 H_2}}{1 - \frac{H_1}{G_3} \cdot \frac{G_1 G_2 G_3}{1 + G_2 G_3 H_2}} \cdot \frac{1 + G_2 G_3 H_2}{1 + G_2 G_3 H_2} = \frac{G_1 G_2 G_3}{1 + G_2 G_3 H_2 - H_1 G_1 G_2}$$



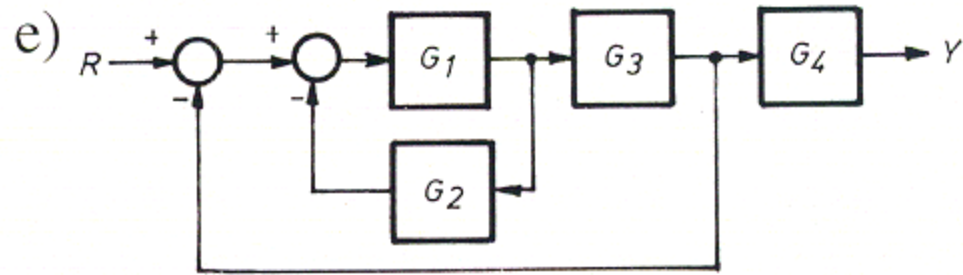
negative feedback

$$\frac{G_1 G_2 G_3}{1 + G_2 G_3 H_2 - H_1 G_1 G_2} \cdot \frac{1 + G_2 G_3 H_2 - H_1 G_1 G_2}{1 + G_2 G_3 H_2 - H_1 G_1 G_2} = \frac{G_1 G_2 G_3}{1 + G_2 G_3 H_2 - H_1 G_1 G_2}$$

8.5 d Blockschemareduktion

$$\frac{\frac{G_1 G_2 G_3}{1 + G_2 G_3 H_2 - H_1 G_1 G_2}}{1 + \frac{G_1 G_2 G_3}{1 + G_2 G_3 H_2 - H_1 G_1 G_2}} = \frac{G_1 G_2 G_3}{1 + G_2 G_3 H_2 - H_1 G_1 G_2 + G_1 G_2 G_3}$$

