

(16.1) a) $y(z) = \underline{\underline{z^{-1} + z^{-2} + z^{-3}}}$

$$y(z) = z \left(\underset{\text{se}(k-1)}{\text{enhetsteg}}; 1 \right) + (-1) \cdot z \left(\underset{\text{se}(k-4)}{\text{enhetsteg}}; 4 \right) =$$

$$= z^{-1} \cdot \frac{z}{z-1} - z^{-4} \cdot \frac{z}{z-1} = \frac{1-z^{-3}}{z-1} \quad \text{fel i facit}$$

b) ramp med lutn. $\frac{1}{4} \Rightarrow y(z) = \frac{z}{4(z-1)^2}$

c) enhets ramp med fördröjning. 2 $\Rightarrow y(z) = z^{-2} \cdot \frac{z}{(z-1)^2} =$

$$= \frac{1}{z(z-1)^2}$$

(16.2) a) $f(k) = 0,5^k \Rightarrow F(z) = \underline{\underline{\frac{z}{z-0,5}}}$

b) $f(k) = 3s_p(k-2) \Rightarrow F(z) = z^{-2} \frac{z}{z-1} = \frac{3}{z(z-1)}$

c) $f(k) = 2p_p(k-3) \Rightarrow F(z) = \underline{\underline{2z^{-3}}}$

d) $f(k) = 1 - 0,3^k \Rightarrow F(z) = \frac{z}{z-1} - \frac{z}{z-0,3} =$

$$= \underline{\underline{\frac{0,7z}{(z-1)(z-0,3)}}}$$

(16.3)

$$f(k) = 2(s_e(k-1) - s_e(k-2) + s_e(k-4) - s_e(k-5) + \dots)$$

$$\Rightarrow F(z) = \frac{2z}{z-1} (z^{-1} - z^{-2} + z^{-4} - z^{-5} + z^{-7} - z^{-8} + \dots) =$$
$$= \frac{2z}{z-1} (z^{-1} + z^{-4} + z^{-7} + \dots) - \frac{2z}{z-1} (z^{-2} + z^{-5} + z^{-8} + \dots)$$

$$= \frac{2z}{z-1} \left(z^{-1} \sum_{k=0}^{\infty} (z^{-3})^k - z^{-2} \sum_{k=0}^{\infty} (z^{-3})^k \right) =$$

$$= \frac{2z}{z-1} \left(z^{-1} \frac{1}{1-z^{-3}} - z^{-2} \frac{1}{1-z^{-3}} \right) =$$

$$= \frac{2z}{z-1} \cdot \frac{z^{-1}}{1-z^{-3}} (1-z^{-1}) = \frac{2(1-z^{-1})}{(z-1)(1-z^{-3})} =$$

$$= \frac{2(1-z^{-1})}{z(1-z^{-1})(1-z^{-3})} = \frac{2z^{-1}}{1-z^{-3}}$$

(16.4)

a) $y(k) = 0,5 y(k-1) + 3u(k-1)$

$$\Rightarrow Y(z)(1-0,5z^{-1}) = 3z^{-1}U(z)$$

$$\Rightarrow H(z) = \frac{3z^{-1}}{1-0,5z^{-1}}$$

$$b) \quad y(k) = 0,8y(k-1) + 0,2y(k-2) + 0,5y(k-3) + u(k-1) + 2u(k-2)$$

$$\Rightarrow Y(z) (1 - 0,8z^{-1} - 0,2z^{-2} - 0,5z^{-3}) = U(z) (z^{-1} + 2z^{-2})$$

$$\Rightarrow H(z) = \frac{z^{-1} + 2z^{-2}}{1 - 0,8z^{-1} - 0,2z^{-2} - 0,5z^{-3}}$$

$$c) \quad y(k) = 0,7y(k-1) + 0,4y(k-2) + 0,7u(k-4) + u(k-5)$$

$$pss: H(z) = \frac{0,7z^{-4} + z^{-5}}{1 - 0,7z^{-1} + 0,4z^{-2}}$$

$$d) \quad y(k) = 2u(k-1) + 3u(k-2) + u(k-3)$$

$$pss: H(z) = 2z^{-1} + 3z^{-2} + z^{-3}$$

$$e) \quad y(k) + 0,8y(k-1) = 2u(k-1) + u(k-2)$$

$$pss: H(z) = \frac{2z^{-1} + z^{-2}}{1 + 0,8z^{-1}}$$

(16.5) a) $H(z) = \frac{z^{-1} + 2z^{-2}}{1 + 0,5z^{-1} + 0,2z^{-2}} = \frac{Y(z)}{U(z)}$

