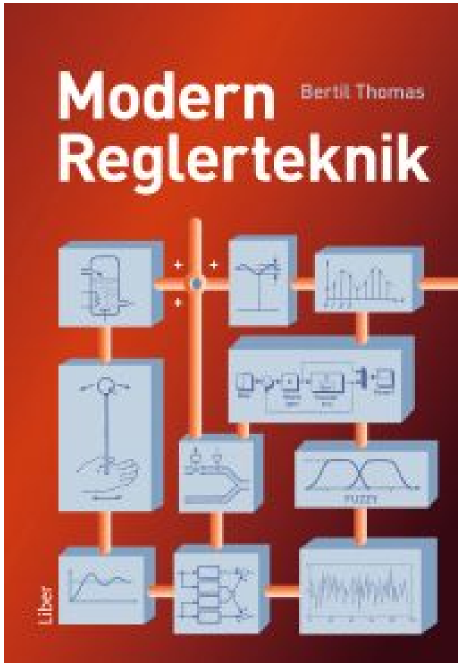
Reglerteknik **5**

Kapitel 9



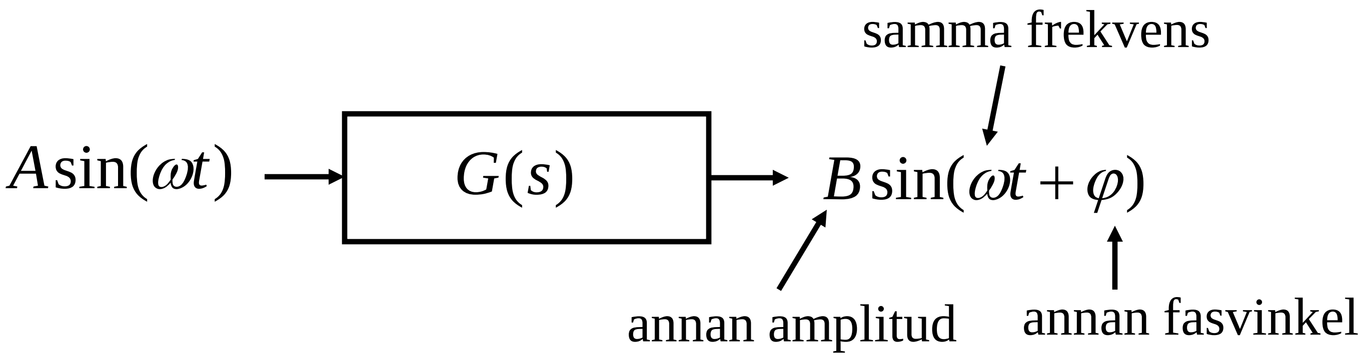
Köp bok och övningshäfte på kårbokhandeln

William Sandqvist william@kth.se

Föreläsning 5 kap 9

- Frekvensanalys

Sinusformade signaler i linjära system



$G(s)$ och $G(j\omega)$

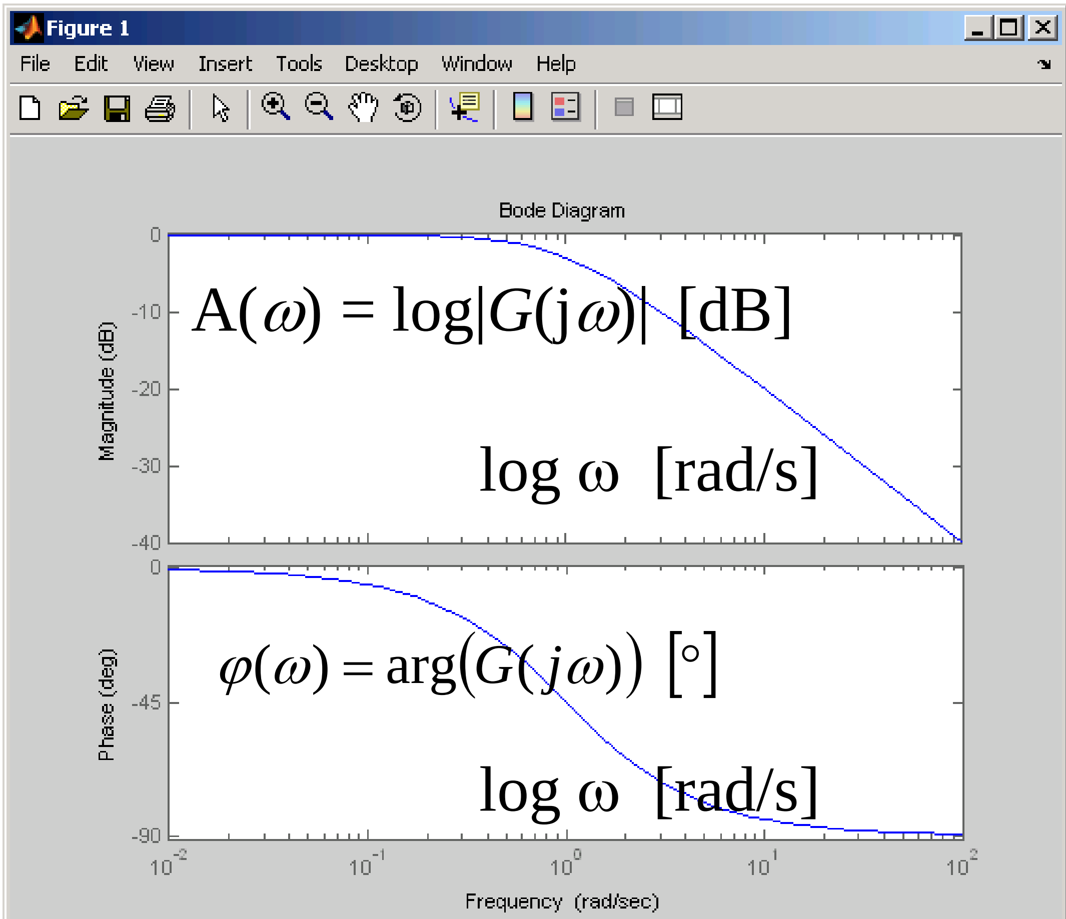
$$G(s)$$

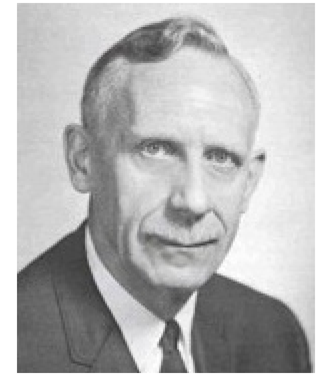
$$A(\omega) = \left| \underline{G}(j\omega) \right|$$

$$\varphi(\omega) = \arg\left(\underline{G}(j\omega)\right) \quad \angle \underline{G}(j\omega)$$

$PHASOR$ $\qquad \boxed{A\angle\varphi}$

Bode-diagram





Hendrik Wade Bode

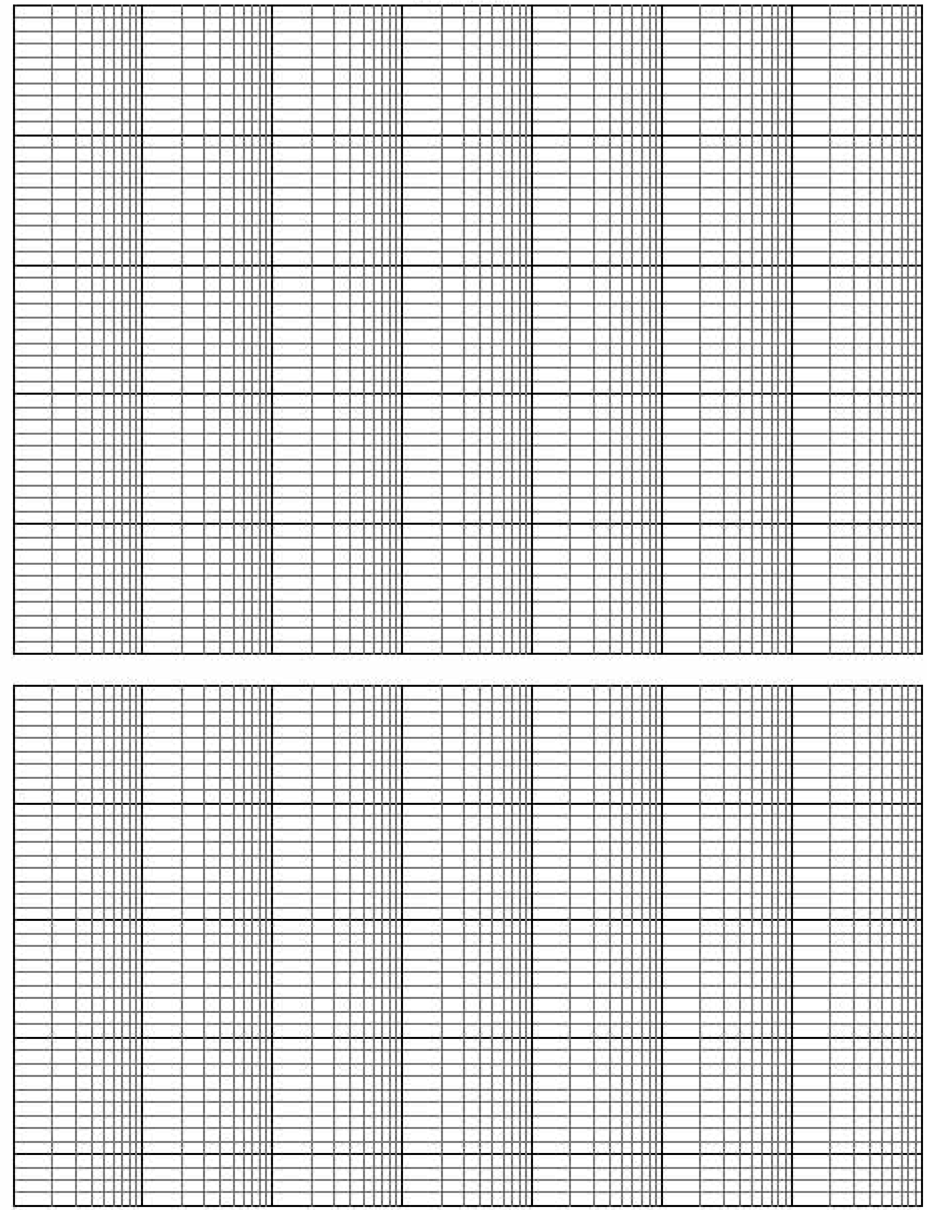
Bode-diagrammet är det *vanligaste* sättet att grafiskt beskriva överföringsfunktioner

Bode-diagram

ω-skalan (x-axeln)
är logaritmisk ([rad/sek]
ibland [Hz])

φ-skalan (y-axel)
är linjär (ofta [°])

A-skalan (y-axel)
är logaritmisk (ofta [dB])

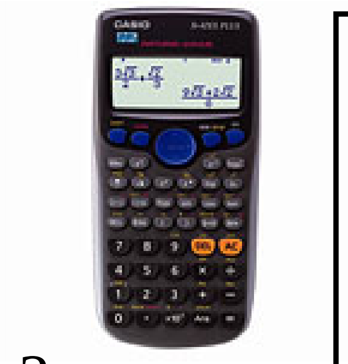


Ex. Bodediagram

$$G(s) = \frac{2}{1+2s}$$

Med teknikräknare:

$$\underline{G}(j\omega) = \frac{2}{1+2j\omega}$$



$$G(\omega) = \frac{2}{\sqrt{1+4\omega^2}}$$

$$\varphi(\omega) = 0° - \arctan\frac{2\omega}{1}$$

$$G(\omega)_{dB} = 20 \cdot {}^{10}\log\frac{2}{\sqrt{1+4\omega^2}}$$

ω[rad]	G	G[dB]	φ[°]
0,1	1,96	5,85	−11,3
0,3	1,72	4,96	−31
0,5	1,41	3,01	−45
0,8	1,06	0,51	−58
1	0,89	−0,97	−64,4
3	0,33	−9,7	−80,5
5	0,20	−14	−84,3
8	0,12	−18,1	−86,4