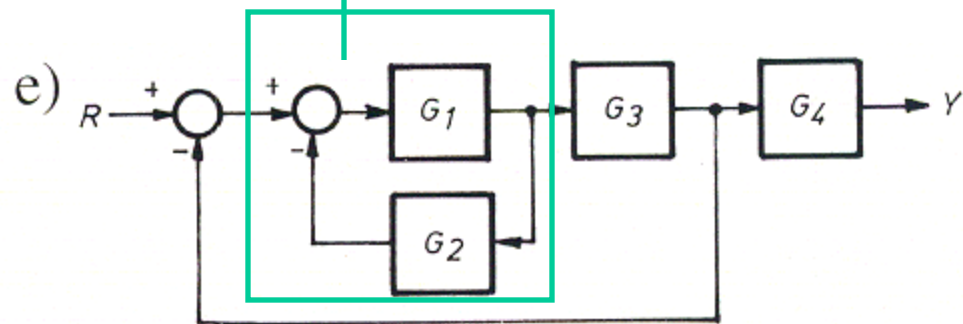
8.5 e Blockschemareduktion



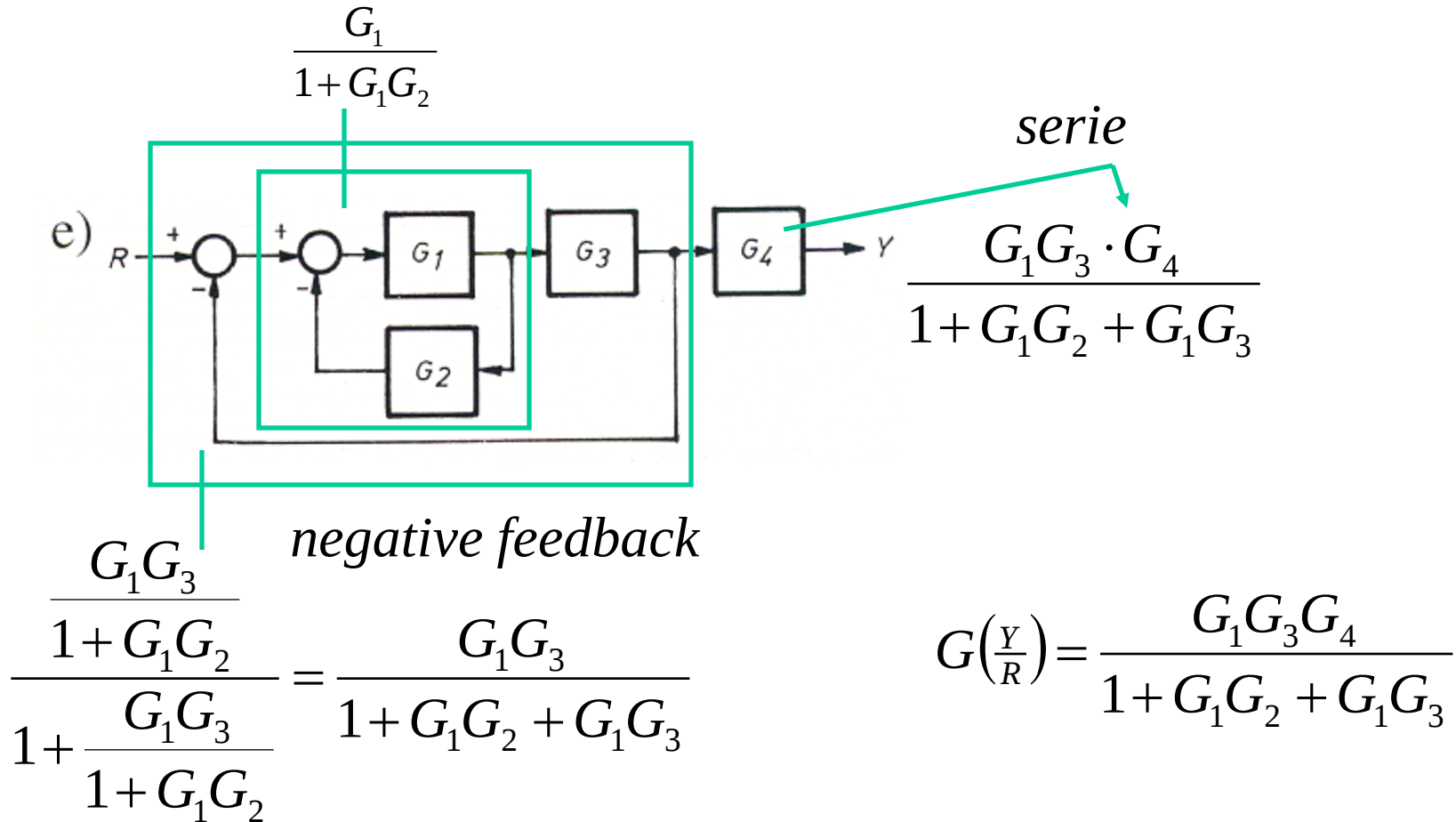
8.5 e Blockschemareduktion

negative feedback

