14.1 a) $y''' + 5y'' + 3y' + 7y = 5u$

Laplace $\Rightarrow$ $Ys^3 + 5Ys^2 + 3Ys + 7Y = 5U$

$$G(s) = \frac{Y}{U} = \frac{5}{s^3 + 5s^2 + 3s + 7}$$

Styrbarkanonisk form:

$$A = \begin{pmatrix} -5 & -3 & -7 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \quad B = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad C = \begin{pmatrix} 0 & 0 & 5 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} x_1' \\ x_2' \\ x_3' \end{pmatrix} = \begin{pmatrix} -5 & -3 & -7 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} u$$

$$y = \begin{pmatrix} 0 & 0 & 5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

b) $\dddot{y} + 2\ddot{y} + 4\dot{y} + 5y = 9\ddot{u} + \dot{u} + 2u$

pss som a

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} -2 & -4 & -5 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} u, \quad y = \begin{pmatrix} 9 & 1 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

c-e ej lösta

$$u \rightarrow \boxed{\frac{2,5}{1+1,5s}} \xrightarrow{v} \boxed{\frac{3}{1+5s}} \rightarrow y$$

14.2

a) $x_1 = v, \; x_2 = y$

$$1,5x_1' + x_1 = 2,5u, \quad 5x_2' + x_2 = 3x_1$$

$$\Downarrow \qquad\qquad\qquad \Downarrow$$

$$x_1' + \frac{1}{1,5}x_1 = \frac{2,5}{1,5}u, \quad x_2' + \frac{1}{5}x_2 = \frac{3}{5}x_1$$

$$\Rightarrow \underline{\underline{\begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = \begin{pmatrix} -\frac{2}{3} & 0 \\ \frac{3}{5} & -\frac{1}{5} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} \frac{5}{3} \\ 0 \end{pmatrix}u, \quad y = \begin{pmatrix} 0 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}}}$$

b) $$G_{TOT} = \frac{2,5 \cdot 3}{(1+1,5s)(1+5s)} = \frac{7,5}{1+1,5s+5s+7,5s^2} =$$

$$= \frac{7,5}{7,5s^2+6,5s+1} = \frac{1}{s^2+\frac{6,5}{7,5}s+\frac{1}{7,5}}$$

$$\Rightarrow \underline{\underline{\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} -\frac{13}{15} & -\frac{2}{15} \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix}u, \quad y = \begin{pmatrix} 0 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}}}$$

c) $$G_{TOT} = [\text{enl. b)}] = \frac{1}{s^2+\frac{13}{15}s+\frac{2}{15}}$$

<u>part bråk uppdeln</u>

$$s^2 + \frac{13}{15}s + \frac{2}{15} = 0 \Rightarrow s = -\frac{13}{30} \pm \sqrt{\frac{169}{900} - \frac{120}{900}} =$$

$$= -\frac{13}{30} \pm \sqrt{\frac{49}{900}} = -\frac{13}{30} \pm \frac{7}{30} = \begin{cases} -\frac{6}{30} = -\frac{1}{5} \\ -\frac{20}{30} = -\frac{2}{3} \end{cases}$$

$$\Rightarrow G_{TOT} = \frac{A}{s+\frac{1}{5}} + \frac{B}{s+\frac{2}{3}}$$

Handpåläggn. $\Rightarrow A = \frac{1}{-\frac{1}{5}+\frac{2}{3}} = \frac{1}{\frac{-3+10}{15}} = \frac{15}{7}$

$$B = \frac{1}{-\frac{2}{3}+\frac{1}{5}} = \frac{1}{\frac{-10+3}{15}} = -\frac{15}{7}$$

$$\Rightarrow G_{TOT} = \frac{\frac{15}{7}}{s+\frac{1}{5}} + \frac{-\frac{15}{7}}{s+\frac{2}{3}}$$

$$\Rightarrow \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} -\frac{1}{5} & 0 \\ 0 & -\frac{2}{3} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} \frac{15}{7} \\ -\frac{15}{7} \end{pmatrix} u \;,\; y = \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

