System planning, EG2050 Probabilistic production cost simulation of electricity markets – lecture 13

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Probabilistic production cost simulation (PPC)

- Development started in the late 1960:th
- Analytical calculations
- A fast method but limitations concerning possibilities to include market details.

PPC model

Assume

- Perfect competition
- Perfect information
- Load is not price sensitive
- Neglect grid losses and limitations
- All scenario parameters can be treated as independent

Some of these assumption can be treated with some specific methods.

Calculation of system index - Basic idea

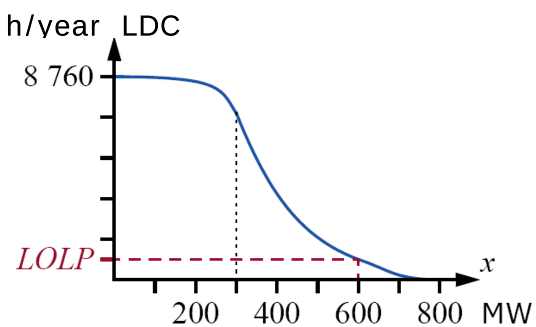
- Assume a system where all power plants are 100 % available.
- In this system it is easy to calculate the system indeces from the load duration curve, LDC

Example

- Load duration curve, LDC
- Two power stations (300 MW, always avialable, incremental operation cost $ß_1$ and $ß_2$ respectively, $ß_1 \leq ß_2$)

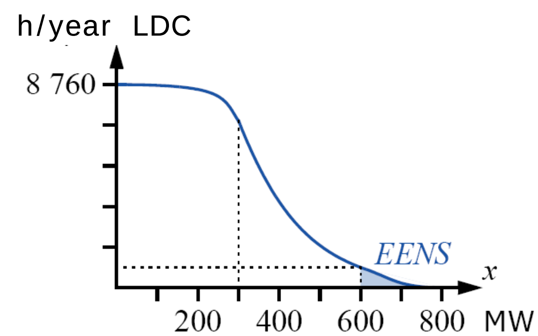
Calculation of system index - Basic idea

LOLP = LDC(600) = 876 h/year = 10%



Calculation of system index - Basic idea

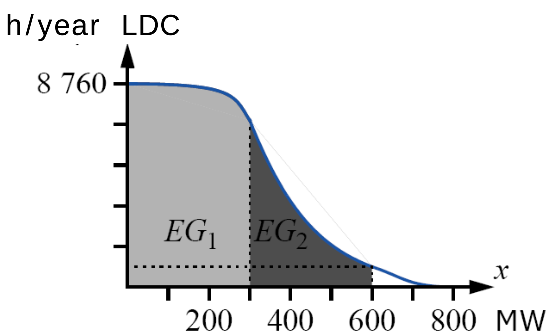
$$EENS = \int_{600}^{\infty} LDC(x)dx$$



Calculation of system index - Basic idea

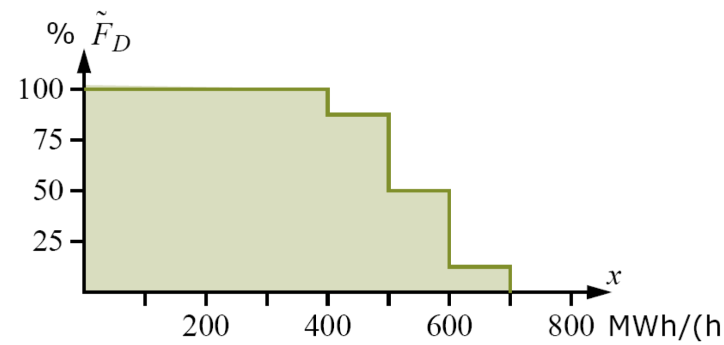
$$ETOC = \beta_1 EG_1 + \beta_2 EG_2$$

Perfect competition $\Rightarrow$ Unit 1 is used first, since $ß_1 \le ß_2$



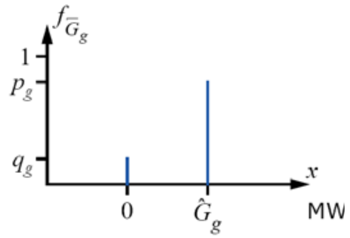
Load model – Alternative 2

Example 6.8:



The area is now changed since the y-axes is divided with T. To get the correct expected value, the area has to be multiplied with T.

