19.1 $\quad G(s) = \dfrac{3}{s}$

a) Euler $\Rightarrow H(z) = \dfrac{3}{\frac{z-1}{zh}} = [h = 1s] = \underline{\underline{\dfrac{3z}{z-1}}}$

b) bilineär $\Rightarrow H(z) = \dfrac{3}{\frac{2(z-1)}{h(z+1)}} = [h = 1s] = \underline{\underline{\dfrac{3z+3}{2z-2}}}$

19.2 $\quad G(s) = \dfrac{1+2s}{1+5s}$

a) Euler $\Rightarrow H(z) = \dfrac{1 + 2\frac{z-1}{zh}}{1 + 5\frac{z-1}{zh}} =$

$= \dfrac{zh+2z-2}{zh+5z-5}$ $\quad \begin{cases} h = 1s \Rightarrow H(z) = \dfrac{3z-2}{6z-5} \\ h = 0{,}1s \Rightarrow H(z) = \dfrac{2{,}1z-2}{5{,}1z-5} \end{cases}$

b) bilineär $\Rightarrow H(z) = \dfrac{1 + 2\frac{2(z-1)}{h(z+1)}}{1 + 5\frac{2(z-1)}{h(z+1)}} =$

$= \dfrac{hz + h + 4z - 4}{hz + h + 10z - 10}$ $\quad \begin{cases} h = 1s \Rightarrow H(z) = \dfrac{5z-3}{11z-9} \\ h = 0{,}1s \Rightarrow H(z) = \dfrac{4{,}1z-3{,}9}{10{,}1z-9{,}9} \end{cases}$

19.3 $G_R(s) = 4\left(1+\frac{1}{20s}\right)$

bilineär, $h=0{,}5\,s \Rightarrow$ sätt $s = 4\frac{z-1}{z+1}$

$$\Rightarrow H_R(z) = 4\left(1+\frac{1}{80\frac{z-1}{z+1}}\right) = 4\left(1+\frac{z+1}{80z-80}\right) =$$

$$= 4\,\frac{80z-80+z+1}{80z-80} = 4\,\frac{81z-79}{80z-80} = \frac{324z-316}{80z-80}$$

$$\Rightarrow U(z)(80z-80) = E(z)(324z-316)$$

$$U(z)(z-1) = E(z)\left(\frac{324}{80}z - \frac{316}{80}\right)$$

$$\Rightarrow u(k+1) = u(k) + 4{,}05\,e(k+1) - 3{,}95\,e(k)$$

19.4 $G(s) = \frac{1}{1+s}$ $\quad h = 1\,s$

a) Impulsinvariant, $H(z) = \mathcal{Z}\left(\mathcal{L}^{-1}(G(s))_{t=kh}\right)$

$$g(t) = e^{-t} \Rightarrow g(kh) = e^{-k} = \left(\frac{1}{e}\right)^k$$

$$\Rightarrow H(z) = \frac{z}{z-\frac{1}{e}} \approx \frac{z}{z-0{,}37}$$

b) Steginvariant, $H(z) = (1 - z^{-1}) Z\left(\mathcal{L}^{-1}\left(\frac{G(s)}{s}\right)_{t=kh}\right)$

$$\frac{G(s)}{s} = \frac{1}{s(s+1)} \Rightarrow \mathcal{L}^{-1}\left(\frac{G(s)}{s}\right) = 1 - e^{-t}$$

$$\Rightarrow \mathcal{L}^{-1}\left(\frac{G(s)}{s}\right)_{t=kh} = 1 - e^{-k}$$

$$\Rightarrow z\left(\mathcal{L}^{-1}\left(\frac{G(s)}{s}\right)_{t=kh}\right) = \frac{z}{z-1} - \frac{z}{z-\frac{1}{e}}$$

$$\Rightarrow H(z) = \frac{z-1}{z-1} - \frac{z-1}{z-\frac{1}{e}} = 1 - \frac{z-1}{z-\frac{1}{e}} =$$

$$= \frac{z - \frac{1}{e} - z + 1}{z - \frac{1}{e}} = \frac{1-\frac{1}{e}}{z-\frac{1}{e}} \approx \underline{\underline{\frac{0,63}{z-0,37}}}$$

c) Rampinvariant, $H(z) = \frac{(z-1)^2}{hz} Z\left(\mathcal{L}^{-1}\left(\frac{G(s)}{s^2}\right)_{t=kh}\right)$

$$\frac{G(s)}{s^2} = \frac{1}{s^2(1+s)} = \frac{As+B}{s^2} + \frac{C}{1+s} =$$

$$\left[\frac{As^2 + Bs + As + B + Cs^2}{s^2(1+s)}\right. = \frac{s^2(A+C) + s(A+B) + B}{s^2(1+s)}$$

$$\Rightarrow \left\{\begin{array}{l} B = 1 \\ A+B=0;\ A=-1 \\ A+C=0;\ C=1 \end{array}\right] = \frac{-s+1}{s^2} + \frac{1}{1+s} = -\frac{1}{s} + \frac{1}{s^2} + \frac{1}{1+s}$$