14,3 $\quad G(s) = C\,(s\cdot \underline{I} - A)^{-1}\cdot B$

a) $\dot{x} = \begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix} x + \begin{pmatrix} 0 \\ 2 \end{pmatrix} u \;,\; y = \begin{pmatrix} 1 & 1 \end{pmatrix} x$

$$s\underline{I} - A = \begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix} - \begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} s-3 & -1 \\ -1 & s-2 \end{pmatrix}$$

$$(s\underline{I} - A)^{-1} = \frac{1}{\det(\;)}\cdot \mathrm{adj}(\;) = \frac{1}{(s-3)(s-2)-1}\cdot \begin{pmatrix} s-2 & 1 \\ 1 & s-3 \end{pmatrix} =$$

$$= \frac{1}{s^2-5s+5} \begin{pmatrix} s-2 & 1 \\ 1 & s-3 \end{pmatrix}$$

$$C\,(s\underline{I} - A)^{-1} = \frac{1}{s^2-5s+5} \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} s-2 & 1 \\ 1 & s-3 \end{pmatrix} = \frac{1}{s^2-5s+5} \begin{pmatrix} s-1 & s-2 \end{pmatrix}$$

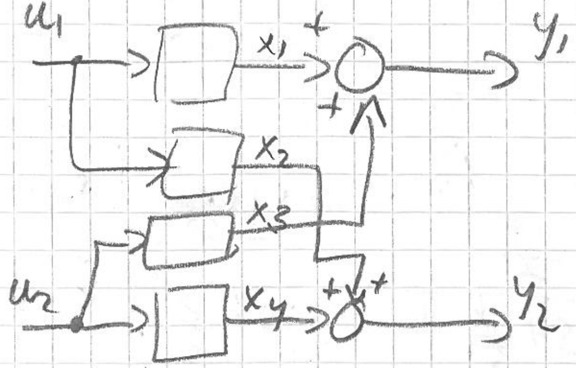
$$G(s) = C(s\underline{I} - A)^{-1} B = \frac{1}{s^2-5s+5} \begin{pmatrix} s-1 & s-2 \end{pmatrix} \begin{pmatrix} 0 \\ 2 \end{pmatrix} =$$

$$= \underline{\underline{\frac{2s-4}{s^2-5s+5}}}$$

b) ej löst

14.4

a)



$$\begin{cases} x_1 = \frac{1}{2s+1} u_1 \\ x_2 = \frac{0{,}8}{5s+1} u_1 \\ x_3 = \frac{0{,}8}{3s+1} u_2 \\ x_4 = \frac{1}{4s+1} u_2 \end{cases} ; \quad \begin{cases} 2\dot{x}_1 + x_1 = u_1 \\ 5\dot{x}_2 + x_2 = 0{,}8 u_1 \\ 3\dot{x}_3 + x_3 = 0{,}8 u_2 \\ 4\dot{x}_4 + x_4 = u_2 \end{cases} ; \quad \begin{cases} \dot{x}_1 = -\frac{1}{2} x + \frac{1}{2} u_1 \\ \dot{x}_2 = -\frac{1}{5} x_2 + \frac{0{,}8}{5} u_1 \\ \dot{x}_3 = -\frac{1}{3} x_3 + \frac{0{,}8}{3} u_2 \\ \dot{x}_4 = -\frac{1}{4} x_4 + \frac{1}{4} u_2 \end{cases}$$

$$\Rightarrow \dot{X} = \begin{pmatrix} -\frac{1}{2} & 0 & 0 & 0 \\ 0 & -\frac{1}{5} & 0 & 0 \\ 0 & 0 & -\frac{1}{3} & 0 \\ 0 & 0 & 0 & -\frac{1}{4} \end{pmatrix} X + \begin{pmatrix} \frac{1}{2} & 0 \\ \frac{0{,}8}{5} & 0 \\ 0 & \frac{0{,}8}{3} \\ 0 & \frac{1}{4} \end{pmatrix} U, \quad Y = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix} X$$

b)

$$\begin{cases} x_1 = \frac{3}{2+s} u \\ x_2 = \frac{5}{4+2s} u \\ x_3 = \frac{4}{3+s}(x_1 + x_2) \end{cases} ; \quad \begin{cases} 2x_1 + \dot{x}_1 = 3u \\ 4x_2 + 2\dot{x}_2 = 5u \\ 3x_3 + \dot{x}_3 = 4x_1 + 4x_2 \end{cases} ; \quad \begin{cases} \dot{x}_1 = -2x_1 + 3u \\ \dot{x}_2 = -2x_2 + \frac{5}{2} u \\ \dot{x}_3 = 4x_1 + 4x_2 - 3x_3 \end{cases}$$