Thermal power station model



- The availability of a thermal power station can not be calculated, but has to be estimated from historical data and forecasts of the future.

Equivalent load

- We have seen that it is simple to calculate the system indices in a system where all power plants are 100 % available
- How is this performed when this is not the case?
- **PPC-method**: Concider outages in power stations as load increase instead of reduced available production capacity.

Equivalent load

Definition 6.10. The equivalent load is given by

$$E_g = D + \sum_{k=1}^{g} O_k.$$

where

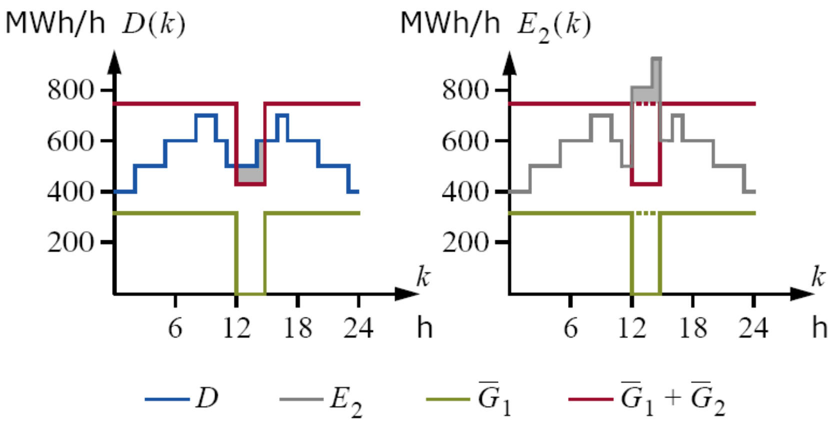
E_g = equivalent load for the power plant next to be dispatched after unit g,
D = actual load,
O_k = outage in unit k.

Equivalent load - example

A capacity deficit problem (LOLO) occurs when:

Available capacity < demand
$\Rightarrow$
Installed capacity – outages < demand
$\Rightarrow$
Installed capacity < demand + outages
$\Rightarrow$
Installed capacity < equivalent load

Equivalent load - example

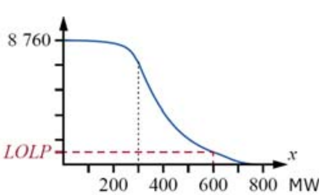


Calculation of system indices

- From the equivalent load duration curve, $\tilde{F}_g(x)$, it is possible to calculate the system indices
- Total available capacity: $$\overline{G}_g^{tot} = \sum_{k=1}^{g} \overline{G}_k,$$
- Total installed capacity: $$\hat{G}_g^{tot} = \sum_{k=1}^{g} \hat{G}_k$$
- Total outage: $$O_g^{tot} = \sum_{k=1}^{g} O_k.$$

Calculation of system indices

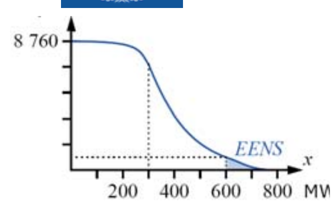
- Risk of capacity deficit = Loss of load probability. Capacity deficit occurs when the deman exceeds available production capacity, i.e.



$$LOLP_g = P(D > \overline{G}_g^{tot}) = P(D > \hat{G}_g^{tot} - O_g^{tot})$$
$$= P(D + O_g^{tot} > \hat{G}_g^{tot}) = P(E_g > \hat{G}_g^{tot})$$
$$= \tilde{F}_g(\hat{G}_g^{tot}).$$