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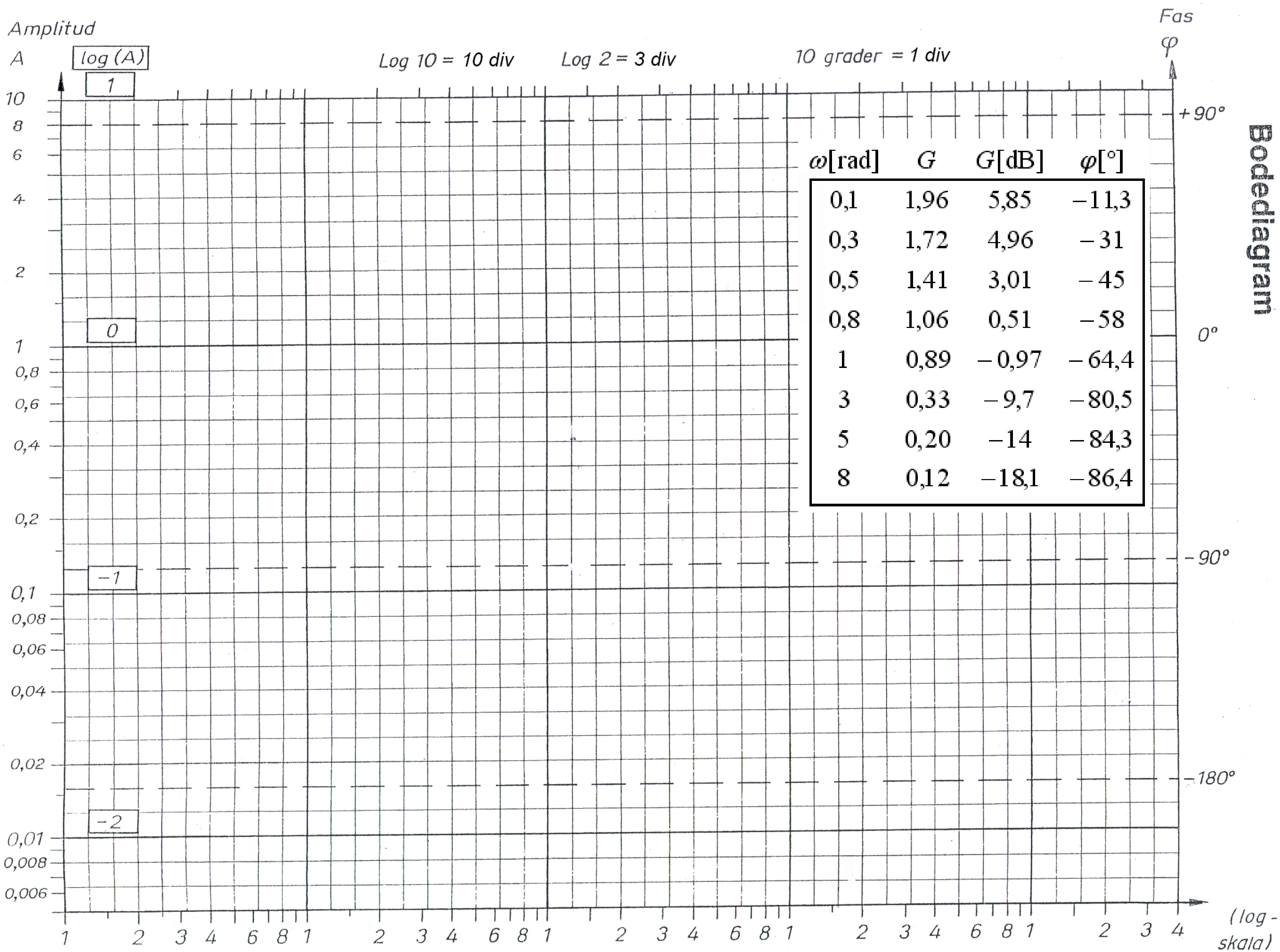

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$$G(s) = \frac{2}{1+2s}$$

Ex. Bodediagram

ω[rad]	G	G[dB]	φ[°]
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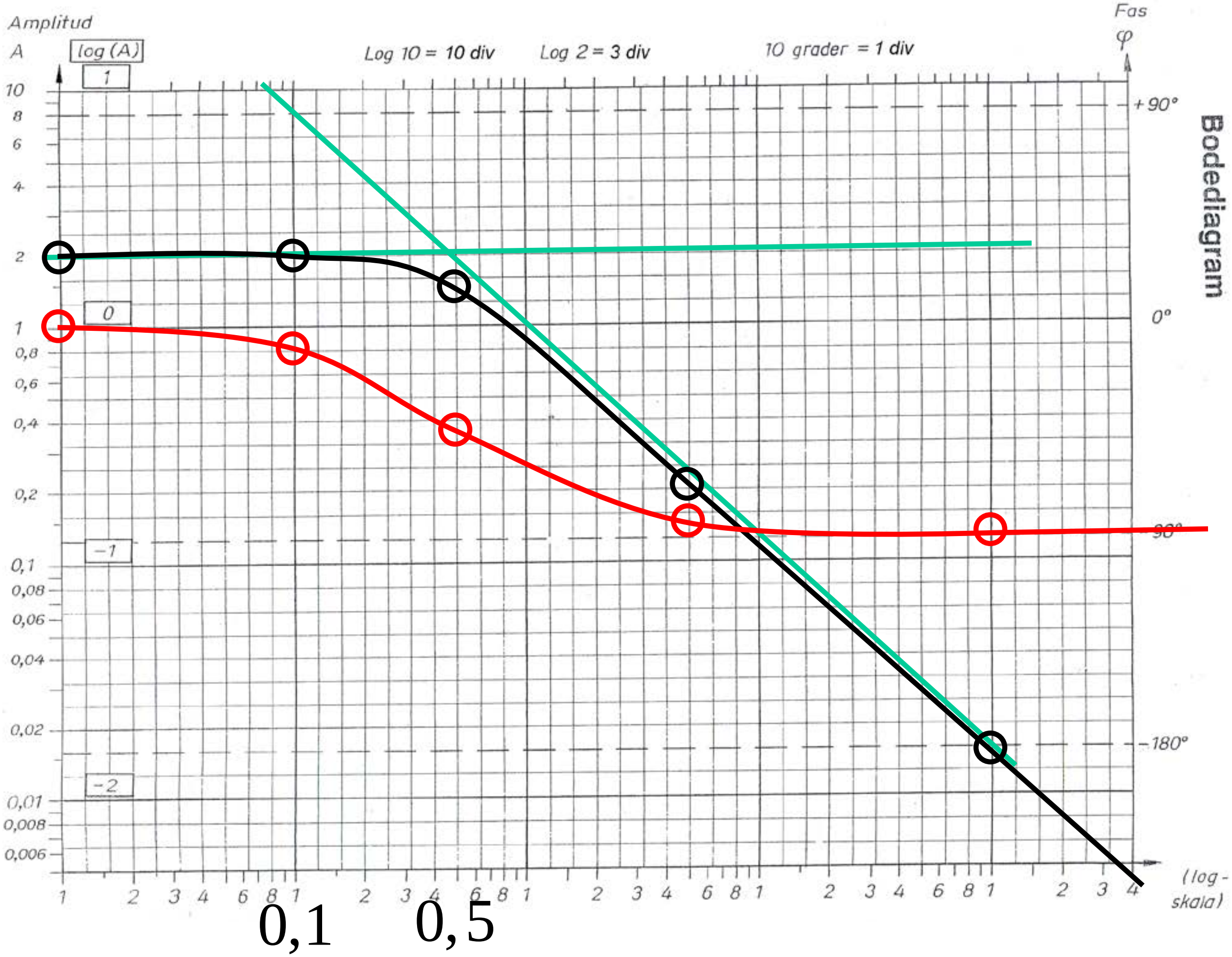
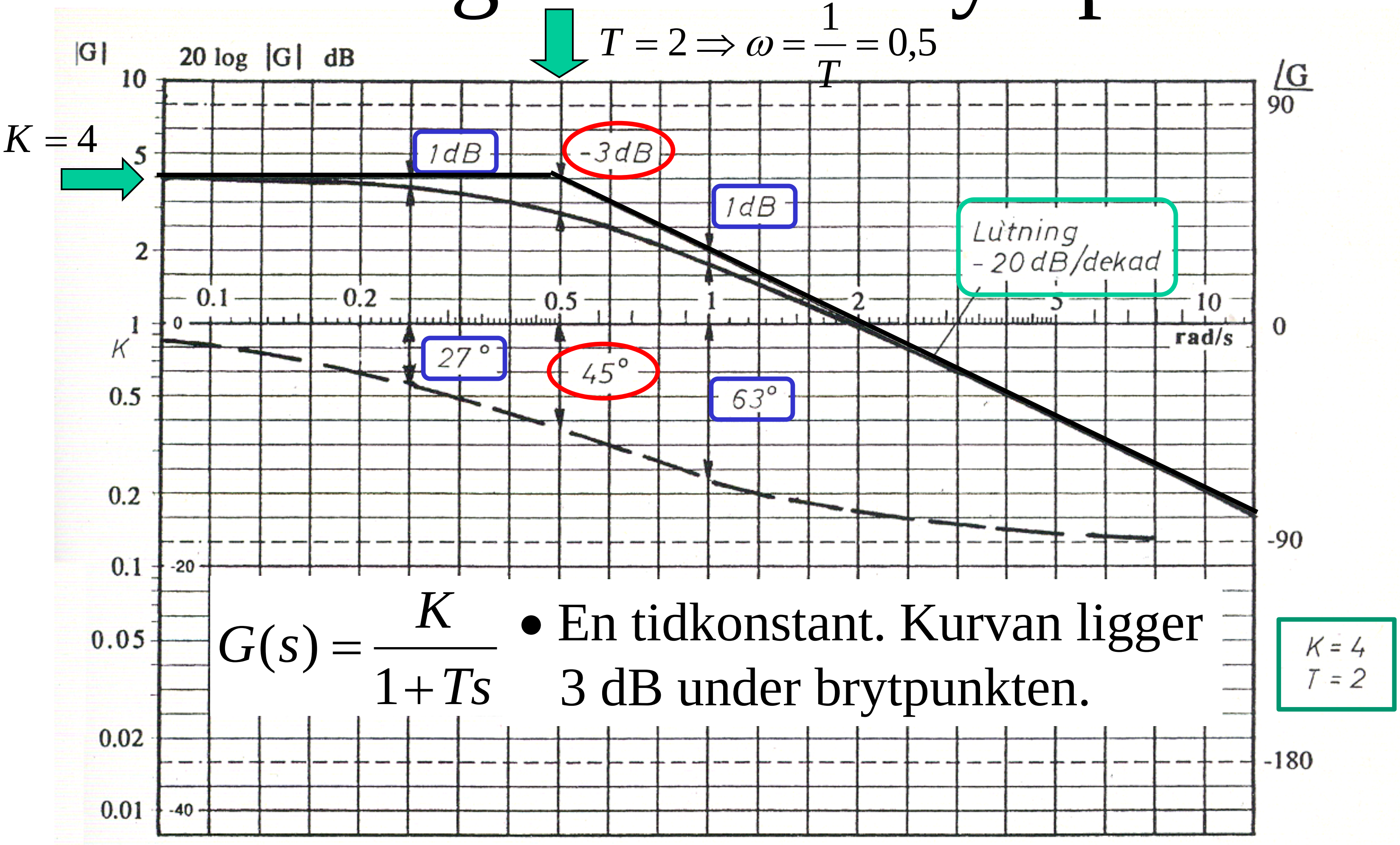


Bodediagrammets asymptoter



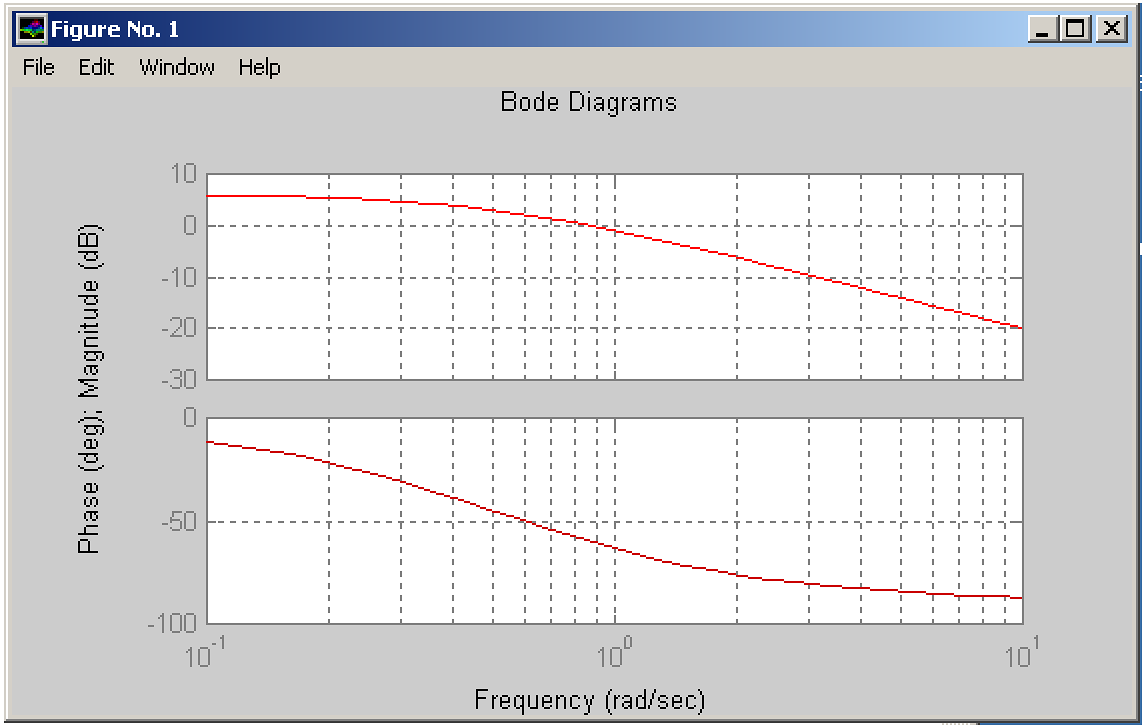
$$G(s) = \frac{K}{1+Ts}$$

- En tidkonstant. Kurvan ligger 3 dB under brytpunkten.