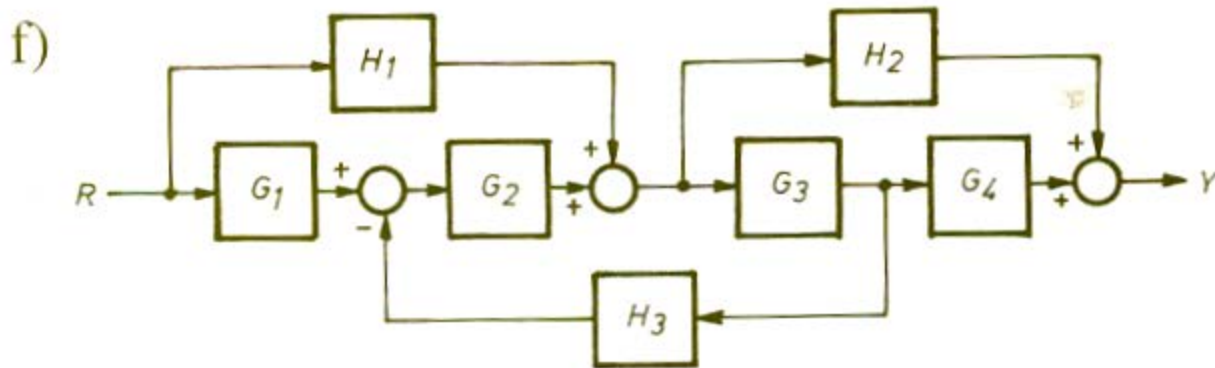
$$\frac{G_1}{1 + G_1 G_2}$$



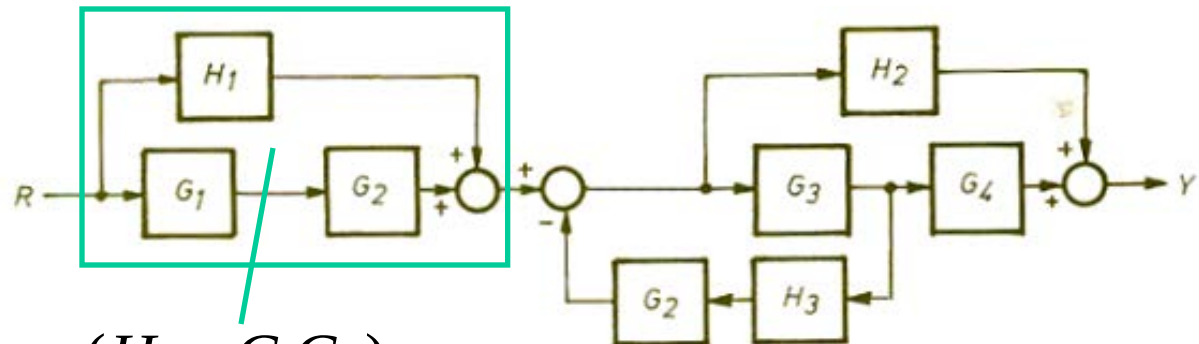
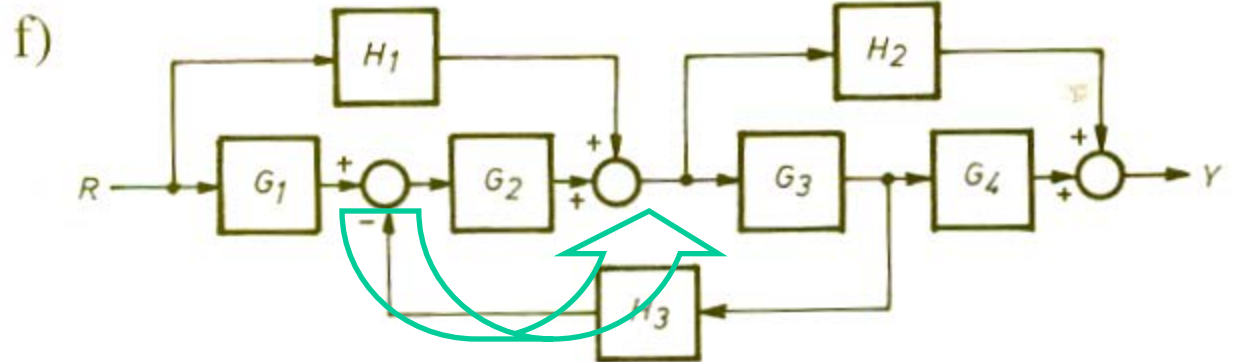
8.5 e Blockschemareduktion