Calculation of system indices

- Expected Energy Not Served, EENS: This means energy that cannot be delivered depending on capacity deficit, i.e., equivalent load > installed capacity



$$EENS_g = T \int_{\hat{G}_g^{tot}}^{\infty} \tilde{F}_g(x)dx.$$

Calculation of system indices

- Expected energy production in power plants, EG: Assume that we do not have unit g. Then the expected unserved energy is $EENS_{g-1}$. When unit g is then installed the expected unserved energy decreases to $EENS_g$. The difference is expected energy production in the unit.

$$EG_g = EENS_{g-1} - EENS_g.$$

Calculation of system indices

- Expected production cost. If energy production in each unit is available, then it is eacy to calculate the production cost.

$$ETOC_g = \sum_{k=1}^{g} \beta_k EG_k$$

- It is also possible to include the cost of disconnected consumers

$$ETOC_g = \sum_{k=1}^{g} \beta_k EG_k + \beta_U EENS_g$$

Calculation of system indices

- ***Example 6.3-6.6.*** In a simple system the load is constantly 200 kW. The system is supplied by a 200 kW power plant having an availability of 80% and the operation cost 1 ¤/kWh. Calculate the *EENS*, *LOLP* and *ETOC* of the system.

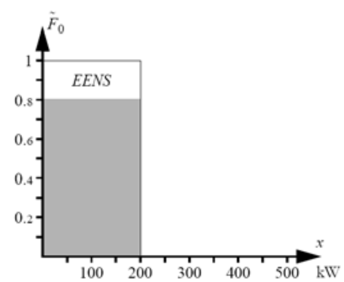


Figure 6.2 Calculation of system indices directly from the load duration curve.

Calculation of system indices

- ***Example 6.4.*** Study the same system as in example 6.3. What does the equivalent load duration curve look like?

- The definition of the equivalent load is:

$$E_g = D + \sum_{k=1}^{g} O_k,$$

- This means convolution since the equivalent load is a sum of independent stochastic variables

Calculation of equivalent load load duration curve, ELDC

- General formula:

$$\tilde{F}_g(x) = p_g \cdot \tilde{F}_{g-1}(x) + q_g \cdot \tilde{F}_{g-1}(x - \hat{G}_g).$$

- Assume an LDC and one power plant with availability $p_1 \Rightarrow$ ELDC:

$$\tilde{F}_1(x) = p_1 \cdot \tilde{F}_0(x) + q_1 \cdot \tilde{F}_0(x - \hat{G}_1).$$

- The equivalent load is > x, when
  - unit available, load is > x, probability p_1
  - outage of G_1, load > x-G_1, probability q_1

Calculation of system indices

- ***Example 6.4-6.5.*** Study the same system as in example 6.3. What does the equivalent load duration curve look like?

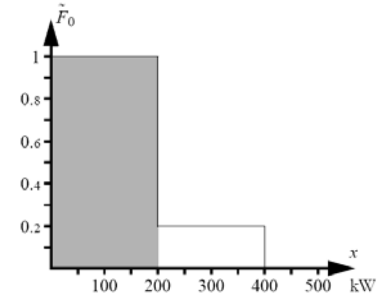


Figure 6.4 Equivalent load duration curve for the system in examples 6.3 and 6.4.

Calculation of system indices

- ***Example 6.6.*** Calculate the unserved energy, loss of load probability and expected operation cost per hour in the system described in figure 6.3.