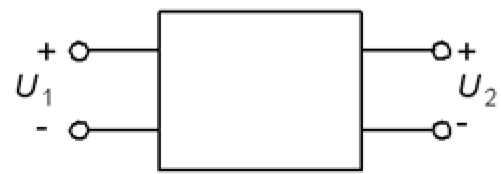
Ex. Matlab Bodediagrams

$$G(s) = \frac{2}{1+2s}$$

```
bode(tf([2],[2,1]))
```



Frekvensanalys

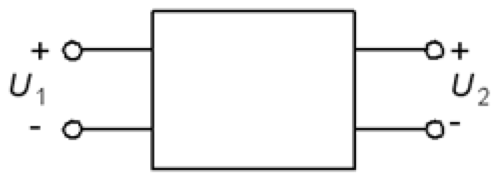


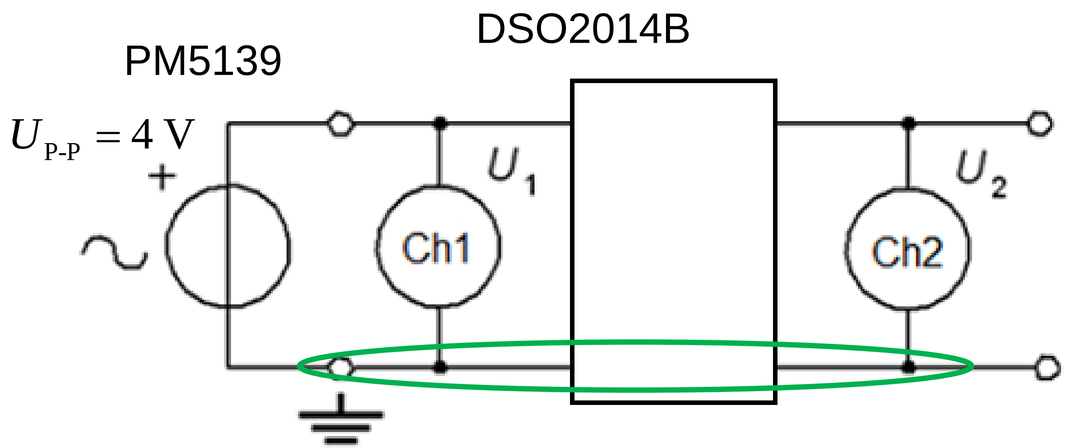
Med relativt enkla mätningar med sinusformade spänningar från en **signalgenerator** och med ett **tvåkanaligt oscilloskop** kan man ”ta fram” de flesta parametrarna för en process överföringsfunktion.

Detta gör frekvensanalysen till en mycket använd mätmetod.

Mätning av överföringsfunktion

Från elläran vet Du hur man ”tar upp” en överföringsfunktion med ett oscillskop.





$$\frac{U_2}{U_1} = \frac{U_{CH2}}{U_{CH1}} \qquad \varphi = \arg\left(\frac{\underline{U}_2}{\underline{U}_1}\right) = Phase(2 \rightarrow 1)$$

ger vinkeln det rätta tecknet!

- Mät och ”plotta” blockets överföringsfunktion på diagrampapper.

Grundfaktorer

Överföringsfunktioner kan delas upp i en produkt av grundfaktorer.

Multiplikation motsvaras av addition i ett logaritmiskt diagram.

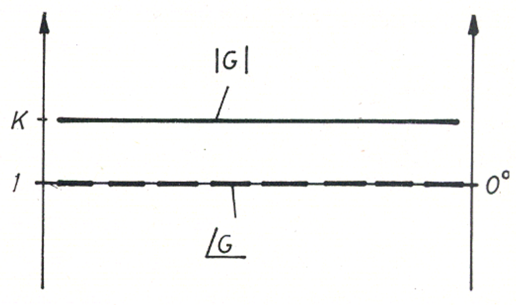
Genom att lära sig vilka kurvor som grundfaktorerna genererar kan man snabbt skissa hur en sammansatt överföringsfunktion bör se ut i Bodediagrammet!

$$G(s) = G_1 \cdot G_2 \cdots G_N$$

Grundfaktorernas asymptoter

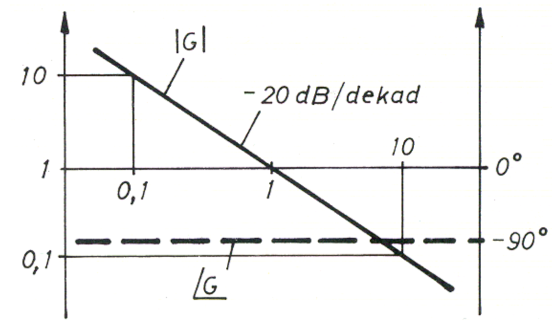
- Konstant förstärkning/dämpning

$$G(s) = K$$
$$G(\omega) = K$$
$$\varphi(\omega) = 0$$



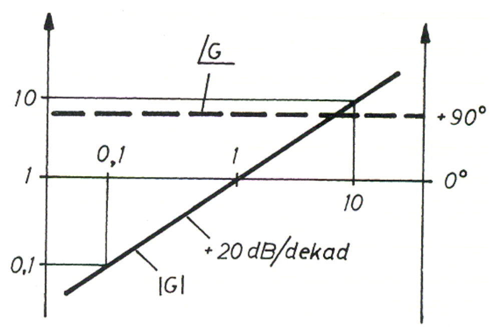
- Integrering

$$G(s) = \frac{1}{s}$$
$$G(\omega) = \frac{1}{\omega}$$
$$\varphi(\omega) = -90°$$



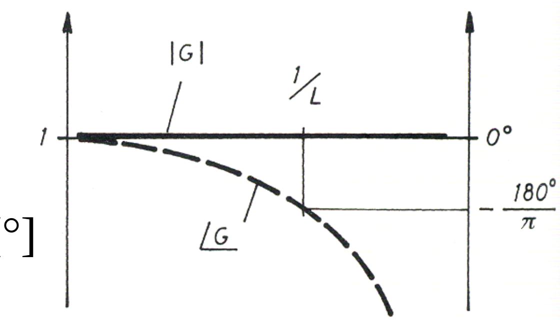
- Derivering

$$G(s) = s$$
$$G(\omega) = \omega$$
$$\varphi(\omega) = +90°$$



- Dödtid

$$G(s) = e^{-Ls}$$
$$G(\omega) = 1$$
$$\varphi(\omega) = -L\omega \cdot \frac{180}{\pi} [°]$$



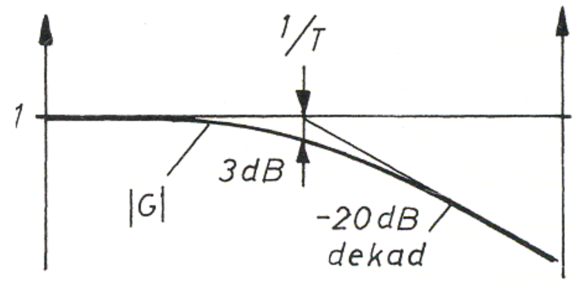
Grundfaktorernas asymptoter

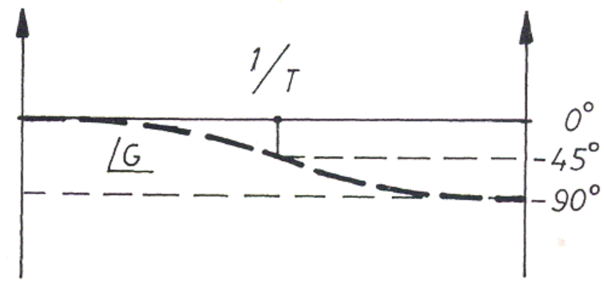
- Tidkonstant nämnaren LP

$$G(s) = \frac{1}{1+Ts}$$

$$G(\omega) = \frac{1}{\sqrt{1+(T\omega)^2}}$$

$$\varphi(\omega) = -\arctan T\omega$$



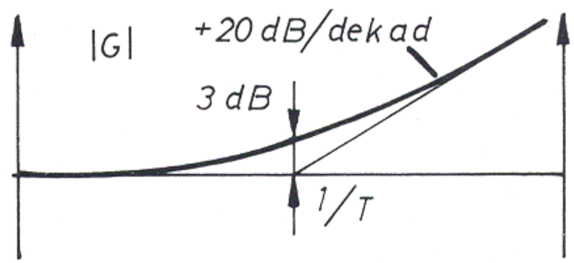


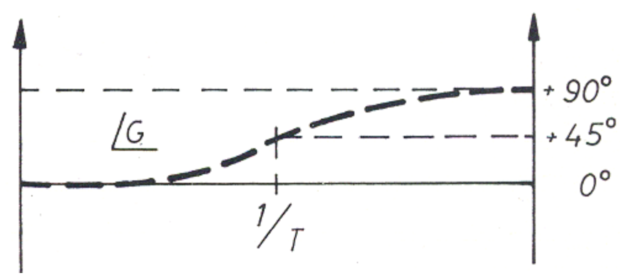
- Tidkonstant täljaren HP

$$G(s) = 1+Ts$$

$$G(\omega) = \sqrt{1+(T\omega)^2}$$

$$\varphi(\omega) = \arctan T\omega$$



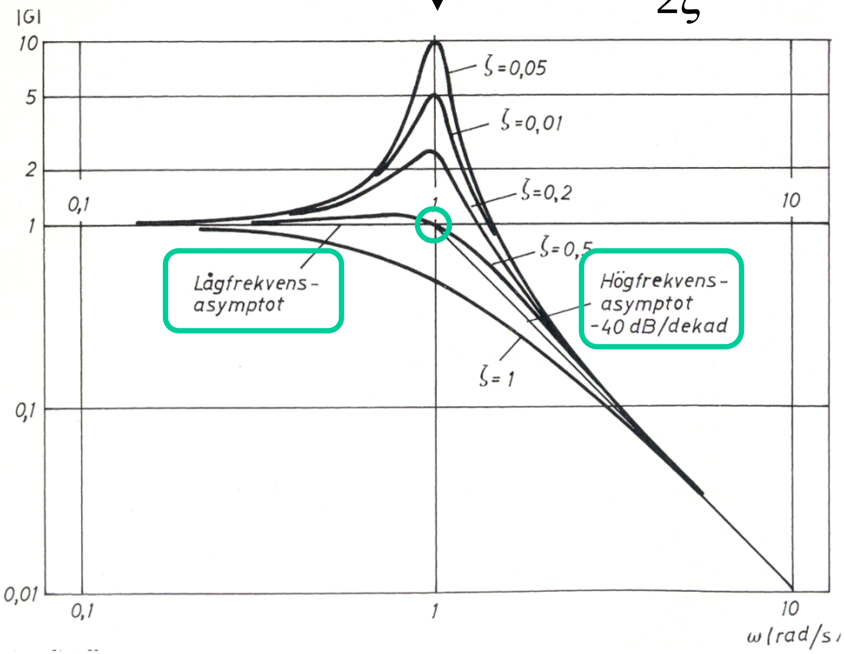


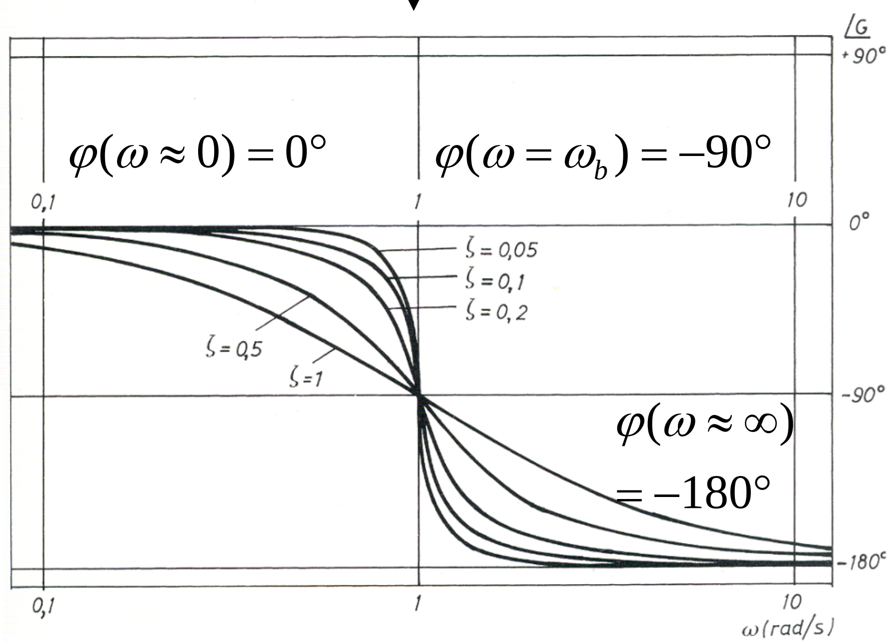
Grundfaktorernas asymptoter

- Andragradsfaktor nämnaren, komplexa rötter

$$G(s)=\frac{1}{bs^2+as+1} \qquad G(s)=\frac{1}{T^2s^2+2\zeta Ts+1} \qquad T=\sqrt{b} \quad \zeta=\frac{a}{2T} \quad \boxed{\omega_b=\frac{1}{T}}$$