$$\Rightarrow \dot{X} = \begin{pmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 4 & 4 & -3 \end{pmatrix} X + \begin{pmatrix} 3 \\ \frac{5}{2} \\ 0 \end{pmatrix} U, \quad Y = \begin{pmatrix} 0 & 0 & 3 \end{pmatrix} X$$

c) ej löst

14.5

$$\begin{cases} x_3 = \dfrac{1}{s+2} w \\ x_4 = \dfrac{0{,}5}{s+3} v \end{cases} ; \quad \begin{cases} \dot{x}_3 + 2x_3 = w \\ \dot{x}_4 + 3x_4 = 0{,}5v \end{cases} ; \quad \begin{cases} \dot{x}_3 = -2x_3 + w \\ \dot{x}_4 = -3x_4 + 0{,}5v \end{cases}$$

Processen: $G_p(s) = \dfrac{s+3}{2s^2+4s+1} = \dfrac{\frac{1}{2}s + \frac{3}{2}}{s^2+2s+\frac{1}{2}}$

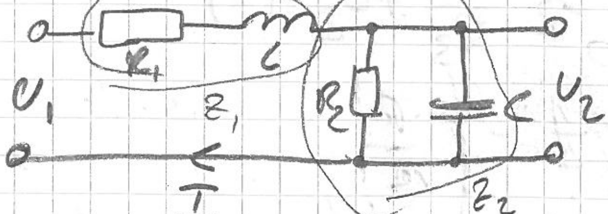
$$\Rightarrow \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} -2 & -0{,}5 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} u \\ x_3 \end{pmatrix}, \quad z = \begin{pmatrix} 0{,}5 & 1{,}5 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

Totalt:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{pmatrix} = \begin{pmatrix} -2 & -0{,}5 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & -3 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0{,}5 & 0 \end{pmatrix}\begin{pmatrix} u \\ v \\ w \end{pmatrix}$$

$$\begin{pmatrix} z \\ y \end{pmatrix} = \begin{pmatrix} 0{,}5 & 1{,}5 & 0 & 0 \\ 0{,}5 & 1{,}5 & 0 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$$

14.6 a)



Tillståndsvariabler: x_1 spänning över kond. $= U_2$

x_2 strömmen genom spolen $= \underline{I}$

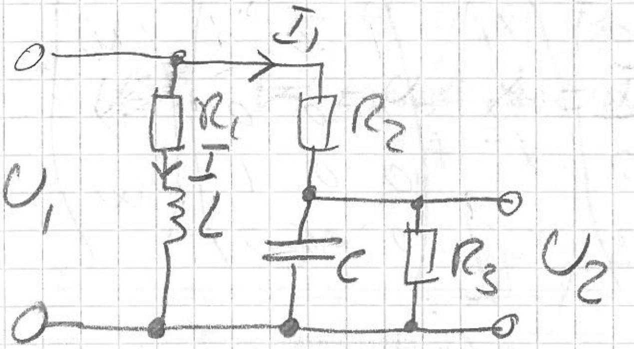
$$\begin{cases} U_1 = U_{R_1} + U_L + U_2 = I R_1 + \dot{I} L + U_2 \\ I = I_C + I_{R_2} = \dot{U}_2 C + \dfrac{U_2}{R_2} \end{cases} ; \quad \begin{cases} \dot{I} L = U_1 - I R_1 - U_2 \\ \dot{U}_2 C = I - \dfrac{U_2}{R_2} \end{cases} ;$$

$$\begin{cases} \dot{I} = \dfrac{U_1}{L} - I \dfrac{R_1}{L} - \dfrac{U_2}{L} \\ \dot{U}_2 = \dfrac{I}{C} - \dfrac{U_2}{R_2 C} \end{cases}$$

$$\begin{cases} x_1 = U_2 \\ x_2 = I \end{cases} \Rightarrow \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} -\dfrac{1}{CR_2} & \dfrac{1}{C} \\ -\dfrac{1}{L} & -\dfrac{R_1}{L} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ \dfrac{1}{L} \end{pmatrix} U_1$$

$$y = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

b)