$$\Rightarrow \mathcal{L}^{-1}\left(\frac{G(s)}{s^2}\right) = -1 + t + e^{-t}$$

$$\Rightarrow \mathcal{L}^{-1}\left(\frac{G(s)}{s^2}\right)_{t=kh} = -1 + k + e^{-k}$$

$$\Rightarrow Z\left(\mathcal{L}^{-1}\left(\frac{G(s)}{s^2}\right)_{t=kh}\right) = -\frac{z}{z-1} + \frac{z}{(z-1)^2} + \frac{z}{z-\frac{1}{e}}$$

$$\Rightarrow H(z) = \frac{(z-1)^2}{z}\left(-\frac{z}{z-1} + \frac{z}{(z-1)^2} + \frac{z}{z-\frac{1}{e}}\right) =$$

$$= -(z-1) + 1 + \frac{(z-1)^2}{z-\frac{1}{e}} =$$

$$= \frac{(1-z)(z-\frac{1}{e}) + z - \frac{1}{e} + z^2 - 2z + 1}{z-\frac{1}{e}} =$$

$$= \frac{\cancel{z} - \cancel{z^2} - \frac{1}{e} + z\frac{1}{e} + \cancel{z} - \frac{1}{e} + \cancel{z^2} - \cancel{2z} + 1}{z-\frac{1}{e}} =$$

$$= \frac{z\frac{1}{e} + 1 - \frac{2}{e}}{z-\frac{1}{e}} \approx \frac{0{,}37z + 0{,}26}{z - 0{,}37}$$

d) bilineär $\Rightarrow H(z) = G(s)_{s=\frac{2}{h}\frac{z-1}{z+1}}$

$$\Rightarrow H(z) = \frac{1}{1+2\frac{z-1}{z+1}} = \frac{z+1}{z+1+2z-2} = \frac{z+1}{3z-1}$$

e) $K_{LF} = H(1)$

Impulsinvariant: $K_{LF} \approx \frac{1}{1-0{,}37} \approx 1{,}6$

Steginvariant: $K_{LF} \approx \frac{0{,}63}{1-0{,}37} = 1{,}0$

Rampinvariant: $K_{LF} \approx \frac{0{,}37+0{,}26}{1-0{,}37} = 1{,}0$

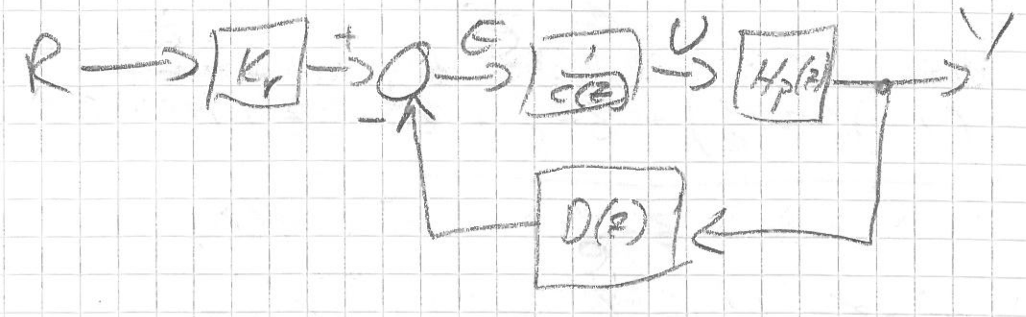
Bilineär: $K_{LF} = \frac{1+1}{3-1} = 1{,}0$

Impulsinvariant ger en dålig approx. av K_{LF}

19.5

$$H_p(z) = \frac{0{,}65z^{-1} + 0{,}35z^{-2}}{1 - 0{,}75z^{-1}} = \frac{B(z)}{A(z)}$$

H_{tot} ska ha dubbelpol i $z=0{,}5$



$$H_{tot} = K_r \frac{\frac{B}{AC}}{1 + \frac{BD}{AC}} = \frac{B}{AC + BD}$$

Dubbelpol i $z=0{,}5 \Rightarrow AC + BD = (0{,}5z^{-1} - 1)^2$

$$(1 - 0{,}75z^{-1})(1 + c_1 z^{-1}) + (0{,}65z^{-1} + 0{,}35z^{-2})(d_0) =$$

$$1 + c_1 z^{-1} - 0{,}75z^{-1} - 0{,}75c_1 z^{-2} + 0{,}65d_0 z^{-1} + 0{,}35d_0 z^{-2} =$$

$$= 0{,}25z^{-2} - z^{-1} + 1 \Rightarrow \begin{cases} -0{,}75c_1 + 0{,}35d_0 = 0{,}25 \; (1) \\ c_1 - 0{,}75 + 0{,}65d_0 = -1 \; (2) \\ 1 = 1 \end{cases}$$

$$(1) \Rightarrow 0{,}35d_0 = 0{,}25 + 0{,}75c_1 \; ; \; d_0 = \frac{0{,}25}{0{,}35} + \frac{0{,}75}{0{,}35}c_1 \; (1')$$