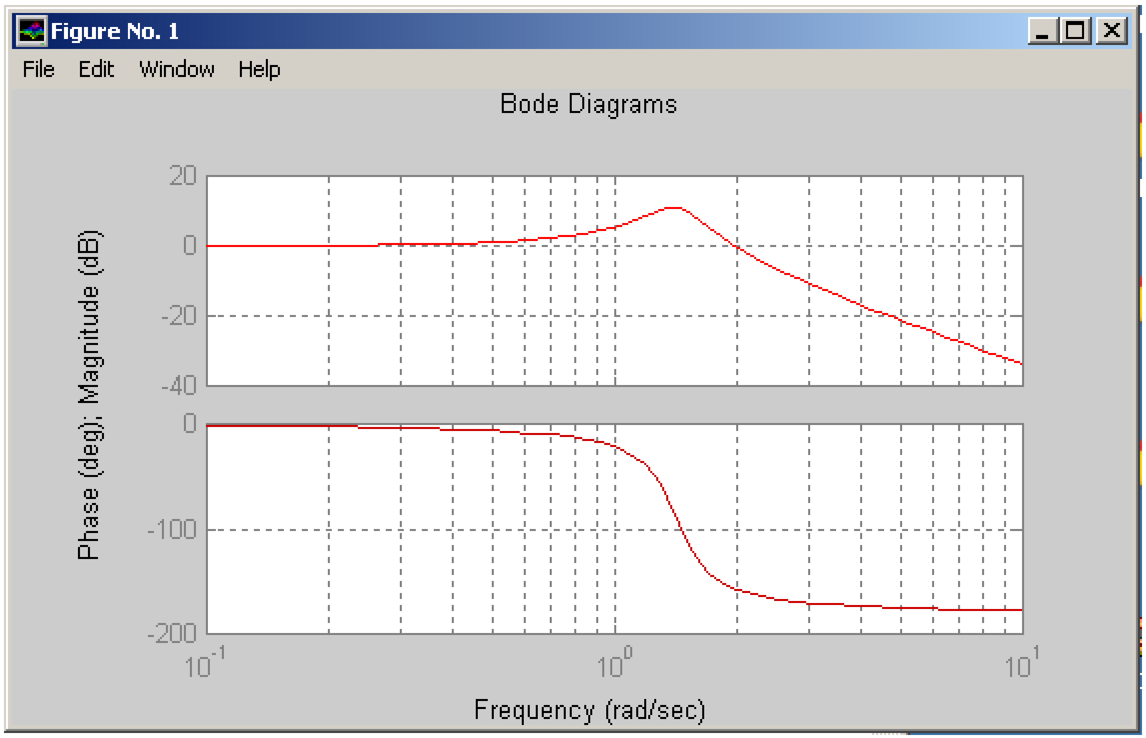
resonanstopp ω_b $G(\omega_b)=\frac{1}{2\zeta}$ $\qquad \omega_b$





Matlab `ord2(Wn,Z)`

```
[num,den]=ord2( 1, 0.05 );
bode( tf( num, den) );
```



Ex. Sammansatt uttryck

$$G(s) = \frac{2{,}5 \cdot (1 + s)}{s^2 + 5{,}2s + 1}$$

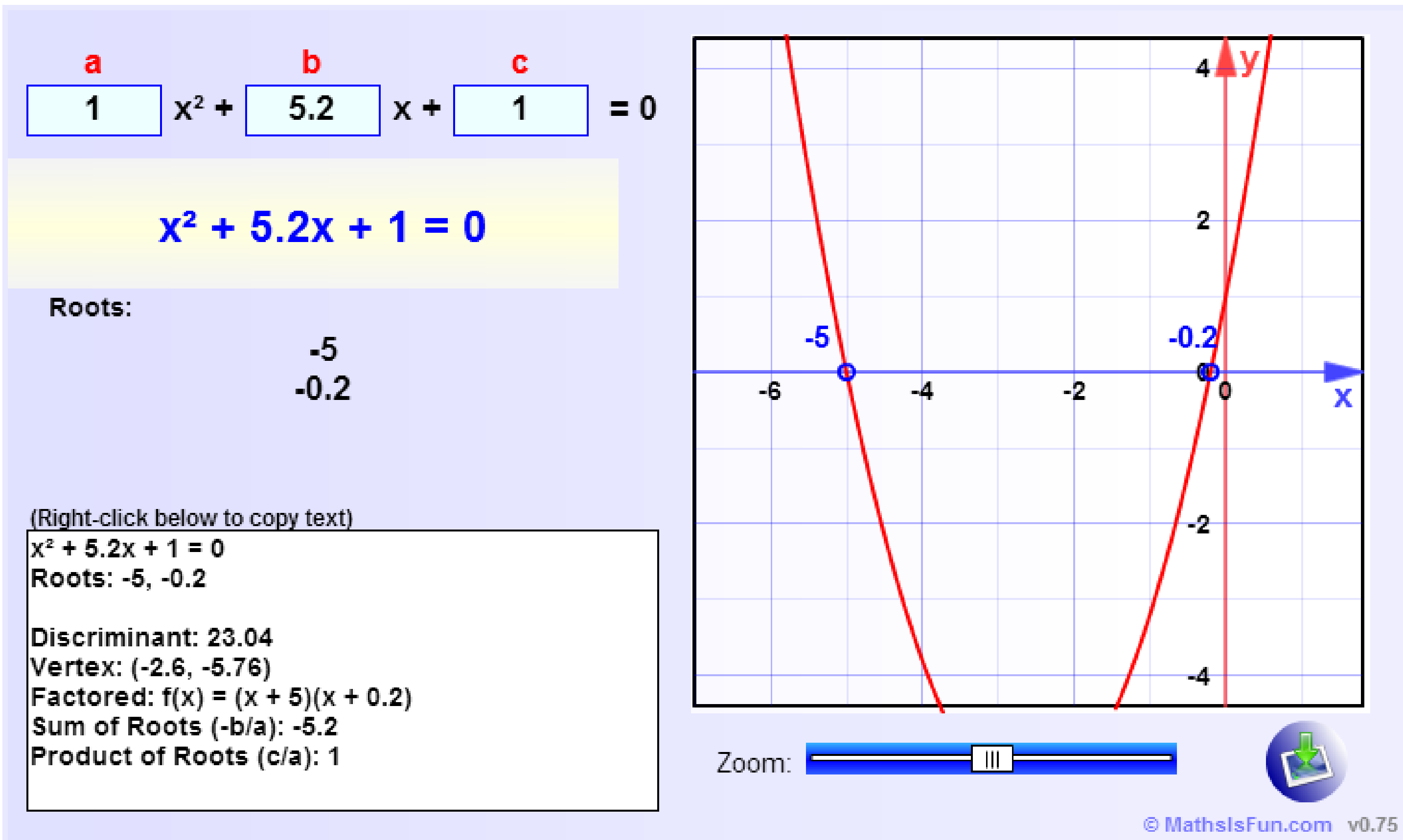
[Quadratic Equation Solver](http://www.mathsisfun.com/quadratic-equation-solver.html)

`http://www.mathsisfun.com/quadratic-equation-solver.html`

$$G(s) = \frac{2{,}5 \cdot (1 + s)}{(5s + 1) \cdot (0{,}2s + 1)}$$

(andragradsekvationen)

http://www.mathsisfun.com/quadratic-equation-solver.html



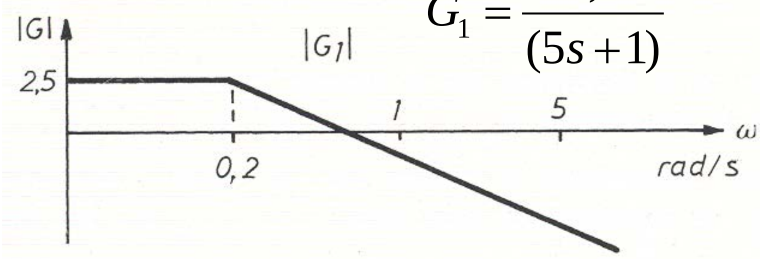
Ex. Sammansatt uttryck

$$G(s) = \frac{2{,}5 \cdot (1+s)}{(5s+1)\cdot(0{,}2s+1)} = G_1 \cdot G_2 \cdot G_3 =$$