Välj $x_1 = U_2$, $x_2 = I$ enl. a)

$$U_1 = U_{R_1} + U_L = I R_1 + \dot{I} L \quad (1)$$

$$I_1 = I_C + I_{R_3} = \dot{U}_2 C + \frac{U_2}{R_3} ; \quad U_1 = I_1 R_2 + U_2 ;$$

$$U_1 = \dot{U}_2 C R_2 + U_2 \frac{R_2}{R_3} + U_2 \quad (2)$$

$$(1) \Rightarrow \dot{I} = U_1 \frac{1}{L} - I \frac{R_1}{L}$$

$$(2) \Rightarrow \dot{U}_2 = U_1 \frac{1}{CR_2} - U_2 \frac{1}{CR_3} - U_2 \frac{1}{CR_2}$$

$$\Rightarrow \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} -\frac{1}{C}\left(\frac{1}{R_3}+\frac{1}{R_2}\right) & 0 \\ 0 & -\frac{R_1}{L} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} \frac{1}{L} \\ \frac{1}{CR_2} \end{pmatrix} v_1$$

$$y = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

14.7 Välj (enl. boken) $x_1 = z,\ x_2 = \dot{z},\ x_3 = y,\ x_4 = \dot{y}$

$$F = m \cdot a = M_1 \ddot{z} = -kz + k(y-z) - b\dot{z}$$
$$F = m \cdot a = M_2 \ddot{y} = -k(y-z) + k(u-y)$$

$$\Rightarrow \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -\frac{2k}{M_1} & -\frac{b}{M_1} & \frac{k}{M_1} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k}{M_2} & 0 & -\frac{2k}{M_2} & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{k}{M_2} \end{pmatrix} u$$

$$\begin{pmatrix} y \\ z \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$$

14.8 a) För varje behållare gäller $A\frac{dh}{dt} = u_{in} - u_{ut}$ (se s.108)

$$A = 1\,m^2 \Rightarrow \frac{dh}{dt} = u_{in} - u_{ut}$$

Tank 1: $\dot{x}_1 = u - q_{12} = u - K(x_1 - x_2)$

Tank 2: $\dot{x}_2 = q_{12} - q_{23} = K(x_1 - x_2) - K(x_2 - x_3)$

Tank 3: $\dot{x}_3 = q_{23} - y = K(x_2 - x_3) - Kx_3$

$$\Rightarrow \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} -K & K & 0 \\ K & -2K & K \\ 0 & K & -2K \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} u = K\begin{pmatrix} -1 & 1 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} u$$

$$y = \begin{pmatrix} 0 & 0 & K \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

b) Vid stationärt tillst. är derivatorna $=0$

$$\Rightarrow \begin{cases} -kx_1+kx_2+u=0 & (1) \\ x_1-2x_2+x_3=0 & (2) \\ x_2-2x_3=0 & (3) \end{cases}$$

$(3) \Rightarrow x_2 = 2x_3 \ (3')$

$(3')$ i $(2) \Rightarrow x_1 - 4x_3 + x_3 = 0$

$x_1 = 3x_3 \ (4')$

$(4')$ & $(3')$ i $(1) \Rightarrow -3kx_3 + 2kx_3 + u = 0$; $x_3 = \frac{u}{k} = 0{,}1m$

Svar: $x_1 = 0{,}3m$, $x_2 = 0{,}2m$, $x_3 = 0{,}1m$

14.9 övre tank: $A_1\dot{h}_1 = u_0 - u_{10} - u_{11} = u_0 - \frac{h_1}{R_{10}} - \frac{h_1}{R_{11}}$

undre tank: $A_2\dot{h}_2 = u_{11} - u_2 = \frac{h_1}{R_{11}} - \frac{h_2}{R_2}$

$$\Rightarrow \begin{pmatrix} \dot{h}_1 \\ \dot{h}_2 \end{pmatrix} = \begin{pmatrix} -\frac{1}{A_1}\left(\frac{1}{R_{10}}+\frac{1}{R_{11}}\right) & 0 \\ \frac{1}{A_2R_{11}} & -\frac{1}{A_2R_2} \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} + \begin{pmatrix} \frac{1}{A_1} \\ 0 \end{pmatrix} u_0$$

$$y = \begin{pmatrix} \frac{1}{R_{10}} & \frac{1}{R_2} \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}$$