$$(1') \text{ i } (2) \Rightarrow c_1 - 0{,}75 + \frac{0{,}65 \cdot 0{,}25}{0{,}35} + \frac{0{,}65 \cdot 0{,}75}{0{,}35}c_1 = -1$$

$$c_1\left(1 + \frac{0{,}65 \cdot 0{,}75}{0{,}35}\right) = -1 + 0{,}75 - \frac{0{,}65 \cdot 0{,}25}{0{,}35} \; ; \; c_1 \approx 0{,}2985 \; (3)$$

$$(3) \,\&\, (1') \Rightarrow d_0 = \frac{0{,}25}{0{,}35} + \frac{0{,}75}{0{,}35} \cdot 0{,}2985 \approx 0{,}0746$$

$$\Rightarrow \underline{\underline{C(z) = 1 - 0{,}30z^{-1}}}, \; \underline{\underline{D(z) = 0{,}075}}$$

$$K_r = \frac{P(1)}{B(1)} = \frac{(0,5-1)^2}{0,65+0,35} = \underline{\underline{0,25}}$$

b) $E = RK_r - YD$, $U = \frac{E}{C}$

$E = 0,25R - 0,075Y$, $U = \frac{EZ}{Z-0,30}$

$$\underline{\underline{\begin{cases} e(k) = 0,25r(k) - 0,075y(k) \\ u(k+1) = 0,30u(k) + e(k+1) \end{cases}}}$$

19.6 a) $y(k) = 0,6y(k-1) + 0,3u(k-1) + 0,7u(k-2)$

$$H_p(z) = \frac{0,3z+0,7}{z^2-0,6z} = \frac{0,3z^{-1}+0,7z^{-2}}{1-0,6z^{-1}}$$

dubbelpol; $z = 0,3 \Rightarrow z-0,3 = 0$; $z = 0,3$; $z^{-1} = \frac{1}{0,3}$

$0,3z^{-1} = 1 \Rightarrow P(z) = (0,3z^{-1}-1)^2$

$AC + BD = P$ (se 19.5) $\Rightarrow$

$(1-0,6z^{-1})(1+c_1z^{-1}) + (0,3z^{-1}+0,7z^{-2})d_0 = (0,3z^{-1}-1)^2$

$1 - 0,6z^{-1} + c_1z^{-1} - 0,67c_1z^{-2} + 0,3d_0z^{-1} + 0,7d_0z^{-2} =$
$= 0,09z^{-2} - 0,6z^{-1} + 1$

$$\Rightarrow \begin{cases} -0,6 + c_1 + 0,3d_0 = -0,6 \\ -0,67c_1 + 0,7d_0 = 0,09 \end{cases} ; \begin{cases} c_1 + 0,3d_0 = 0 \\ -0,67c_1 + 0,7d_0 = 0,09 \end{cases}$$

MatLab $\Rightarrow$ $\underline{\underline{c_1 \approx -0,030}}$, $\underline{\underline{d_0 \approx 0,10}}$
$(A\backslash Y)$

$$K_r = \frac{P(1)}{B(1)} = \frac{(0{,}3-1)^2}{0{,}3+0{,}7} \approx 0{,}49$$

$$\Rightarrow \underline{C(z) = 1 - 0{,}030z^{-1} \quad D(z) = 0{,}10 \quad K_r = 0{,}49}$$

b) $$H_p = \frac{B}{A} \Rightarrow H_{tot} = K_r \frac{\frac{B}{AC}}{1 + \frac{BD}{AC}} = \frac{K_r B}{AC+BD} =$$

$$= \frac{0{,}49(0{,}3z^{-1} + 0{,}7z^{-2})}{(1-0{,}6z^{-1})(1-0{,}030z^{-1}) + 0{,}10(0{,}3z^{-1}+0{,}7z^{-2})}$$

Sage
(expr.simplify_full()) $\Rightarrow$ $$H_{tot} = \frac{0{,}147z + 0{,}343}{z^2 - 0{,}6z + 0{,}088}$$

19.7 a) $$H_p = \frac{B}{A} = \frac{0{,}7}{z-0{,}8} = \frac{0{,}7z^{-1}}{1-0{,}8z^{-1}}$$

antal poler $n_p = n_a + b_b - 1 = 1+1-1 = 1$

pol i $z = 0{,}3 \Rightarrow z^{-1} = \frac{1}{0{,}3}$; $0{,}3z^{-1} = 1$; $P(z) = 0{,}3z^{-1} - 1$

$$AC + BD = P \Rightarrow (1-0{,}8z^{-1}) \cdot 1 + 0{,}7z^{-1} \cdot d_0 = -0{,}3z^{-1} + 1$$

$$\Rightarrow -0{,}8 + 0{,}7d_0 = -0{,}3 ; \; d_0 = \frac{-0{,}3+0{,}8}{0{,}7} \approx 0{,}71$$