$$= \frac{2{,}5}{(5s+1)} \cdot \frac{1}{(0{,}2s+1)} \cdot (1+s)$$

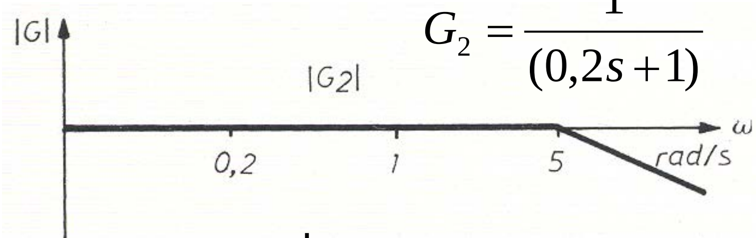
Ex. Sammansatt uttryck

$$G_1 = \frac{2,5}{(5s+1)}$$



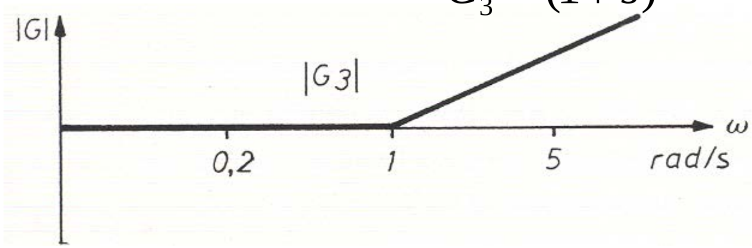
+

$$G_2 = \frac{1}{(0,2s+1)}$$



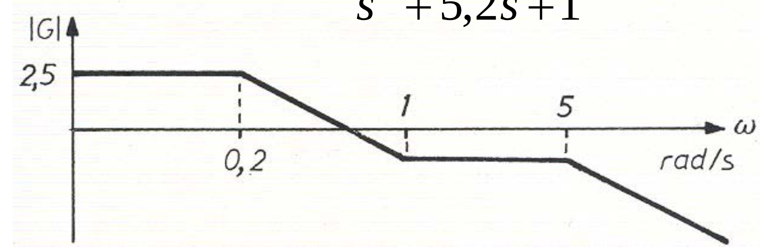
+

$$G_3 = (1+s)$$

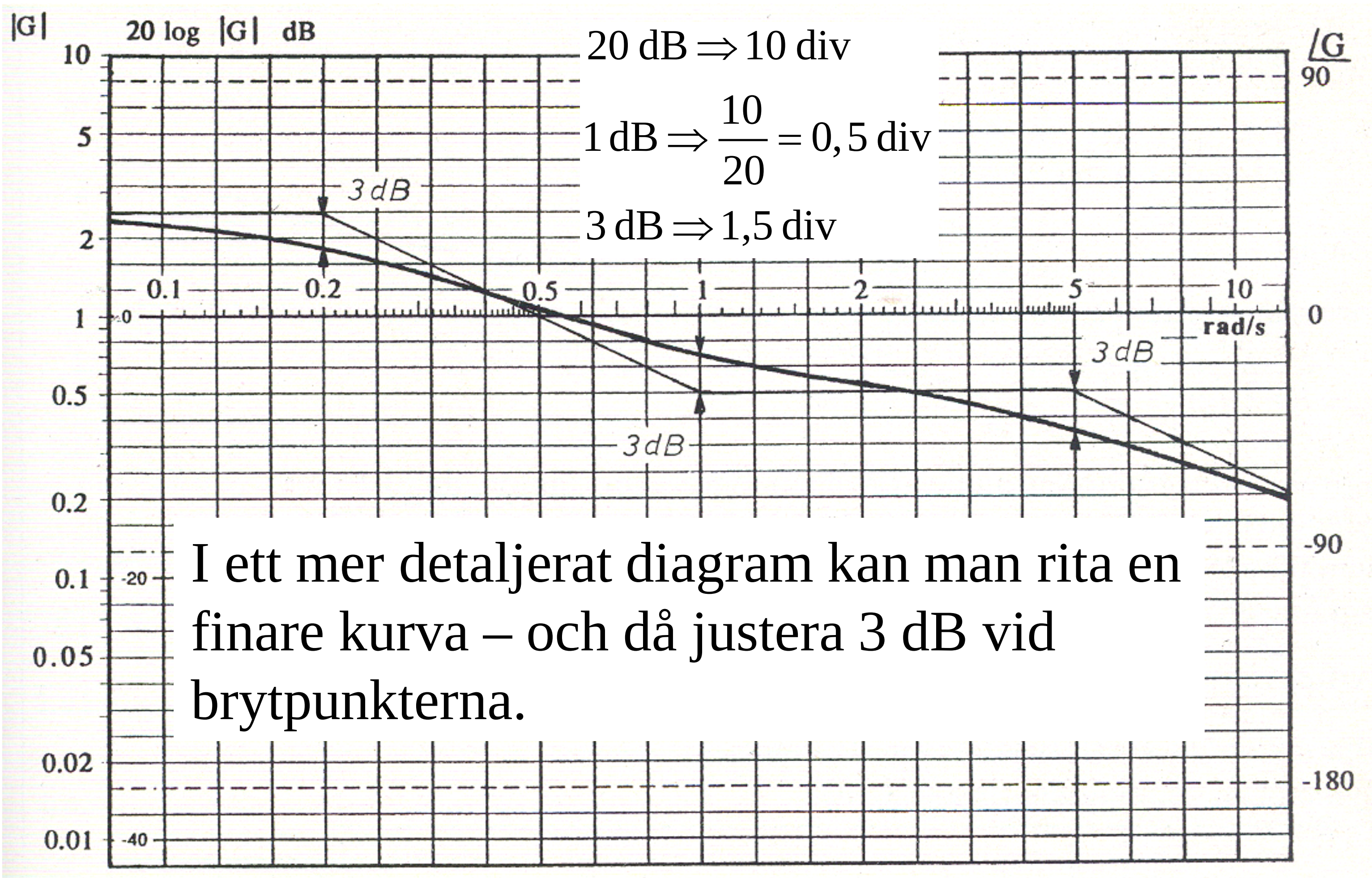


=

$$G(s) = \frac{2,5\cdot(1+s)}{s^2+5,2s+1}$$



Ex. Sammansatt uttryck



$20\ \text{dB} \Rightarrow 10\ \text{div}$

$$1\ \text{dB} \Rightarrow \frac{10}{20} = 0{,}5\ \text{div}$$

$3\ \text{dB} \Rightarrow 1{,}5\ \text{div}$

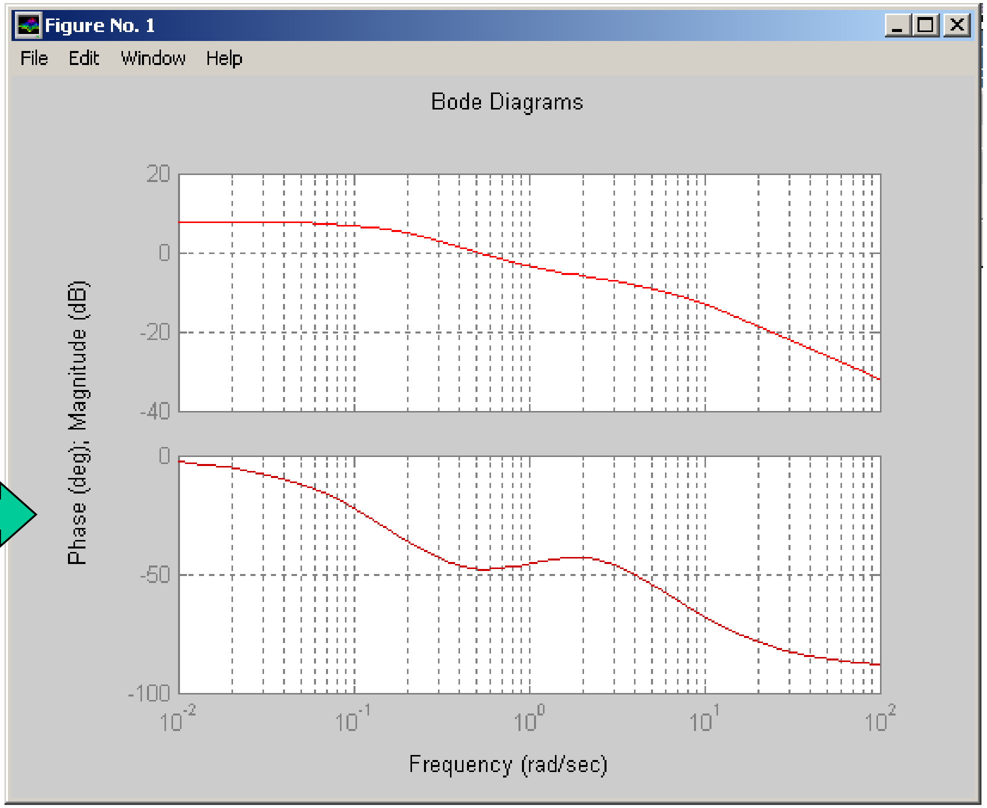
I ett mer detaljerat diagram kan man rita en finare kurva – och då justera 3 dB vid brytpunkterna.

(Med Matlab)

```
bode(tf([2.5,2.5],[1,5.2,1]))
```

$$G(s)=\frac{2{,}5\cdot(1+s)}{s^2+5{,}2s+1}$$

Också fasvinkel-värden kan adderas i Bodediagrammet.



9.2 Amplitud och Fas- värden

$$A(\omega) = ? \quad \varphi(\omega) = ?$$

a) $U \longrightarrow \boxed{\dfrac{4}{3+2s}} \longrightarrow Y$ $\quad \omega = 1{,}5$ rad/sek

b) $U \longrightarrow \boxed{\dfrac{4}{3+2s}} \longrightarrow Y$ $\quad \omega = 6$ rad/sek

c) $U \longrightarrow \boxed{\dfrac{4}{1+6s}} \longrightarrow Y$ $\quad \omega = 1$ rad/sek

d) $U \longrightarrow \boxed{\dfrac{1+3s}{1+s}} \longrightarrow Y$ $\quad \omega = 0{,}5$ rad/sek

e) $U \longrightarrow \boxed{\dfrac{2(1+3s)}{1+s}} \longrightarrow Y$ $\quad \omega = 2$ rad/sek

9.2 a lösn. Amplitud och Fas

$$A(\omega) = ? \quad \varphi(\omega) = ?$$

a) $U \longrightarrow \boxed{\dfrac{4}{3+2s}} \longrightarrow Y$ $\quad \omega = 1{,}5 \text{ rad/sek}$

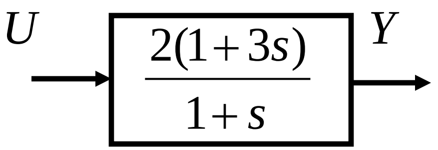
$$\frac{4}{3+2s} \quad j\omega = s \quad \frac{4}{3+2j\omega} \quad \Rightarrow \quad A(\omega) = \frac{4}{\sqrt{9+4\omega^2}} \quad \varphi(\omega) = 0 - \arctan\frac{2\omega}{3}$$

$$A(1{,}5) = \frac{4}{\sqrt{9+4\cdot 1{,}5^2}} = 0{,}94 \quad \varphi(1{,}5) = -\arctan\frac{2\cdot 1{,}5}{3} = -45°$$

9.2 e lösn. Amplitud och Fas

$$A(\omega) = ? \quad \varphi(\omega) = ?$$

e)



$\omega = 2$ rad/sek

$$A(\omega) = \frac{2\sqrt{1+9\omega^2}}{\sqrt{1+\omega^2}} \quad \varphi(\omega) = \arctan 3\omega - \arctan \omega$$

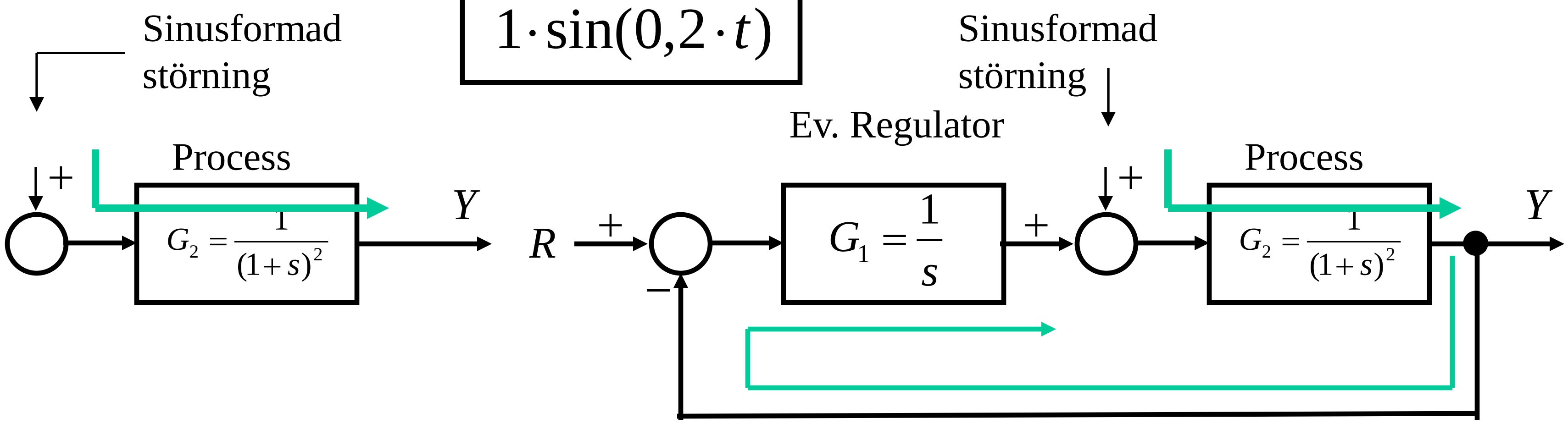
$$A(2) = \frac{2\sqrt{1+9\cdot 2^2}}{\sqrt{1+2^2}} \approx 5{,}44$$

$$\varphi(2) = \arctan 3\cdot 2 - \arctan 2 \approx 80{,}5 - 63{,}4 = 17°$$

9.4 sinusformad störning

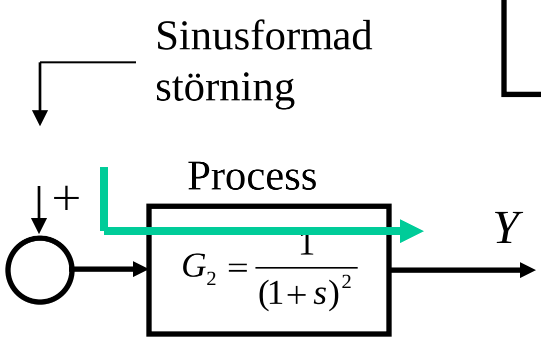
- *utan regulator*
- *med regulator*

$$1 \cdot \sin(0{,}2 \cdot t)$$



9.4 lösn. sinusformad störning

utan regulator



$$\boxed{1 \cdot \sin(0{,}2 \cdot t)}$$

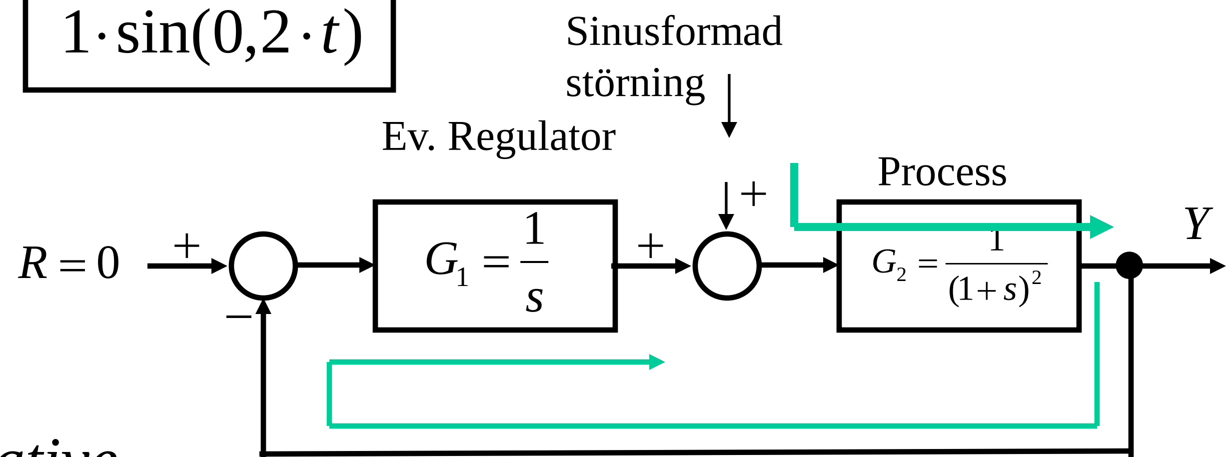
$$\frac{1}{(1+s)^2} \qquad j\omega = s \qquad A(\omega) = \frac{1}{\left(\sqrt{1+\omega^2}\right)^2} = \frac{1}{1+\omega^2}$$

$$A(0{,}2) = \frac{1}{1+0{,}2^2} = \boxed{0{,}96}$$

9.4 lösn. sinusformad störning

med regulator

$$1 \cdot \sin(0{,}2 \cdot t)$$



negative feedback

$$G(s) = \frac{\frac{1}{(1+s)^2}}{1 + \frac{1}{s} \cdot \frac{1}{(1+s)^2}} = \frac{s}{s(1+s)^2 + 1} = \frac{s}{s^3 + 2s^2 + s + 1}$$

9.4 lösn. sinusformad störning

$$G(s)=\frac{s}{s^3+2s^2+s+1} \quad j\omega = s \quad \underline{G}(j\omega)=\frac{j\omega}{j\omega - j\omega^3+1-2\omega^2}$$

$$A(\omega)=\frac{\omega}{\sqrt{(\omega-\omega^3)^2+(1-2\omega^2)^2}}$$

$$A(0,2)=\frac{0,2}{\sqrt{(0,2-0,2^3)^2+(1-2\cdot 0,2^2)^2}}\approx \boxed{0,21}$$

Med regulatorn inkopplad reduceras sinusstörningens inverkan med 79 %!