Calculation of system indices

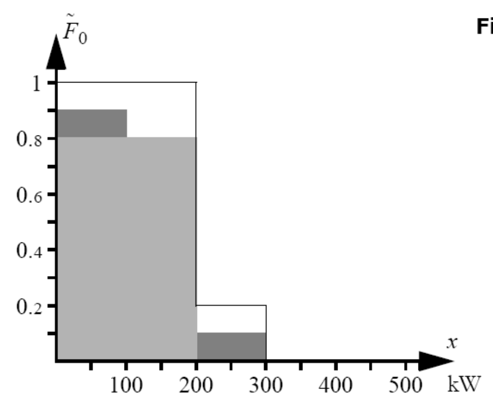


Figure 6.3 A system with two power plants and two possible load levels. The generation in the least expensive unit (capacity 200 kW, availability 80%, operation cost 1 ¤/kWh) is shown by the lighter shadowed area. There is also a backup unit (capacity 100 kW, availability 50%, operation cost 2 ¤/kWh) which is used when the base unit is unavailable or when the load is high (indicated by the two darker areas). The expectation value of unserved energy is indicated by the white areas below the load duration curve.

Example 6.6

Solution: The load duration curve is given in the figure:

$$\tilde{F}_0(x) = \begin{cases} 1 & x < 200, \\ 0.2 & 200 \le x < 300, \\ 0 & 300 \le x. \end{cases}$$

The larger power plant has the lower operation cost and should therefore be added first:

$$\tilde{F}_1(x) = 0.8\tilde{F}_0(x) + 0.2\tilde{F}_0(x-200) =$$

$$= \begin{cases} 0.8 \cdot 1 + 0.2 \cdot 1 = 1 & x < 200, \\ 0.8 \cdot 0{,}2 + 0.2 \cdot 1 = 0.36 & 200 \le x < 300, \\ 0.8 \cdot 0 + 0.2 \cdot 1 = 0.2 & 300 \le x < 400, \\ 0.8 \cdot 0 + 0.2 \cdot 0.2 = 0.04 & 400 \le x < 500, \\ 0 & 500 \le x. \end{cases}$$

Example 6.6

Then the second power plant is added:

$$\tilde{F}_2(x) = 0.5\tilde{F}_1(x) + 0.5\tilde{F}_1(x-100) =$$

$$= \begin{cases} 0.5 \cdot 1 + 0.5 \cdot 1 = 1 & x < 200, \\ 0.5 \cdot 0.36 + 0.5 \cdot 1 = 0.68 & 200 \le x < 300, \\ 0.5 \cdot 0.2 + 0.5 \cdot 0.36 = 0.28 & 300 \le x < 400, \\ 0.5 \cdot 0.04 + 0.5 \cdot 0.2 = 0.12 & 400 \le x < 500, \\ 0.5 \cdot 0 + 0.5 \cdot 0.04 = 0.02 & 500 \le x < 600, \\ 0 & 600 \le x. \end{cases}$$

$$EENS_0 = 1 \cdot \int_0^\infty \tilde{F}_0(x)dx = 1 \cdot 200 + 0.2 \cdot 100 = 220 \text{ kWh},$$

$$EENS_1 = 1 \cdot \int_{200}^\infty \tilde{F}_1(x)dx = 0.36 \cdot 100 + 0.2 \cdot 100 + 0.04 \cdot 100 = 60 \text{ kWh},$$

$$EENS_2 = 1 \cdot \int_{300}^\infty \tilde{F}_2(x)dx = 0.28 \cdot 100 + 0.12 \cdot 100 + 0.02 \cdot 100 = 42 \text{ kWh},$$

$$EG_1 = EENS_0 - EENS_1 = 160 \text{ kWh},$$

$$EG_2 = EENS_1 - EENS_2 = 18 \text{ kWh}.$$

It is now easy to calculate the expected operation cost per hour:

$$ETOC = \beta_1 \cdot EG_1 + \beta_2 \cdot EG_2 = 1 \cdot 160 + 2 \cdot 18 = 196 \text{ ¤/h}.$$

The loss of load probability is given by

$$LOLP = \tilde{F}_2(300) = 28\%$$

and the unserved energy is 42 kWh/h, as we already have calculated