8.5 f Blockschemareduktion



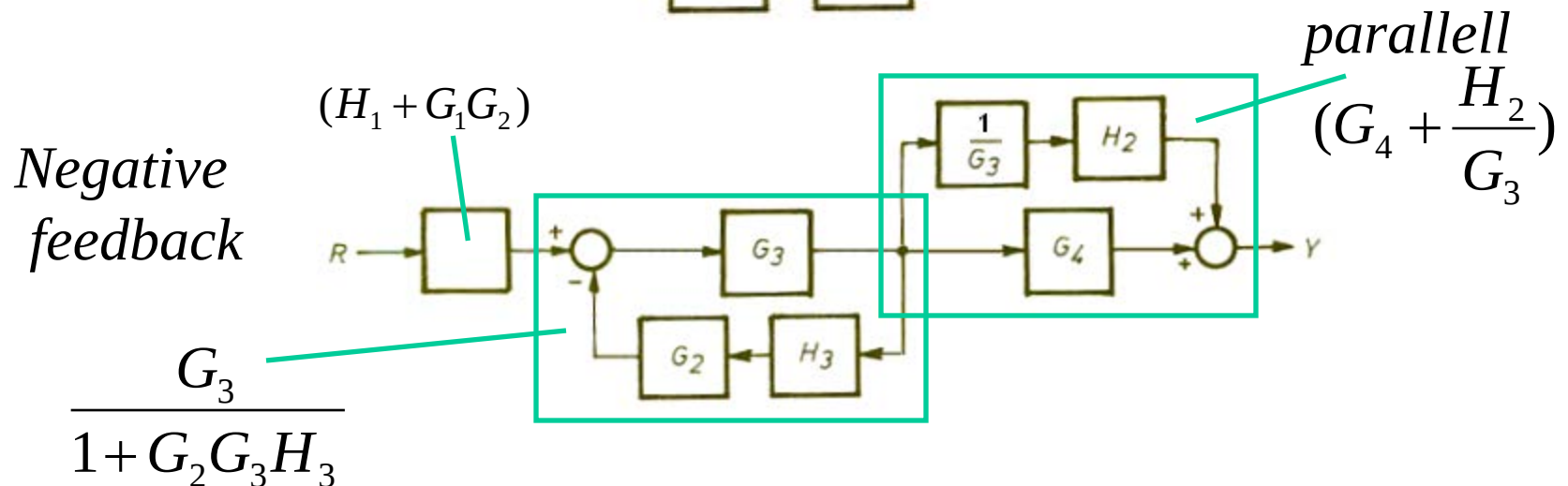
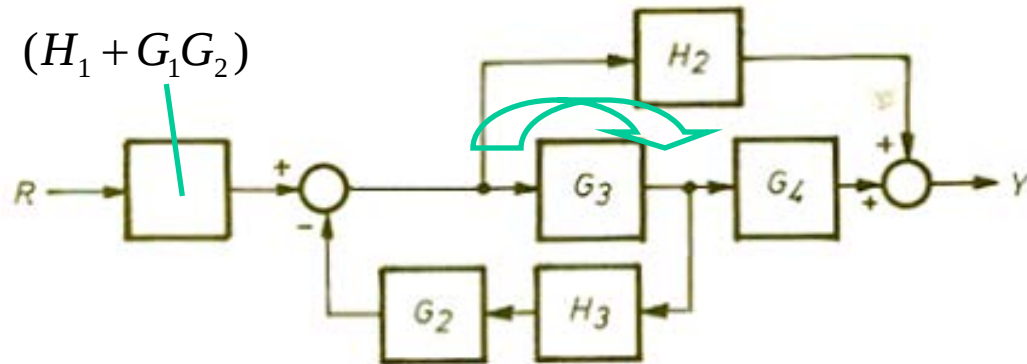
8.5 f Blockschemareduktion