9.4 med Matlab, sinus-störning

$$G(s) = \frac{1}{(1+s)^2} = \frac{1}{s^2+2s+1} \qquad G_{REG}(s) = \frac{s}{s^3+2s^2+s+1}$$

```
abs(freqresp(tf([1],[1,2,1]),0.2))
>> ans = 0.9615

abs(freqresp(tf([1,0],[1,2,1,1]),0.2))
>> ans = 0.2128
```

Med Matlab går det snabbare att ta reda på samma sak.

9.5 parameterbestämning

Process:

$$G(s)=\frac{K\cdot e^{-L\cdot s}}{1+Ts}$$

Två mätningar med sinusstorheter på processen:

ω[rad/s]	$A(\omega)$[ggr]	$\varphi(\omega)[°]$
1	5	-100
2	3,6	$-$

Beräkna processparametrarna K L T.

9.5 lösn. parametrar

$$G(s) = \frac{K \cdot e^{-L \cdot s}}{1 + Ts}$$

ω[rad/s]	$A(\omega)$[ggr]	$\varphi(\omega)$[°]
1	5	−100
2	3,6	–

$$\underline{G}(j\omega) = \boxed{\frac{K}{1 + Tj\omega}} \cdot \boxed{e^{-L \cdot j\omega}}$$

$$A(\omega) = \boxed{\frac{K}{\sqrt{1 + T^2\omega^2}}} \cdot \boxed{1} \quad \varphi(\omega) = \boxed{0 - \arctan T\omega} - \boxed{L\omega}$$

9.5 lösn. parametrar

ω[rad/s]	$A(\omega)$[ggr]	$\varphi(\omega)$[°]
1	5	-100
2	3,6	$-$

$$A(\omega) = \frac{K}{\sqrt{1+T^2\omega^2}} \cdot 1 \quad K = A \cdot \sqrt{1+T^2\omega^2} \quad \varphi = -\arctan \omega T - \omega L$$

(1) $\quad K = 5 \cdot \sqrt{1+T^2}$

(2) $\quad K = 3{,}6 \cdot \sqrt{1+4T^2}$

(3) $\quad -100 \cdot \dfrac{\pi}{180} = -\arctan 1 \cdot T - 1 \cdot L$

$[°] \rightarrow [\text{rad}]$

$(1) = (2)$

$$5 \cdot \sqrt{1+T^2} = 3{,}6 \cdot \sqrt{1+4T^2}$$

$$25 \cdot (1+T^2) = 12{,}96 \cdot (1+4T^2)$$

$$25 + 25T^2 = 12{,}96 + 51{,}84T^2$$

$$26{,}84 \cdot T^2 = 12{,}04 \quad \boxed{T = 0{,}67}$$

$$K = 5 \cdot \sqrt{1+T^2} = 5 \cdot \sqrt{1+0{,}67^2} \approx \boxed{6}$$

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9.5 lösn. parametrar

ω[rad/s]	$A(\omega)$[ggr]	$\varphi(\omega)$[°]
1	5	−100
2	3,6	–

$$A(\omega)=\frac{K}{\sqrt{1+T^2\omega^2}}\cdot 1 \quad K=A\cdot\sqrt{1+T^2\omega^2} \quad \varphi=-\arctan\omega T-\omega L$$

$$(3) \quad -100\cdot\frac{\pi}{180}=-\arctan 1\cdot T-1\cdot L$$

$$\Rightarrow \quad L=100\cdot\underbrace{\frac{\pi}{180}}_{[°]\rightarrow[\mathrm{rad}]}-\arctan T=100\cdot\frac{\pi}{180}-\arctan 0{,}67\approx\boxed{1{,}16}$$

Med mätning med sinus-storheter vid två frekvenser kunde alla processparametrarna beräknas.

Ofta använd testfrekvens **159** Hz

f = 0,16 Hz ⇒ ω = 1 rad/s
f = 1,59 Hz ⇒ ω = 10 rad/s
f = 15,9 Hz ⇒ ω = 100 rad/s
$\boxed{f = 159\ \text{Hz} \Rightarrow \omega = 1000\ \text{rad/s}}$

Eftersom ingenjörer är för lata för att räkna …

f = 318 Hz ⇒ ω = 2000 rad/s

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9.8 rita Bodediagram

$a)\quad \dfrac{5}{(1+s)(1+10s)}$

$b)\quad \dfrac{1+5s}{(1+2s)(1+10s)}$

$c)\quad \dfrac{1}{s(1+s)^2}$

$d)\quad \dfrac{2(1+s)}{s}$

$e)\quad \dfrac{3{,}2\cdot e^{-2s}}{(1+3s)}$

$f)\quad \dfrac{e^{-s}}{s}$

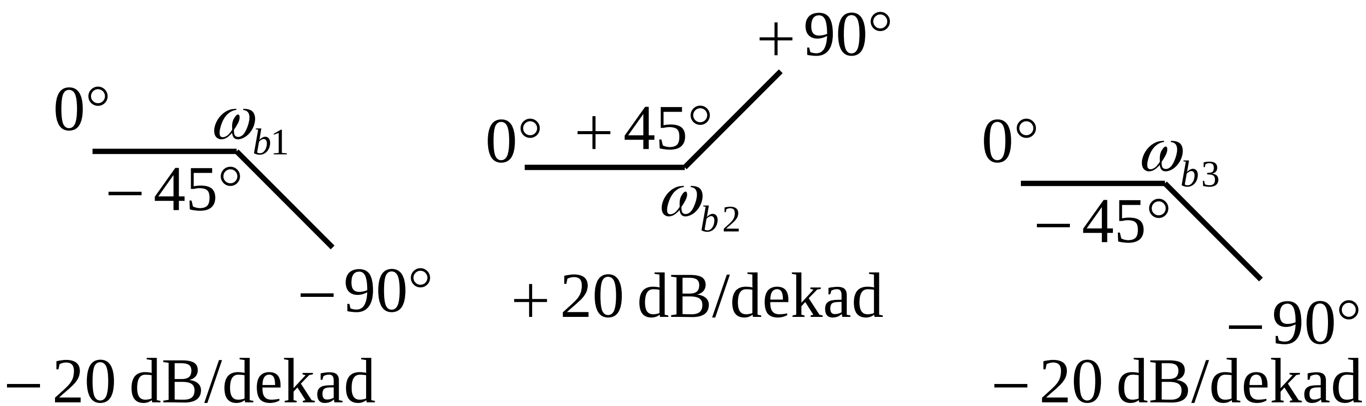
9.8 b rita Bodediagram

$$b) \quad \frac{1+5s}{(1+2s)(1+10s)}$$

9.8 b rita Bodediagram

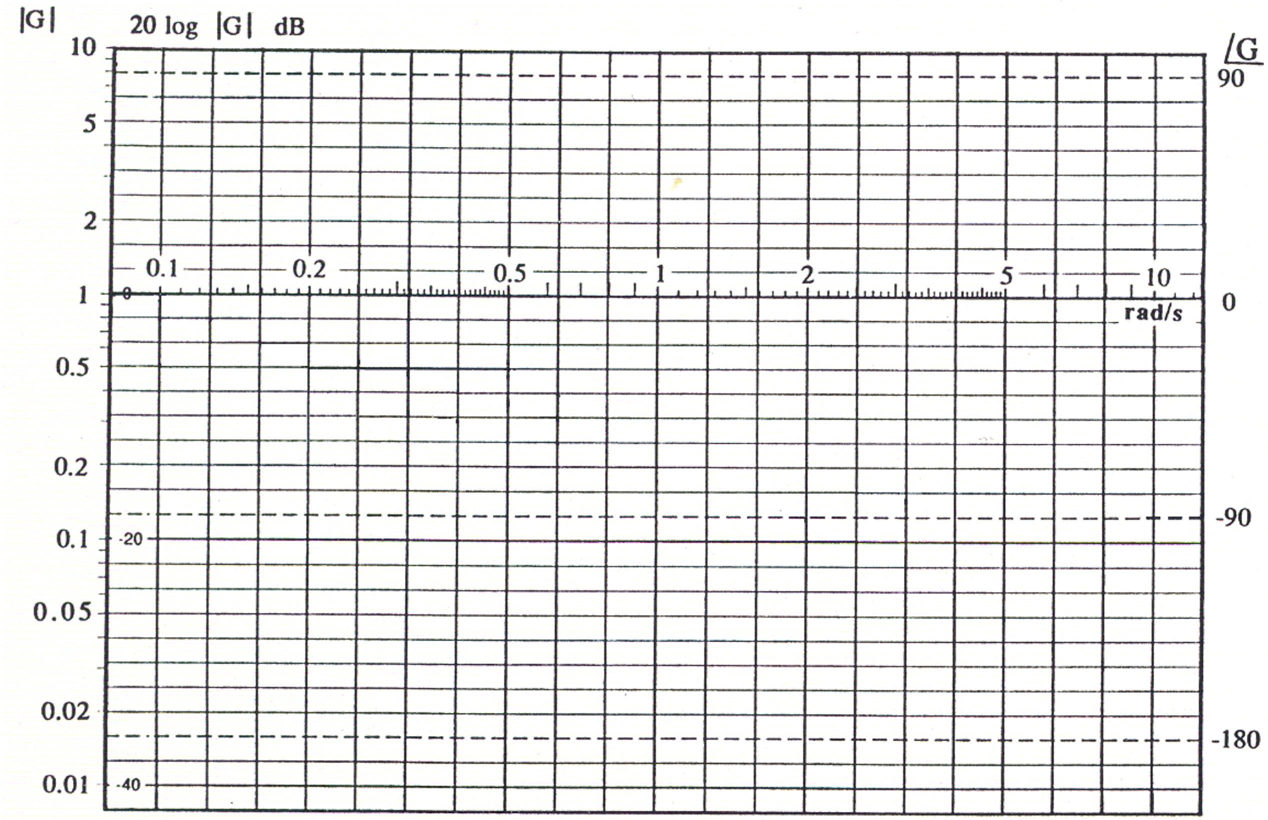
$$\frac{1+5s}{(1+2s)(1+10s)} = \frac{1}{(1+2s)} \cdot (1+5s) \cdot \frac{1}{(1+10s)}$$

$$\omega_{b1} = \frac{1}{T_1} = \frac{1}{2} = 0{,}5 \quad \omega_{b2} = \frac{1}{T_2} = \frac{1}{5} = 0{,}2 \quad \omega_{b3} = \frac{1}{T_3} = \frac{1}{10} = 0{,}1$$

