EXERCISE 4.6 Average error probability for Rayleigh fading

Compute the average probability of error for a Rayleigh fading channel given the error probability of AWGN channel model.

EXERCISE 4.7 Detection in a Rayleigh fading channel

In a Rayleigh fading channel the detection of symbol x from y is based on the sign of the real sufficient statistic

$$r = |h|x + z,$$

where $z \sim \mathcal{N}(0, N_0/2)$. It means that, If the transmitted symbol is $x = \pm a$, then, for a given value of h, the error probability of detecting x is

$$\mathbf{Q}\left(\frac{a|h|}{\sqrt{N_0/2}}\right) = \mathbf{Q}\left(\sqrt{2|h|^2 \text{SNR}}\right),$$

where SNR $= a^2/N_0$ is the average received signal-to-noise ratio per symbol time (note that we normalized the channel gain such that $\mathbf{E}[|\mathbf{h}|^\mathbf{2}] = \mathbf{1}$.) For Rayleigh fading when $|h|$ has Rayleigh distribution with mean 0 and variance 1, calculate the average probability of error. Approximate the solution for high SNR regions.

EXERCISE 4.8 Average error probability for log-normal fading

Consider a log-normal wireless channel with AWGN receiver noise. We know that the probability of error in AWGN is

$$\mathbf{Q}(\gamma) = \Pr\{\mathbf{x} > \gamma\} = \int_\gamma^\infty \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt$$

The average probability of error with respect to the log-normal distribution is the average of $\mathbf{Q}(\gamma)$ with respect to the log-normal distribution. It is difficult to compute because $\mathbf{Q}$ is highly non linear. Suppose to perform a Stirling approximation of the $\mathbf{Q}$ function, which is

$$\mathbf{E}\left\{f(\theta)\right\} \sim \frac{2}{3} f(\mu) + \frac{1}{6} f(\mu + \sqrt{3}\sigma) + \frac{1}{6} f(\mu - \sqrt{3}\sigma)$$

where $f(\theta)$ is any function of a random variable θ having mean μ and variance σ^2. Compute the average probability of error of log-normal channel by using the Stirling approximation.

EXERCISE 4.9 Probability of error at the message level

In a WSN communication platform, consider a Rayleigh Channel over a AWGN receiver noise. The message is a frame of size f bits and is composed of the preamble, network payload, and a CRC code.

(a) Compute p the probability that the message is correctly received.

(b) Assume that the received signal level at the receiver decays inversely with the squared of the distance, i.e.,

$$\text{SNR} \approx \frac{\alpha E_b}{N_0 d^2}.$$

For messages of size 10 bits and the values $E_b/N_0 = 100$ and $\alpha = 0.1$, compute the farthest distance to deploy a receiver such that the probability of successfully message reception is at least $p = 0.9^{10} \approx 0.35$.