$$\Rightarrow \underline{C(z) = 1}, \; \underline{D(z) = 0{,}71}$$

$$K_r = \frac{P(1)}{B(1)} = \frac{0{,}3-1}{0{,}7} = \underline{1{,}0}$$

b) Integralverkan $\Rightarrow A^* = A(1-z^{-1})$

$$\Rightarrow H_{TOT} = \frac{B}{A^*C+BD} = \frac{B}{AC(1-z^{-1})+BD}$$

Placera båda polerna i $z=0,3$

$$\Rightarrow P(z) = (1-0,3z^{-1})^2$$

$$n_c = n_b - 1 = 0 \Rightarrow C(z) = 1$$

$$n_d = n_a - 1 = 1 \Rightarrow D(z) = d_0 - d_1 z^{-1}$$

Polplaceringsekv, $A^*C+BD = P$:

$$(1-0,8z^{-1})(1-z^{-1}) + 0,7z^{-1}(d_0 - d_1 z^{-1}) = (1-0,3z^{-1})^2$$

$$1-0,8z^{-1} - z^{-1} + 0,8z^{-2} + 0,7d_0 z^{-1} - 0,7d_1 z^{-2} =$$

$$= 1 - 0,6z^{-1} + 0,09z^{-2}$$

$$\Rightarrow \begin{cases} -0,8-1+0,7d_0 = -0,6 \\ 0,8-0,7d_1 = 0,09 \end{cases} ; \begin{cases} d_0 \approx 1,7 \\ d_1 \approx 1,0 \end{cases}$$

$$\Rightarrow \underline{\underline{C(z) = 1,\ D(z) = 1,7 - 1,0z^{-1}}}$$

$$K_r = \frac{P(1)}{B(1)} = \frac{(1-0,3)^2}{0,7} = \underline{\underline{0,7}}$$

(19.8) $y(k+1) = 0{,}6\,y(k) + 2u(k)$

$$\Rightarrow H_p(z) = \frac{2}{z-0{,}6} = \frac{2z^{-1}}{1-0{,}6z^{-1}}$$

Inget kvarst. fel $\Rightarrow$ integrering

$$\Rightarrow H_p^*(z) = \frac{2z^{-1}}{(1-0{,}6z^{-1})(1-z^{-1})} = \frac{B(z)}{A^*(z)}$$

$$H_{TOT} = \frac{B}{A^*C+BD}$$

antal poler, $n_p = n_a^* + n_b - 1 = 2+1-1 = 2$

både polerna i $z = 0{,}2 \Rightarrow P(z) = (1-0{,}2z^{-1})^2$

$n_c = n_b - 1 = 1-1 = 0 \Rightarrow C(z) = 1$ (eller $1-z^{-1}$ om integr. tas med)

$n_d = n_a^* - 1 = 2-1 = 1 \Rightarrow D(z) = d_0 + d_1 z^{-1}$

polplaceringsekv: $(1-0{,}6z^{-1})(1-z^{-1}) + 2z^{-1}(d_0 + d_1 z^{-1}) = (1-0{,}2z^{-1})^2$

$1-0{,}6z^{-1} - z^{-1} + 0{,}6z^{-2} + 2d_0 z^{-1} + 2d_1 z^{-2} = 1 - 0{,}4z^{-1} + 0{,}04z^{-2}$

$$\Rightarrow \begin{cases} -0{,}6-1+2d_0 = -0{,}4 \\ 0{,}6+2d_1 = 0{,}04 \end{cases} ; \begin{cases} d_0 = 0{,}6 \\ d_1 = -0{,}28 \end{cases}$$

$$\Rightarrow \underline{\underline{C(z) = 1-z^{-1},\ D(z) = 0{,}6-0{,}28z^{-1}}}$$

$$K_r = \frac{P(1)}{B(1)} = \frac{(1-0{,}2)^2}{2} = \underline{\underline{0{,}32}}$$

(19.9)

alla poler: $z=0$, $h=1s$,
inga kvarst. fel på stegf. störn
$\Rightarrow$ integrering

fördröjn. = 1s
tidskonst: $0{,}63 \cdot y_{ss} = 0{,}63 \cdot 3 \approx 1{,}9$
(s. 111)
$y = 1{,}9 \Rightarrow t \approx 3s$
1s dödtid $\Rightarrow$ Tidskonst, $T = 3-1 = 2$
$K = y_{ss} = 3$

$$\Rightarrow G_p(s) = \frac{Ke^{-Ls}}{1+Ts} = \frac{3e^{-s}}{1+2s}$$

Diskretisering enl tabell s314 (dvs steg-invariant transform)

$$\Rightarrow H_p(z) = z^{-1} \cdot \frac{3(1-e^{-\frac{1}{2}})}{z-e^{-\frac{1}{2}}} = \frac{3(1-e^{-\frac{1}{2}})z^{-2}}{1-e^{-\frac{1}{2}}z^{-1}}$$

$$H(z) \approx \frac{1{,}2z^{-2}}{1-0{,}61z^{-1}}$$

$$\Rightarrow H^*(z) = \frac{1{,}2z^{-2}}{(1-z^{-1})(1-0{,}61z^{-1})} = \frac{B(z)}{A^*(z)}$$

antal poler, $n_p = n_a + n_b - 1 = 2+2-1 = 3$

$n_c = n_b - 1 = 2-1 = 1 \Rightarrow C(z) = (1+c_1 z^{-1})\underbrace{(1-z^{-1})}_{\text{integr.}}$