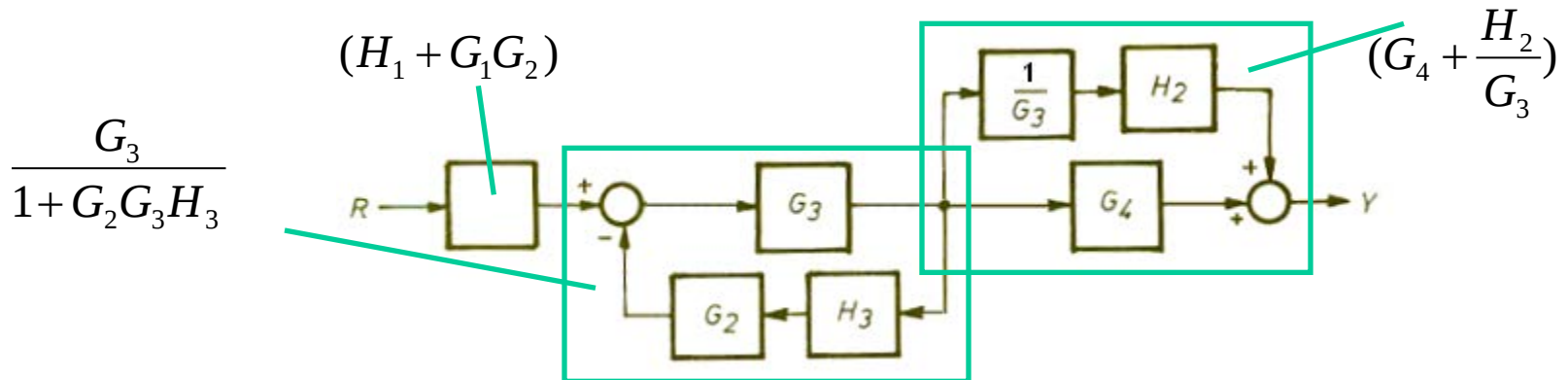
$$(H_1 + G_1 G_2)$$

parallell

8.5 f Blockschemareduktion



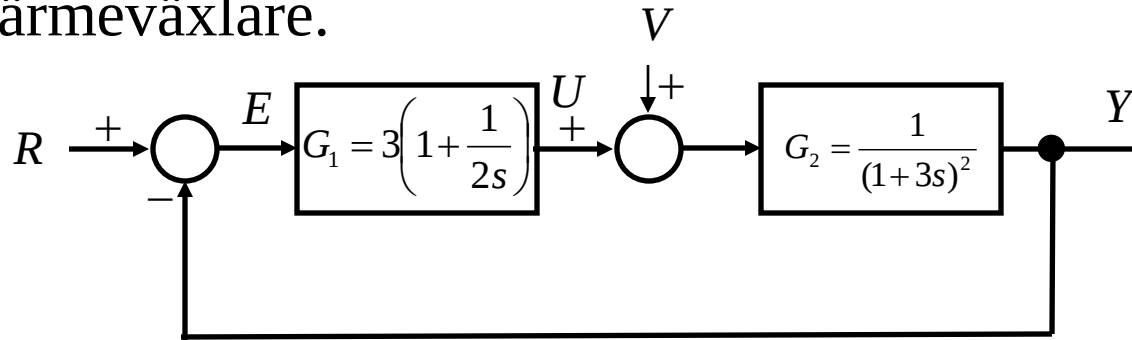
8.5 f Blockschemareduktion



$$\begin{aligned}
 & (H_1 + G_1G_2) \cdot \frac{G_3}{1 + G_2G_3H_3} \cdot \left(G_4 + \frac{H_2}{G_3}\right) = \\
 & = \frac{(H_1 + G_1G_2)(G_4G_3 + H_2)}{1 + G_2G_3H_3}
 \end{aligned}$$

8.7 Styrsignaler

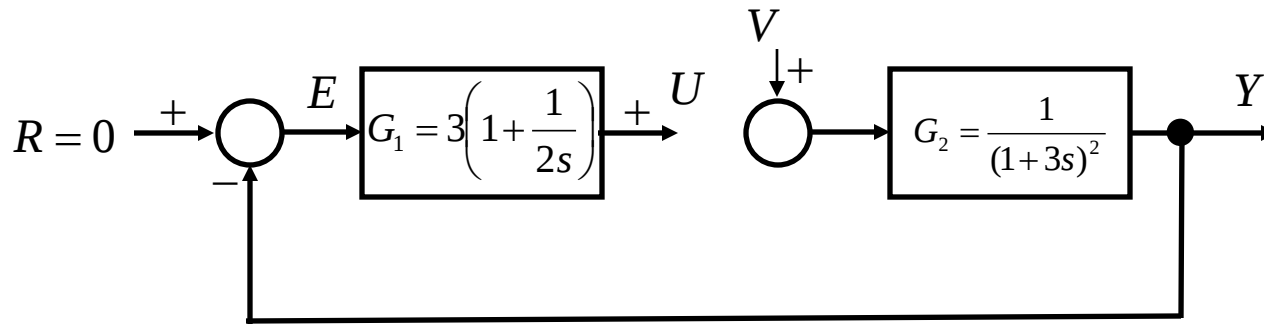
Värmeväxlare.



Hur stora styrsignaler U krävs vid olika störningar V ?
För att svara på det behöver man överföringsfunktionen från $V \rightarrow U$.

$$G\left(\frac{U}{V}\right) = ?$$

8.7 Styrsignaler



$$\begin{aligned} G\left(\frac{U}{V}\right) &= \frac{1}{(1+3s)^2} \cdot (-1) \cdot 3\left(1 + \frac{1}{2s}\right) = \\ &= \frac{-3(2s+1)}{(1+3s)^2 2s} = \frac{-(6s+3)}{18s^3 + 12s^2 + 8s + 3} \end{aligned}$$