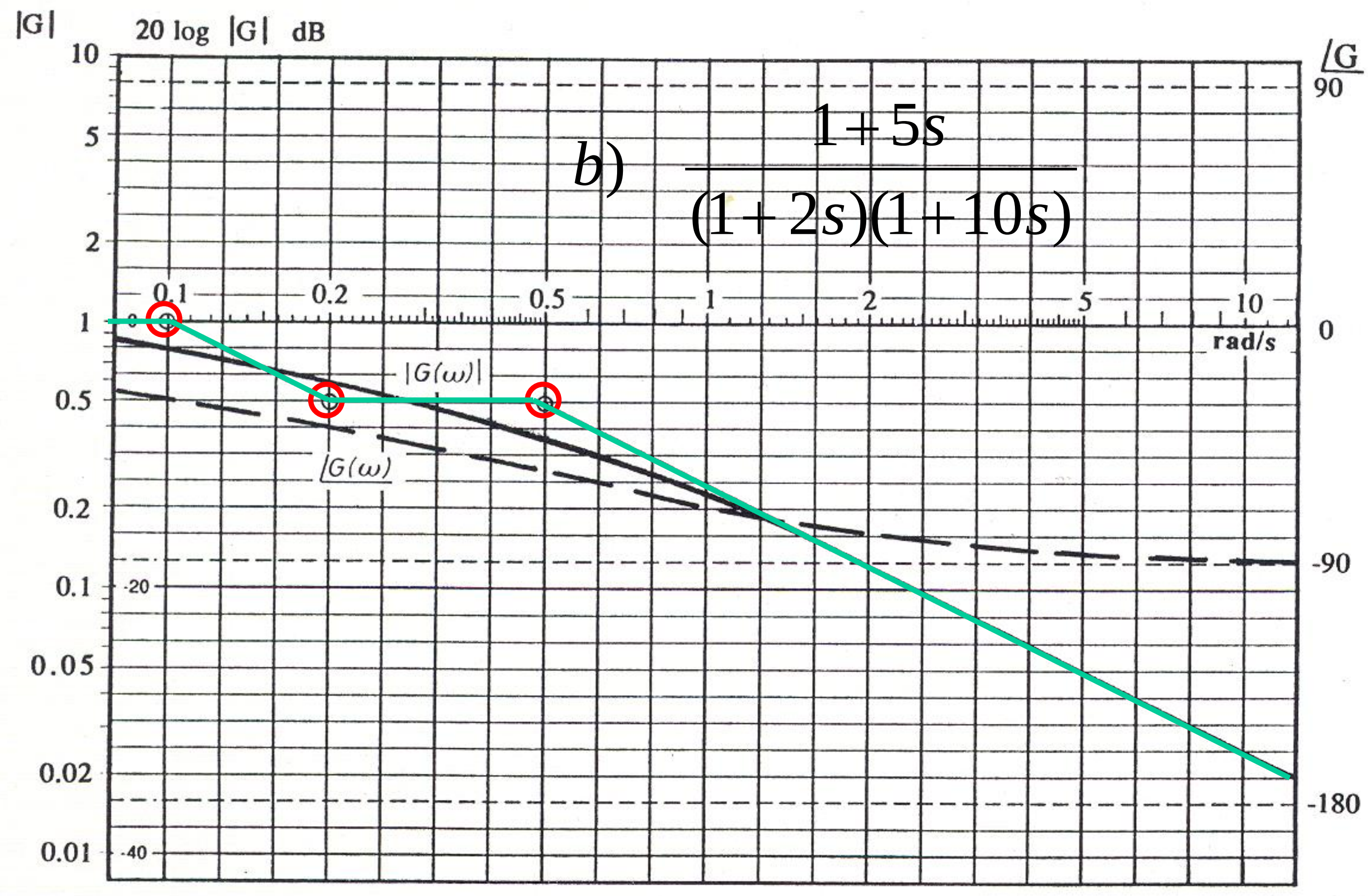


9.8 b rita Bodediagram

$$\omega_{b1} = \frac{1}{T_1} = \frac{1}{2} = 0{,}5 \quad \omega_{b2} = \frac{1}{T_2} = \frac{1}{5} = 0{,}2 \quad \omega_{b3} = \frac{1}{T_3} = \frac{1}{10} = 0{,}1$$



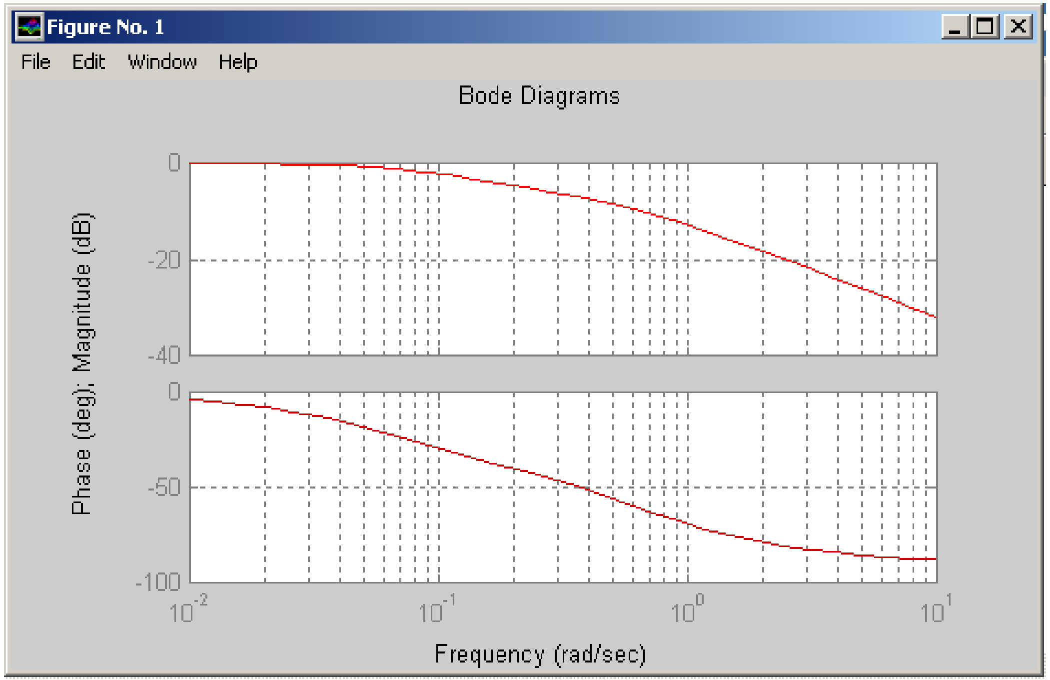
9.8 b facit rita Bodediagram



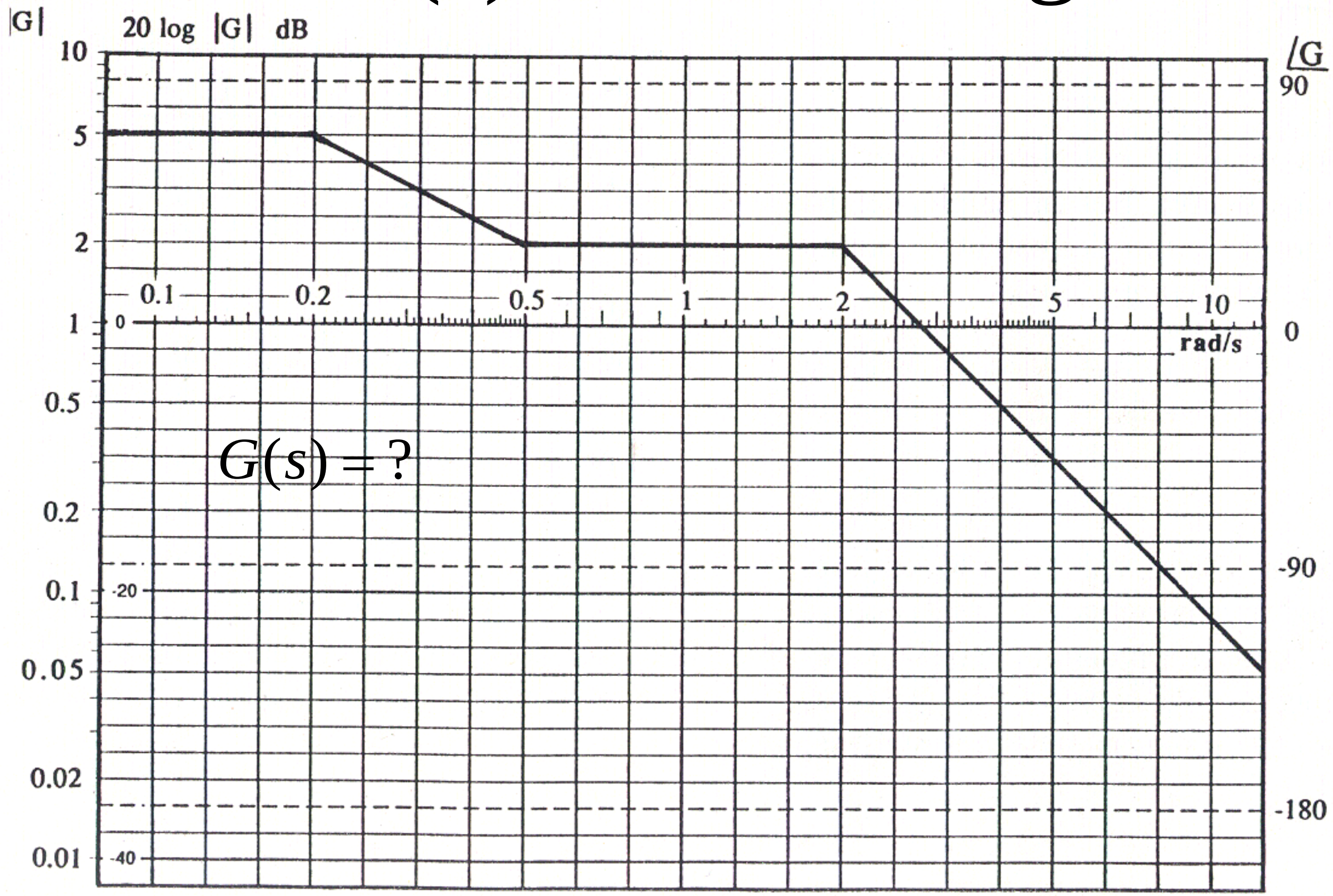
9.8 b facit Matlab Bode

$$\frac{1+5s}{1+2s} \cdot \frac{1}{1+10s}$$

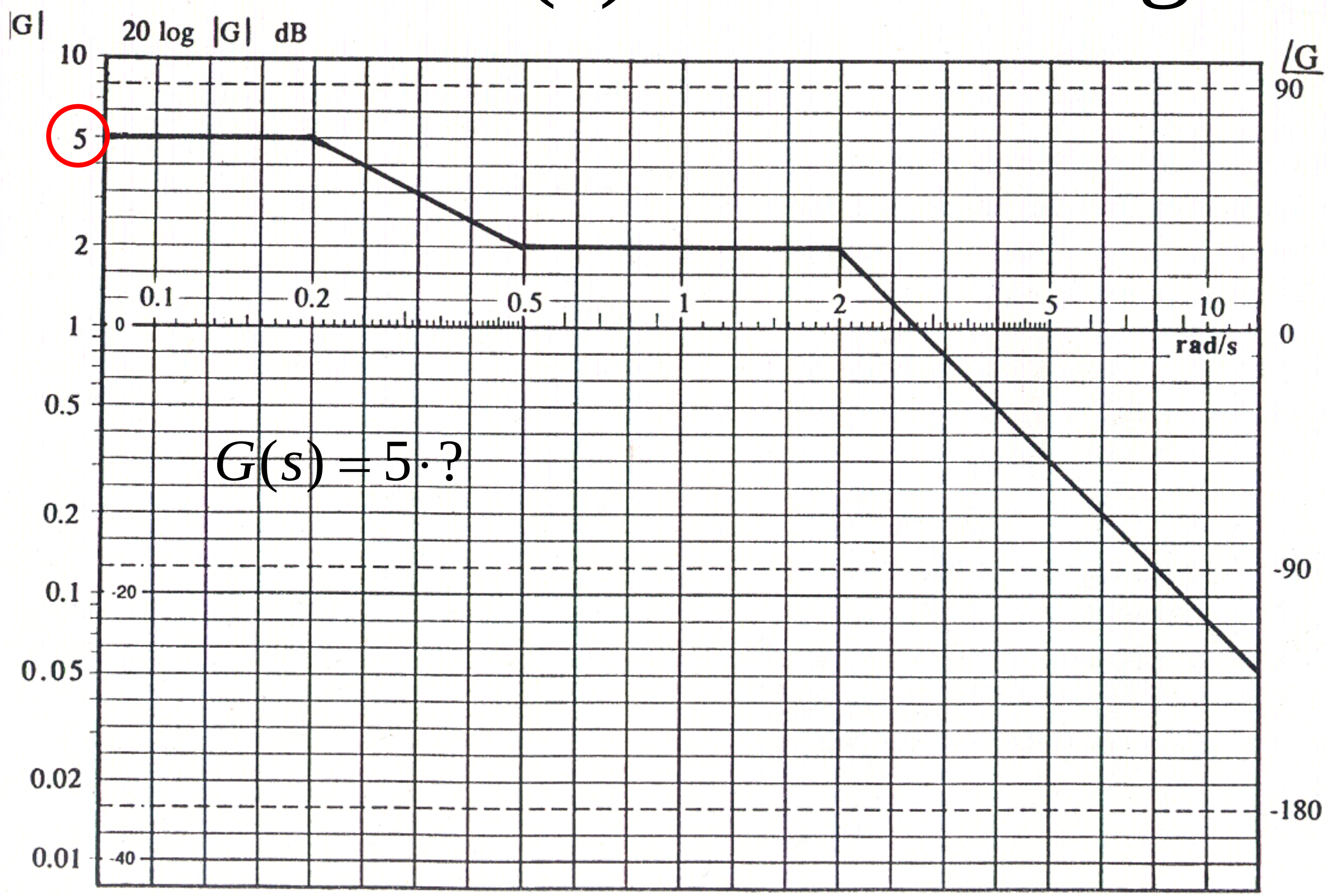
```
G1=tf([5,1],[2,1]);
G2=tf([1],[10,1]);
G=G1*G2;
bode(G);
```