$n_d = n_a - 1 = 2-1 = 1 \Rightarrow D(z) = d_0 + d_1 z^{-1}$

tre poler i origo $\Rightarrow P(z) = (1-0\cdot z^{-1})^3 = 1$

polplaceringsekv: $A^*C + BD = P$

eftersom A^*C+BD har högsta koeff z^{-3} betyder 1 egentligen z^3, dvs tre poler

$$(1-z^{-1})(1-0{,}61z^{-1})(1+c_1z^{-1})+1{,}2z^{-2}(d_0+d_1z^{-1})=1$$

$$(1-z^{-1}-0{,}61z^{-1}+0{,}61z^{-2})(1+c_1z^{-1})+1{,}2d_0z^{-2}+1{,}2d_1z^{-3}=1$$

$$1-z^{-1}-0{,}61z^{-1}+0{,}61z^{-2}+c_1z^{-1}-c_1z^{-2}-0{,}61c_1z^{-2}+0{,}61c_1z^{-3}+$$
$$+1{,}2d_0z^{-2}+1{,}2d_1z^{-3}=1$$

$$\begin{cases}-1-0{,}61+c_1=0\\ 0{,}61-c_1-0{,}61c_1+1{,}2d_0=0\\ 0{,}61c_1+1{,}2d_1=0\end{cases};\quad \begin{cases}c_1=1{,}61\\ -1{,}61c_1+1{,}2d_0=-0{,}61\\ 0{,}61c_1+1{,}2d_1=0\end{cases}$$

Matlab $A\backslash B \Rightarrow c_1 \approx 1{,}61,\ d_0\approx 1{,}65,\ d_1\approx -0{,}82$

$$\Rightarrow C(z)=(1+1{,}61z^{-1})(1-z^{-1})$$
$$\underline{\underline{D(z)=1{,}65-0{,}82z^{-1}}}$$

$$K_r=\frac{P(1)}{B(1)}=\frac{1}{1{,}2}\approx\underline{\underline{0{,}83}}$$

(19,10) $G_p(s)=\dfrac{K}{s(s+a)}$

inte integrering
Alla poler i origo
3 poler

a) $G_p(s)=\dfrac{K}{s(s+a)}=\dfrac{\frac{K}{a}}{s(1+\frac{1}{a}s)}$

Tabell s 314 $\Rightarrow H(z)=\dfrac{B(z)}{A(z)}=\dfrac{K}{a}\cdot\dfrac{1}{a}\,\dfrac{(e^{-ah}-1+ah)z^{-1}+(1-e^{-ah}(1+ah))z^{-2}}{1-(1+e^{-ah})z^{-1}+e^{-ah}z^{-2}}$

ekv(1)

$$\frac{U}{R}=K_r\,\frac{\frac{1}{C}}{1+\frac{1}{C}\cdot\frac{B}{A}\cdot D}=\frac{K_rA}{AC+BD};\quad U=\frac{K_rA}{AC+BD}R$$

$$R=\frac{1}{1-z^{-1}}\Rightarrow U=\frac{K_rA}{(AC+BD)(1-z^{-1})}$$

Alla poler i origo $\Rightarrow P(z) \equiv 1$, dvs $AC + BD \equiv 1$

$$\Rightarrow U = \frac{K_r A}{1 - z^{-1}}$$

u_{max} inträffar vid första samplet. Då är alla z^{-x}-termer $= 0$. Enligt (1) är då $A = 1$

$$K_r = \frac{P(1)}{B(1)} = \frac{1}{\frac{K}{a^2}\left(\cancel{e^{-ah}} \cancel{-1} + ah \cancel{+1} \cancel{-e^{-ah}} - ahe^{-ah}\right)} =$$

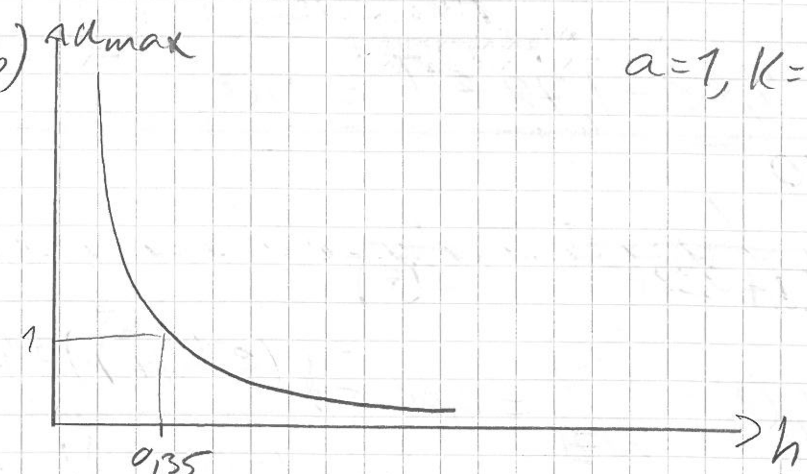
$$= \frac{1}{\frac{K}{a^2} \cdot ah\left(1 - e^{-ah}\right)} = \frac{1}{\frac{Kh}{a}\left(1 - e^{-ah}\right)} = \frac{a}{Kh(1 - e^{-ah})}$$