14.10 Materialbalans: $\frac{dV}{dt} = u_1 + u_2 - u_3 \quad (1)$

energibalans: $\frac{dE}{dt} = \rho c T_1 u_1 + \rho c T_2 u_2 - \rho c T u_3 \quad (2)$

$E = \rho c V T$, där V & T är variabla

$$\Rightarrow \frac{dE}{dt} = \rho c \frac{d(VT)}{dt} = \rho c \left(V\frac{dT}{dt} + T\frac{dV}{dt} \right) \quad (3)$$

$$(1)\&(3) \Rightarrow \frac{dE}{dt} = \rho c\left(V\frac{dT}{dt} + T(u_1+u_2-u_3)\right) \quad (4)$$

$$(2)\&(4) \Rightarrow \rho c T_1 u_1 + \rho c T_2 u_2 - \rho c T u_3 = \rho c\left(V\frac{dT}{dt} + T(u_1+u_2-u_3)\right)$$

$$T_1 u_1 + T_2 u_2 - \cancel{T u_3} = VT' + Tu_1 + Tu_2 - \cancel{Tu_3}$$

$$V = A\cdot h \Rightarrow AhT' = T_1 u_1 + T_2 u_2 - T(u_1+u_2)$$

$$T' = -\frac{T}{Ah}(u_1+u_2) + \frac{T_1 u_1}{Ah} + \frac{T_2 u_2}{Ah}$$

$$(1) \Rightarrow h' = \frac{u_1}{A} + \frac{u_2}{A} - \frac{u_3}{A}$$

14.11 ej löst

14.12 a)

$$\frac{dE_1}{dt} = c_1\frac{dT_1}{dt} = k_{12}(T_2-T_1) - k_{14}(T_1-T_4) \quad (1)$$

$$\frac{dE_2}{dt} = c_2\frac{dT_2}{dt} = P - k_{12}(T_2-T_1) - k_{23}(T_2-T_3) - k_{24}(T_2-T_4) \quad (2)$$

$$\frac{dE_3}{dt} = c_3\frac{dT_3}{dt} = k_{23}(T_2-T_3) - k_{34}(T_3-T_4) \quad (3)$$

$$(1) \Rightarrow T_1' = -T_1\frac{k_{12}+k_{14}}{c_1} + T_2\frac{k_{12}}{c_1} + T_4\frac{k_{14}}{c_1}$$

$$(2) \Rightarrow T_2' = T_1\frac{k_{12}}{c_2} - T_2\frac{k_{12}+k_{23}+k_{24}}{c_2} + T_3\frac{k_{23}}{c_2} + T_4\frac{k_{24}}{c_2} + \frac{P}{c_2}$$

$$(3) \Rightarrow T_3' = T_2\frac{k_{23}}{c_3} - T_3\frac{k_{23}+k_{34}}{c_3} + T_4\frac{k_{34}}{c_3}$$

$$\Rightarrow \begin{pmatrix} \bar{T}_1' \\ \bar{T}_2' \\ \bar{T}_3' \end{pmatrix} = \begin{pmatrix} \frac{-(k_{12}+k_{14})}{c_1} & \frac{k_{12}}{c_1} & 0 \\ \frac{k_{12}}{c_2} & \frac{-(k_{12}+k_{23}+k_{24})}{c_2} & \frac{k_{23}}{c_2} \\ 0 & \frac{k_{23}}{c_3} & \frac{-(k_{23}+k_{34})}{c_3} \end{pmatrix} \begin{pmatrix} \bar{T}_1 \\ \bar{T}_2 \\ \bar{T}_3 \end{pmatrix} +$$

$$+ \begin{pmatrix} 0 & \frac{k_{14}}{c_1} \\ \frac{1}{c_2} & \frac{k_{24}}{c_2} \\ 0 & \frac{k_{34}}{c_3} \end{pmatrix} \begin{pmatrix} P \\ \bar{T}_4 \end{pmatrix}, \quad \bar{T}_2 = \begin{pmatrix} 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \bar{T}_1 \\ \bar{T}_2 \\ \bar{T}_3 \end{pmatrix}$$