8.7 Styrsignaler med Matlab

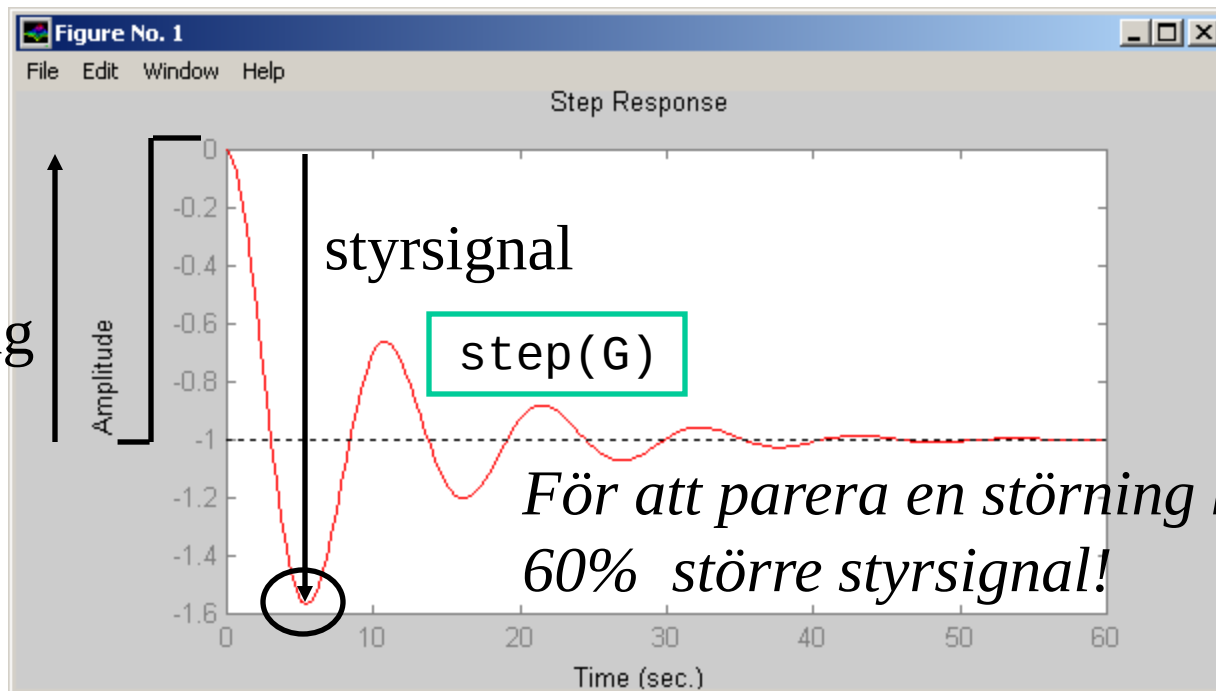
$$G\left(\frac{U}{V}\right) = \frac{-(6s + 3)}{18s^3 + 12s^2 + 8s + 3}$$

```
G=tf([-6, -3],[18,12,8,3])
```

Transfer function:

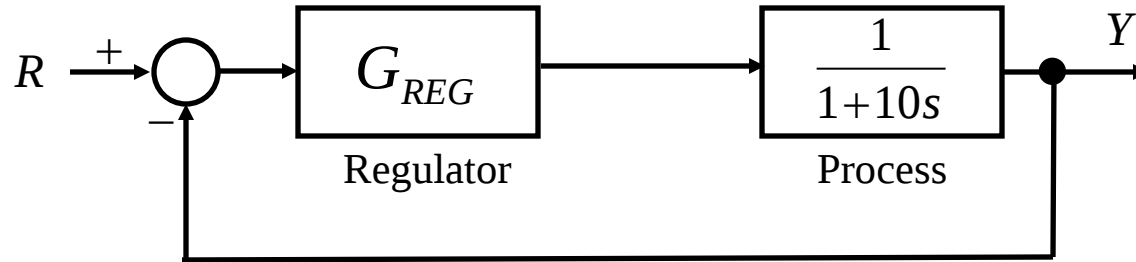
$$\frac{-6s - 3}{18s^3 + 12s^2 + 8s + 3}$$

steg-
störning



För att parera en störning krävs en 60% större styrsignal!