$$\Rightarrow U_{max} = K_r \Rightarrow u_{max} = K_r$$

$$u_{max} = \frac{a}{Kh(1 - e^{-ah})}$$

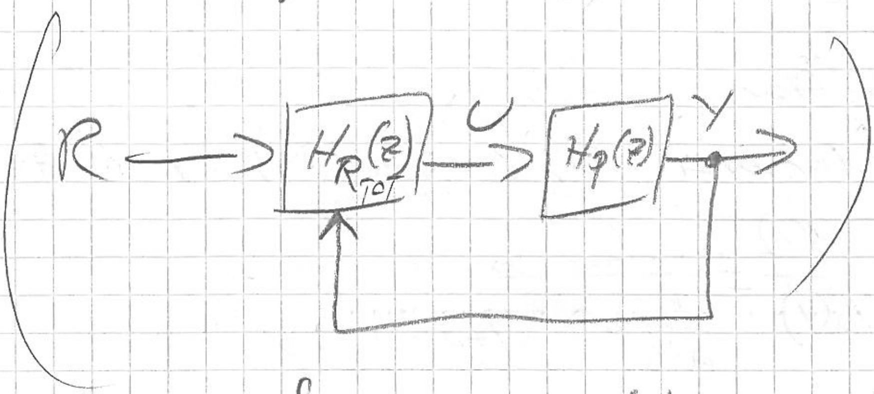
b)

$$a = 1,\ K = 10 \Rightarrow u_{max} = \frac{1}{10h(1 - e^{-h})}$$



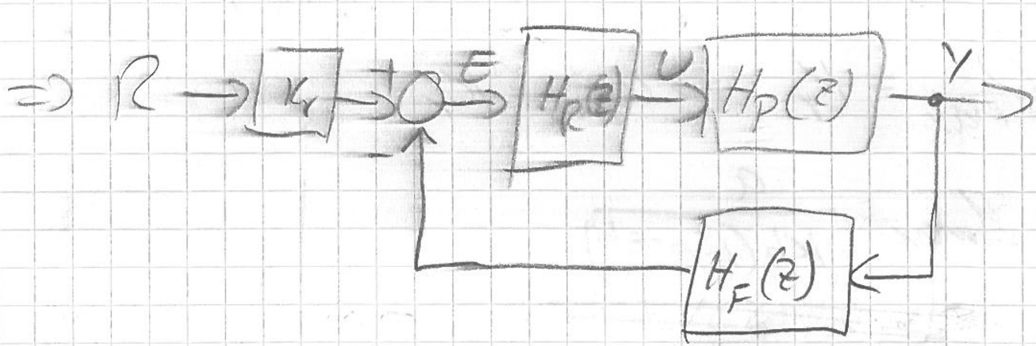
(19.11) $G_p(s) = \dfrac{Y}{U} = \dfrac{0{,}05}{s(1+5s)}$ $\quad h = 5s$

U styckvis konst $\Rightarrow$ använd steginv. diskretisering



$$H_R(z): \begin{cases} f(k) = 4{,}88y(k) - 1{,}78y(k-1) \\ e(k) = 3{,}10r(k) - f(k) \\ u(k) = e(k) - 0{,}32u(k-1) \end{cases}$$

$\Rightarrow$



$H_F(z) = 4{,}88 - 1{,}78z^{-1} = D(z)$

$K_r = 3{,}10$

$H_R(z) = \dfrac{1}{1+0{,}32z^{-1}} = \dfrac{1}{C(z)}$

Tab s. 516 $\Rightarrow H_p(z) = \dfrac{B(z)}{A(z)} = 0{,}05 \cdot 5 \cdot \dfrac{(e^{-1} - \cancel{1+1})z^{-1} + (1 - e^{-1}(1+1))z^{-2}}{1-(1+e^{-1})z^{-1} + e^{-1}z^{-2}}$

$\approx \dfrac{0{,}092z^{-1} + 0{,}0661z^{-2}}{1 - 1{,}3679z^{-1} + 0{,}3679z^{-2}}$

Kar. ekv.: $AC + BD = 0$:

$(1 - 1{,}3679z^{-1} + 0{,}3679z^{-2})(1 + 0{,}32z^{-1}) +$

$+ (0{,}092z^{-1} + 0{,}0661z^{-2})(4{,}88 - 1{,}78z^{-1}) = 0$

$$1-0{,}5989z^{-1}+0{,}089z^{-2}+0{,}00007z^{-3}=0$$

stämmer med kar. ekv. i facit

Lösn. ger alltså två poler i 0,3 och en i origo

b) $$Y=\frac{K_r B}{AC+BD}R\approx$$

$$\approx\frac{3{,}10(0{,}092z^{-1}+0{,}0661z^{-2})}{1-0{,}5989z^{-1}+0{,}089z^{-2}+0{,}00007z^{-3}}R\approx$$

$$\approx\frac{0{,}2852z^{-1}+0{,}2049z^{-2}}{\ldots}R$$

$$\Rightarrow y(k)\approx0{,}60y(k-1)-0{,}090y(k-2)+0{,}000070y(k-3)+$$
$$+0{,}29r(k-1)+0{,}20r(k-2)$$

$$k\to\infty\Rightarrow y(k)=K_{LF}=H_{TOT}(1)=\frac{0{,}2852+0{,}2049}{1-0{,}5989+0{,}089+0{,}00007}$$

$$\approx 1{,}0$$

19.12 a) $$\ddot{\varphi}=\frac{1}{J}\cdot M \quad \text{Laplace}\Rightarrow s^2\varphi=\frac{1}{J}M$$

$$\Rightarrow G(s)=\frac{1}{Js^2}=\frac{1}{1250s^2}$$

b) s. 516 $$\Rightarrow H(z)=\frac{5^2}{2\cdot1250}\cdot\frac{z^{-1}+z^{-2}}{1-2z^{-1}+z^{-2}}=$$

$$=10^{-2}\frac{z^{-1}+z^{-2}}{1-2z^{-1}+z^{-2}}$$

c) $H_{TOT}(z) = K_r \dfrac{B}{AC+BD}$ Alla poler i origo $\Rightarrow P(z)=1$

$n_p = n_a + n_b - 1 = 2+2-1=3$

$n_c = n_a - 1 = 2-1 = 1 \Rightarrow C(z) = 1 + c_1 z^{-1}$

$n_d = n_b - 1 = 2-1=1 \Rightarrow D(z) = d_0 + d_1 z^{-1}$

polplaceringsekv: $(1-2z^{-1}+z^{-2})(1+c_1 z^{-1}) + 10^{-2}(z^{-1}+z^{-2})(d_0+d_1 z^{-1}) = 1$

$1-2z^{-1}+z^{-2}+c_1 z^{-1} - 2c_1 z^{-2} + c_1 z^{-3} + 10^{-2}d_0 z^{-1} + 10^{-2}d_0 z^{-2} + 10^{-2}d_1 z^{-2} + 10^{-2}d_1 z^{-3} = 1$

$$\Rightarrow \begin{cases} z^{-1}: -2 + c_1 + 10^{-2}d_0 = 0 & (1) \\ z^{-2}: 1 - 2c_1 + 10^{-2}d_0 + 10^{-2}d_1 = 0 & (2) \\ z^{-3}: c_1 + 10^{-2}d_1 = 0 & (3) \end{cases}$$

$(1) \Rightarrow d_0 = 10^2(2-c_1)$ $(1')$ $\quad (3) \Rightarrow d_1 = -10^2 c_1$ $(3')$

$(1')$ & $(3')$ i $(2) \Rightarrow 1-2c_1 + 10^{-2}\cdot 10^2 \cdot (2-c_1) + 10^{-2}\cdot(-10^{-2}c_1) = 0$

$1-2c_1+2-c_1-c_1 = 0;\ -4c_1 = -3;\ c_1 = 0{,}75$ (4)

(4) i $(1) \Rightarrow -2+0{,}75+10^{-2}d_0 = 0;\ 10^{-2}d_0 = 1{,}25;\ \underline{d_0 = 125}$

(4) i $(3) \Rightarrow 0{,}75 + 10^{-2}d_1 = 0;\ \underline{d_1 = -75}$

$K_r = \dfrac{P(1)}{B(1)} = \dfrac{1}{10^{-2}(1+1)} = 10^2 \cdot 0{,}5 = 50$

<u>Svar:</u> $K_r = 50$, $C(z) = 1+0{,}75z^{-1}$, $D(z) = 125 - 75z^{-1}$

d) $\dfrac{\psi}{\psi_{ref}} = H_{TOT}(z) = K_r \dfrac{B}{AC+BD} =$

$= 50 \dfrac{10^{-2}(z^{-1}+z^{-2})}{(1-2z^{-1}+z^{-2})(1+0{,}75z^{-1}) + 10^{-2}(z^{-1}+z^{-2})(125-75z^{-1})} =$

$$= \frac{0{,}5z^{-1}+0{,}5z^{-2}}{1-2z^{-1}+z^{-2}+0{,}75z^{-1}-1{,}5z^{-2}+0{,}75z^{-3}+1{,}25z^{-1}+1{,}25z^{-2}-0{,}75z^{-2}-0{,}75z^{-3}} =$$