---

b) Stationär $\Leftrightarrow \bar{T}_1' = \bar{T}_2' = \bar{T}_3' = 0$

$C_1 = C_2 = C_3 = 2 \cdot 10^6 \, J/°C$, $P = 4000 \, W$, $\bar{T}_4 = 0°C$

$k_{12} = k_{23} = 200 \, W/°C$, $k_{14} = k_{34} = 100 \, W/°C$, $k_{24} = 50 \, W/°C$

$$\Rightarrow \begin{cases} -1{,}5 \cdot 10^{-4} \bar{T}_1 + 1 \cdot 10^{-4} \bar{T}_2 + 0{,}5 \cdot 10^{-4} \bar{T}_4 = 0 \\ 1 \cdot 10^{-4} \bar{T}_1 - 2{,}25 \cdot 10^{-4} \bar{T}_2 + 1 \cdot 10^{-4} \bar{T}_3 + 0{,}25 \cdot 10^{-4} \bar{T}_4 + 20 \cdot 10^{-4} = 0 \\ 1 \cdot 10^{-4} \bar{T}_2 - 1{,}5 \cdot 10^{-4} \bar{T}_3 + 0{,}5 \cdot 10^{-4} \bar{T}_4 = 0 \end{cases}$$

dela med 10^{-4}, $\bar{T}_4 = 0$

$$\Rightarrow \begin{cases} -1{,}5 \bar{T}_1 + \bar{T}_2 = 0 \\ \bar{T}_1 - 2{,}25 \bar{T}_2 + \bar{T}_3 = -20 \\ \bar{T}_2 - 1{,}5 \bar{T}_3 = 0 \end{cases}, \quad \text{Matlab}(A \backslash B) \Rightarrow \begin{cases} \bar{T}_1 \approx 15°C \\ \bar{T}_2 \approx 22°C \\ \bar{T}_3 \approx 15°C \end{cases}$$

(14.13)

Rum 1: $C_1 \frac{dT_1}{dt} = P - k_{10}(T_1 - T_0) - k_{12}(T_1 - T_2) + \rho c Q T_{in} - \rho c Q T_1$

Rum 2: $C_2 \frac{dT_2}{dt} = k_{12}(T_1 - T_2) - k_{20}(T_1 - T_0) + \rho c Q T_1 - \rho c Q T_2$

$T_{in} = T_0 + 0{,}75(T_2 - T_0)$

Fortsätt som l. 14.12

(14.14) $\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} -3 & -2 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u$, $y = \begin{pmatrix} 3 & 4 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$

a) $\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases}$ Laplace $\Rightarrow \begin{cases} sX = AX + BU & (1) \\ Y = CX & (2) \end{cases}$

$(1) \Rightarrow s\underline{I}X = AX + BU$; $(s\underline{I} - A)X = BU$; $X = (s\underline{I} - A)^{-1}BU \quad (1')$

enhets-matris

$(1') \& (2) \Rightarrow Y = C(s\underline{I} - A)^{-1}BU$

$\Rightarrow G(s) = C(s\underline{I} - A)^{-1}B$

$s\underline{I} - A = \begin{pmatrix} s+3 & 2 \\ -1 & s \end{pmatrix}$; $(s\underline{I} - A)^{-1} = \frac{1}{s(s+3)+2}\begin{pmatrix} s & -2 \\ 1 & s+3 \end{pmatrix}$

$C(s\underline{I} - A) = \frac{1}{s^2+3s+2}\begin{pmatrix} 3 & 4 \end{pmatrix}\begin{pmatrix} s & -2 \\ 1 & s+3 \end{pmatrix} = \frac{1}{s^2+3s+2}\begin{pmatrix} 3s+4 & 4s+12-6 \end{pmatrix}$