8.8 Regulatorer



a) $G\left(\frac{Y}{R}\right) = ?$ $G_{REG} = 5$ P-regulator

b) $G\left(\frac{Y}{R}\right) = ?$ $G_{REG} = 3\left(2 + \frac{1}{10s}\right)$ PI-regulator

8.8 a med Matlab, P-Regulator

a) $G\left(\frac{Y}{R}\right) = ?$ $G_{REG} = 5$

P-regulator

```
Greg=tf(5)
```

Transfer function:

5

static gain

```
G=Greg*Gp
```

Transfer function:

5

10 s + 1

```
Gp=tf([1],[10,1])
```

Transfer function:

1

10 s + 1

```
Gclosed=feedback(G,1,-1)
```

Transfer function:

5

10 s + 6

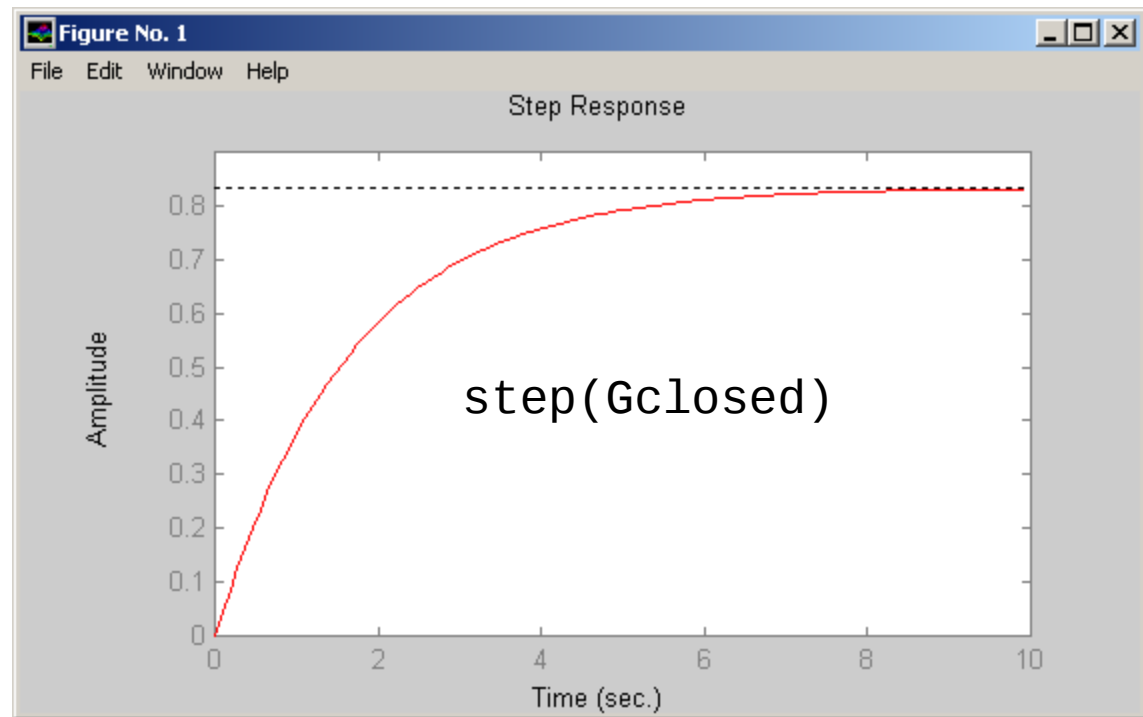
8.8 a med Matlab, P-Regulator

```
Gclosed=feedback(G,1,-1)
```

Transfer function:

5

10 s + 6



8.8 b med Matlab, PI-Regulator

b) $G\left(\frac{Y}{R}\right) = ?$ $G_{REG} = 3\left(2 + \frac{1}{10s}\right)$
 PI-regulator

$$G_{REG} = 3\left(2 + \frac{1}{10s}\right) = \frac{60s + 3}{10s}$$

```
Gp=tf([1],[10,1])
Transfer function:
      1
-----
10 s + 1
```

```
Greg=tf([60,3],[10,0])
Transfer function:
60 s + 3
-----
10 s
G=Greg*Gp
Transfer function:
60 s + 3
-----
100 s^2 + 10 s
```

```
Gclosed=feedback(G,1,-1)
Transfer function:
60 s + 3
-----
100 s^2 + 70 s + 3
```

8.8 b med Matlab, PI-Regulator

```
Gclosed=feedback(G,1,-1)
```

Transfer function:

$$60s + 3$$

$$100s^2 + 70s + 3$$

Visst kan en väl inställd regulator
göra nytta?

(Ingenjörer kan göra skillnad)