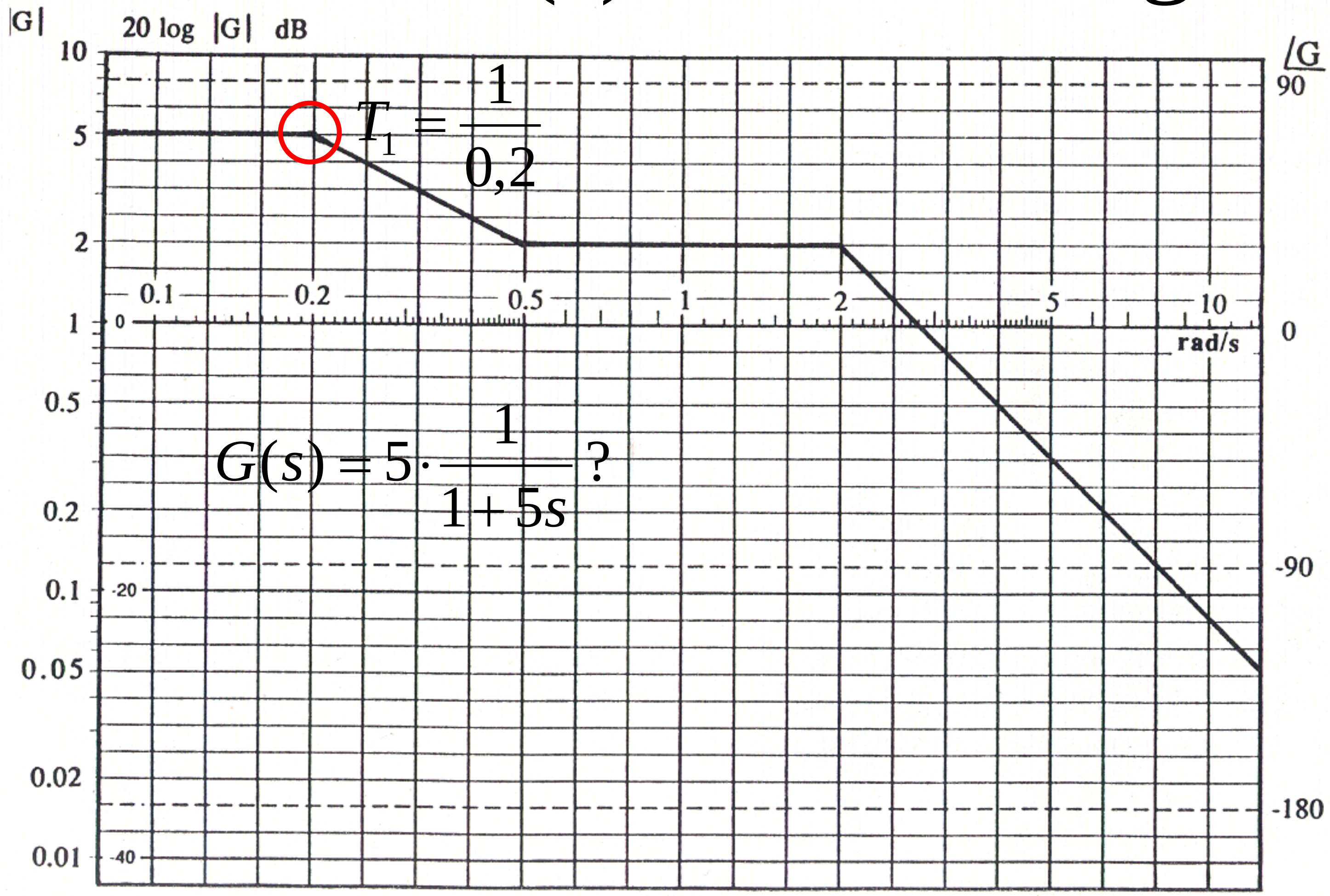
9.10 $G(s)$ ur Bodediagram



9.10 lösn. $G(s)$ ur Bodediagram



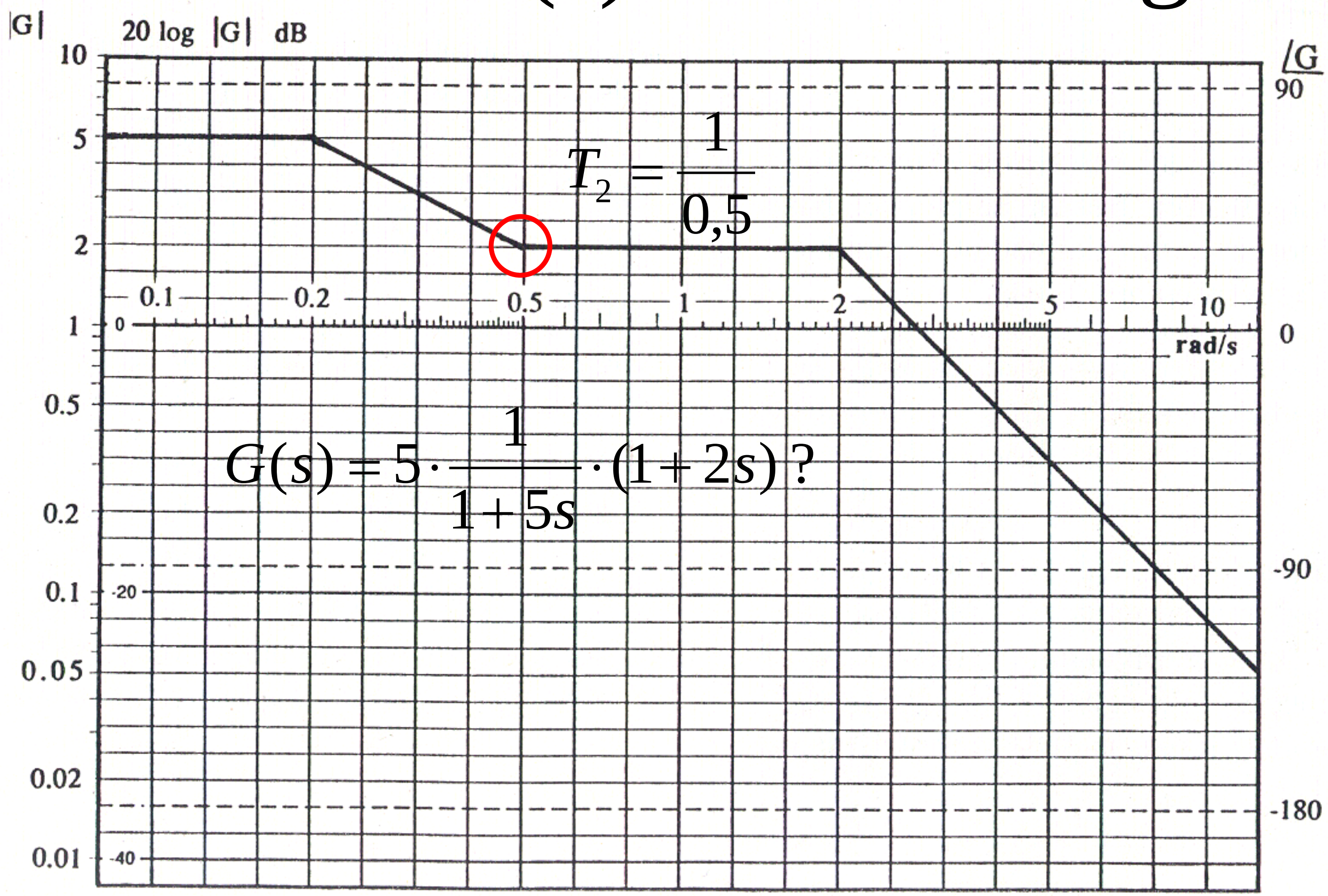
9.10 lösn. $G(s)$ ur Bodediagram



$$T_1 = \frac{1}{0{,}2}$$

$$G(s) = 5 \cdot \frac{1}{1+5s} ?$$

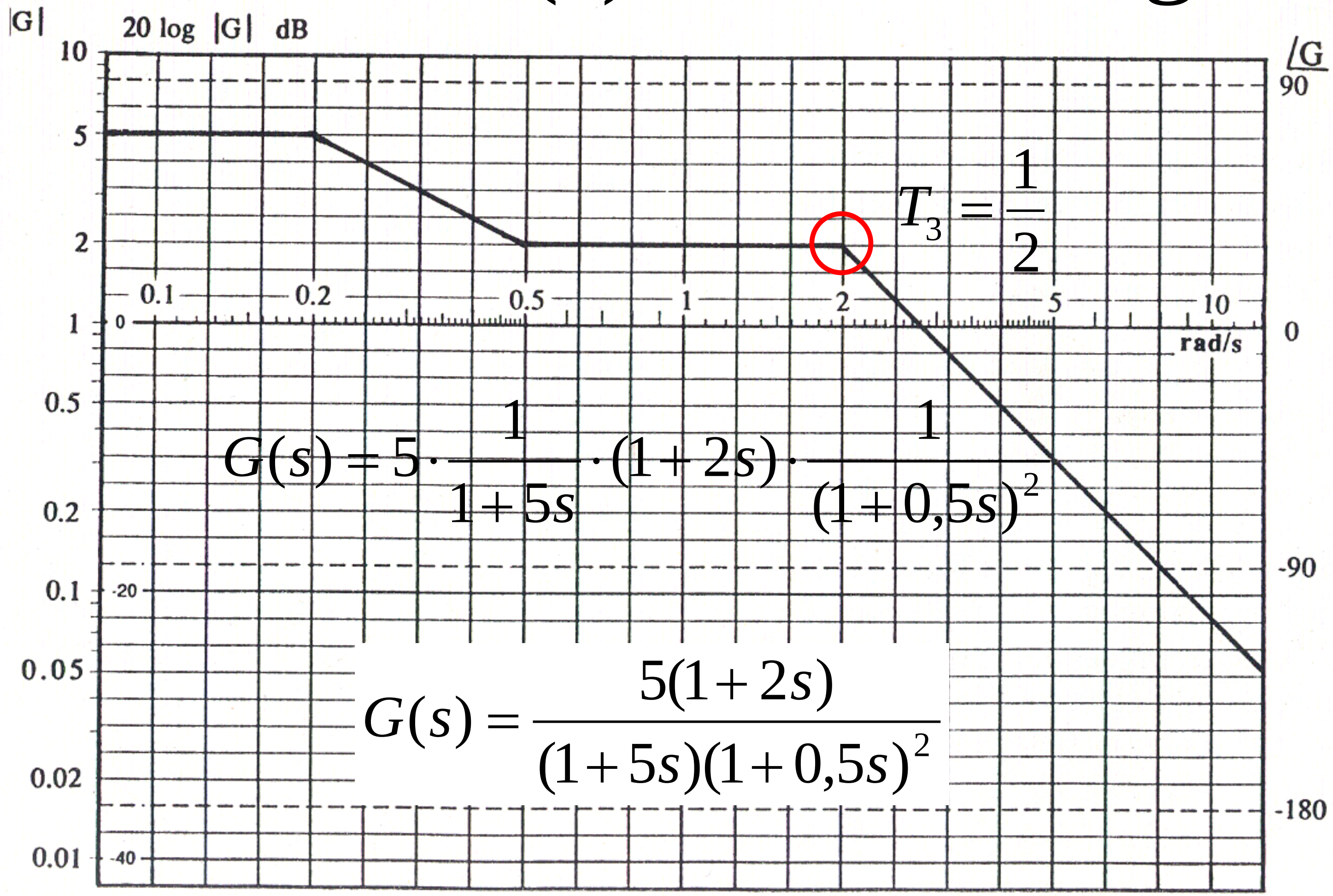
9.10 lösn. $G(s)$ ur Bodediagram



$$T_2 = \frac{1}{0,5}$$

$$G(s) = 5\cdot\frac{1}{1+5s}\cdot(1+2s)\,?$$

9.10 lösn. $G(s)$ ur Bodediagram



$$T_3 = \frac{1}{2}$$

$$G(s) = 5 \cdot \frac{1}{1+5s} \cdot (1+2s) \cdot \frac{1}{(1+0{,}5s)^2}$$

$$G(s) = \frac{5(1+2s)}{(1+5s)(1+0{,}5s)^2}$$

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