$G(s) = C(s\underline{I} - A)B = \frac{1}{s^2+3s+2}\begin{pmatrix} 3s+4 & 4s+6 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \underline{\underline{\frac{3s+4}{s^2+3s+2}}}$

b) ej löst

14.15

$$\begin{pmatrix} \dot{T}_1 \\ \dot{T}_2 \end{pmatrix} = \begin{pmatrix} -0{,}3 & 0{,}2 \\ 0{,}2 & -0{,}3 \end{pmatrix}\begin{pmatrix} T_1 \\ T_2 \end{pmatrix} + \begin{pmatrix} 0{,}1 & 0 \\ 0{,}1 & 1 \end{pmatrix}\begin{pmatrix} T_u \\ P \end{pmatrix}, \quad y = (0\ 1)\begin{pmatrix} T_1 \\ T_2 \end{pmatrix}$$

byt tillst. var. till $T_m = \frac{T_1 + T_2}{2}$, $T_s = T_2 - T_1$

Nya matriser: $\begin{cases} \tilde{A} = TAT^{-1} \\ \tilde{B} = TB \\ \tilde{C} = CT^{-1} \end{cases}$ där T är transformationsmatrisen från de gamla till de nya tillst. var.

$$\begin{pmatrix} T_m \\ T_s \end{pmatrix} = \begin{pmatrix} 0{,}5 & 0{,}5 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} T_1 \\ T_2 \end{pmatrix} \Rightarrow T = \begin{pmatrix} 0{,}5 & 0{,}5 \\ -1 & 1 \end{pmatrix}$$

Matlab $\Rightarrow \tilde{A} = \begin{pmatrix} -0{,}1 & 0 \\ 0 & -0{,}5 \end{pmatrix}$, $\tilde{B} = \begin{pmatrix} 0{,}1 & 0{,}5 \\ 0 & 1 \end{pmatrix}$, $C = (1\ \ 0{,}5)$

14.16 $\quad G(s) = \frac{10}{(1+2s)(1+s)} = \frac{20}{1+2s} + \frac{-10}{1+s} = \frac{10}{s+\frac{1}{2}} + \frac{-10}{s+1}$

Diagonalform: $\begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = \begin{pmatrix} -0{,}5 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 10 \\ -10 \end{pmatrix} u$, $y = (1\ \ 1)\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$

Polplacering: $P(s) = \det(sI - A + BL) =$

$$\det\left| \begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix} - \begin{pmatrix} -0{,}5 & 0 \\ 0 & -1 \end{pmatrix} + \begin{pmatrix} 10 \\ -10 \end{pmatrix}(l_1\ \ l_2) \right| =$$

$$= \det\left|\begin{pmatrix} s+0{,}5 & 0 \\ 0 & s+1 \end{pmatrix} + \begin{pmatrix} 10l_1 & 10l_2 \\ -10l_1 & -10l_2 \end{pmatrix}\right| =$$

$$= \begin{vmatrix} s+0{,}5+10l_1 & 10l_2 \\ -10l_1 & s+1-10l_2 \end{vmatrix} = (s+0{,}5+10l_1)(s+1-10l_2)+100l_1l_2 =$$

$$= s^2+s-10sl_2+0{,}5s+0{,}5-5l_2+10sl_1+10l_1-\cancel{100l_1l_2}+\cancel{100l_1l_2} =$$

$$= s^2+s(1-10l_2+0{,}5+10l_1)+0{,}5-5l_2+10l_1 \quad (1)$$

Alla poler: $s=-2 \Rightarrow P(s)=(s+2)^2 = s^2+4s+4$ (2)

$$(1)\,\&\,(2) \Rightarrow \begin{cases} s^2: 1=1 \\ s^1: 1-10l_2+0{,}5+10l_1 = 4; \\ s^0: 0{,}5-5l_2+10l_1 = 4 \end{cases} \begin{cases} 10l_1-10l_2 = 2{,}5 \; (3) \\ 10l_1-5l_2 = 3{,}5 \; (4) \end{cases}$$

$(3) \Rightarrow 10l_1 = 2{,}5+10l_2 \; (3')$

$(3')\,\&\,(4) \Rightarrow 2{,}5+10l_2-5l_2=3{,}5;\quad 5l_2=1;\quad \underline{l_2=0{,}2}$

$\Rightarrow 10l_1-2=2{,}5;\quad \underline{l_1=0{,}45}$

$\Rightarrow \underline{L=\begin{pmatrix} 0{,}45 & 0{,}2 \end{pmatrix}}$

$$K_r = \frac{1}{C(-A+BL)^{-1}B}$$

blockschema, se sid 288

Dessa värden stämmer. Kanske har facit använt andra tillst. var.

14.17-18 Ej löst