$$= \frac{0{,}5z^{-1}+0{,}5z^{-2}}{1+z^{-1}(-2+0{,}75+1{,}25)+z^{-2}(1-1{,}5+1{,}25-0{,}75)+z^{-3}(0{,}75-0{,}75)}$$

$$= \frac{0{,}5z^{-1}+0{,}5z^{-2}}{1} \leftarrow \text{stämmer!} \quad = 0{,}5z^{-1}+0{,}5z^{-2}$$

$$\Rightarrow \psi = (0{,}5z^{-1}+0{,}5z^{-2})\psi_{ref} \Rightarrow \underline{\underline{\psi(k) = 0{,}5\psi_{ref}(k-1)+0{,}5\psi_{ref}(k-2)}}$$

$$\frac{M}{\psi_{ref}} = K_r \frac{\frac{1}{C}}{1+\frac{1}{C}\cdot\frac{B}{A}\cdot D} = K_r \frac{A}{AC+BD} = [AC+BD=1] =$$

$$= K_r A = 50(1-2z^{-1}+z^{-2}) = 50-100z^{-1}+50z^{-2}$$

$$\Rightarrow M = (50-100z^{-1}+50z^{-2})\psi_{ref}$$

$$\Rightarrow \underline{\underline{M(k) = 50\psi_{ref}(k)-100\psi_{ref}(k-1)+50\psi_{ref}(k-2)}}$$

e)

K	ψ_{ref} [rad]	M [Nm]	ψ [rad]
1	0,1745	8,7	0
2	0,1745	−8,7	0,087
3	0,1745	0	0,1745
4	0,1745	0	0,1745
5	0,1745	0	0,1745

formler i svaren (d) ⇒

$F = ma$

$N = \frac{kgm}{s^2}$

$\ddot{\psi} = \frac{M}{J}$

$\frac{1}{s^2} = \frac{kgm^2}{\cancel{kgm^2}s^2}$

OK!

19.13

$$G_p = \frac{K_p}{1+as} \qquad G_R = K_R\left(1+\frac{1}{Ts}\right) = K_R\frac{1+Ts}{Ts}$$

$$G_{TOT} = \frac{\frac{K_pK_R(1+Ts)}{Ts(1+as)}}{1+\frac{K_pK_R(1+Ts)}{Ts(1+as)}} = \frac{K_pK_R(1+Ts)}{Ts(1+as)+K_pK_R(1+Ts)} =$$

$$= \frac{K_pK_R(1+Ts)}{Ts+Tas^2+K_pK_R+K_pK_RTs} = \frac{K_pK_R+K_pK_RTs}{K_pK_R+s(T+K_pK_RT)+Tas^2}$$

dubbelpol i $s=-1 \Rightarrow P(s) = (s+1)^2 = s^2+2s+1$

polplaceringsekv: $K_pK_R+s(T+K_pK_RT)+s^2\cdot aT = s^2+2s+1$

$$\Rightarrow \begin{cases} K_pK_R = 1 \\ T+K_pK_RT = 2 \\ aT = 1 \end{cases} \quad \begin{cases} K_p = 3 \\ a = 2 \end{cases} \Rightarrow \begin{cases} 3K_R = 1 \\ T+3K_RT = 2 \\ 2T = 1 \end{cases}$$ Saknar lösning

Sätt $P'(s) = 3P(s) \Rightarrow$
(För att K_pK_R ska bli 1) vilket eliminerar en (löser) av de tre ekv.

$$\begin{cases} 3K_R = 3 & (1) \\ T+3K_RT = 6 & (2) \\ 2T = 3 & (3) \end{cases}$$

$(1) \Rightarrow \underline{K_R = 1}$ $(1')$ $(3) \Rightarrow \underline{T = 1{,}5}$ $(3')$

$(1')$ & $(3')$ i $(2) \Rightarrow 1{,}5+3\cdot 1\cdot 1{,}5 = 6$ OK!

Svar: $G_R = 1+\frac{1}{1{,}5s} \Rightarrow G_{TOT} = \frac{3(3+4{,}5s)}{3+6s+3s^2}$

(9.14) a) För att utvärdet ska vara lika med referensvärdet

b) Den ger snabbast möjliga insvängn.-förlopp.

+: snabb , -: stora styrsignaler

c) ss-Felet vid stegformade störningar elimineras

d) +: Ofta bättre egenskaper, speciellt vid svårreglerade processer som t.ex sådana med lång dödtid.

Enkel att kombinera med param. identifiering, vilken ger systemets diskreta överf.funk.

Lämpar sig väl för datorstödd reglering

-: Svårt bestämma polernas placering

Svårt få tag i enkelt regulatorer

(9.15) a) Olika regulatorer används för olika reglerområden. Reglerområdet väljs m.h.a en styrande parameter

b) Uppskattn. av $H_p(z)$ beräknas om efter varje sampel. Utifrån $H_p(z)$ beräknas även $H_R(z)$ efter varje sampel

c) +: Styrande parameter ej nödvändig

Färre beräkningar

-: Långsam