$\Rightarrow Y(z)(1 + 0,5z^{-1} + 0,2z^{-2}) = U(z)(z^{-1} + 2z^{-2})$

$\Rightarrow y(k) + 0,5y(k-1) + 0,2y(k-2) = u(k-1) + 2u(k-2)$
 $y(k) = -0,5y(k-1) - 0,2y(k-2) + u(k-1) + 2u(k-2)$

b) ej löst-

c) $H(z) = \frac{z + 0,7}{z^2 + 0,8z - 0,5}$; $z^{-2} \cdot H(z) = \frac{z^{-1} + 0,7z^{-2}}{1 + 0,8z^{-1} - 0,5z^{-2}}$

pss sam a) $\Rightarrow y(k) = -0,8y(k-1) + 0,5y(k-2) + u(k-1) + 0,7u(k-2)$

d-e) ej löst

f) $H(z) = \frac{Y(z)}{U(z)} = \frac{2z^{-1} + z^{-2} + 0,5z^{-3}}{1}$

pss sam a) $\Rightarrow y(k) = 2u(k-1) + u(k-2) + 0,5u(k-3)$

(16.6) a) $y(k) = y(k-1) + 2u(k-1) \Rightarrow Y(z) = z^{-1}Y(z) + 2z^{-1}U(z)$

$\Rightarrow H(z) = \frac{Y(z)}{U(z)} = \frac{2z^{-1}}{1 - z^{-1}}$

$Y(z) = H(z)U(z) = \frac{2z^{-1}}{1 - z^{-1}} \cdot \frac{1}{1 - z^{-1}} = \frac{2z^{-1}}{(1 - z^{-1})^2}$

$\Rightarrow y(k) = 2k$ (Ausramp), $k > 0$
 $S_p(z)$
 diskontinuierlicher
 Stufen

$$b) \quad y(k) = 0,5y(k-1) + 2u(k-1)$$

$$pss \Rightarrow H(z) = \frac{2z^{-1}}{1-0,5z^{-1}}$$

$$pss \quad Y(z) = \frac{2z^{-1}}{1-0,5z^{-1}} \cdot \frac{1}{1-z^{-1}} = \frac{2z}{(z-0,5)(z-1)}$$

$$\text{part. brøk oppdeln: } Y(z) = \frac{-2}{z-0,5} + \frac{4}{z-1} =$$

$$= -2z^{-1} \cdot \frac{z}{z-0,5} + 4z^{-1} \cdot \frac{z}{z-1}$$

$$\Rightarrow y(k) = -2(0,5)^{k-1} + 4 \cdot 1^{k-1} = 4(1 - 0,5(0,5)^{k-1}) =$$

$$= \underline{\underline{4(1-0,5^k)}}, \quad k > 0$$

$$c) \quad y(k) = 5u(k-3) \Rightarrow H(z) = 5z^{-3}$$

$$\Rightarrow Y(z) = 5z^{-3} \cdot \frac{1}{1-z^{-1}} = \frac{5z^{-3}}{1-z^{-1}}$$

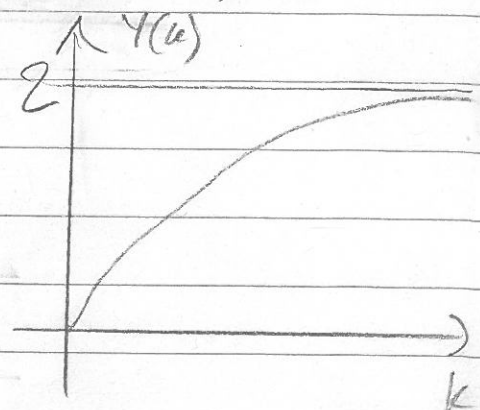
$$\Rightarrow y(k) = 5 \cdot 1^{k-3} = 5, \quad k > 2$$

(6,7) a)

$$y(k) = 0,5y(k-1) + u(k-1)$$

k	u(k-1)	y(k-1)	y(k)
0	0	0	0
1	1	0	1
2	1	1	1,5
3	1	1,5	1,75
4	1	1,75	1,875
5	1	1,875	1,9375
6	1	1,9375	1,9687
7	1	1,9687	1,9844

$$y(k) = 2(1-0,5^k)$$



16.7 b - 16.9 ej lösta