Example 1: Harmonic frequencies in a quarter wave plate

Consider a system that emits primarily at a frequency f, but also at the harmonics $2f$, $3f$, $4f$… We will now study how the polarization of the different wave components change when passing through a quarter wave plate.

a) Derive the dispersion relation and eigenvectors for the O-mode and X-mode, when $\mathbf{k}$ is in the x-direction and the dielectric tensor is

$$K = \begin{bmatrix} K_\perp & 0 & 0 \\ 0 & K_\perp & 0 \\ 0 & 0 & K_\parallel \end{bmatrix}$$

b) When entering the wave plate the polarization of all harmonic components is

$$\mathbf{E} \propto [1 \quad 1 \quad 0]$$

Express this polarization using the O-mode and X-mode eigenvectors

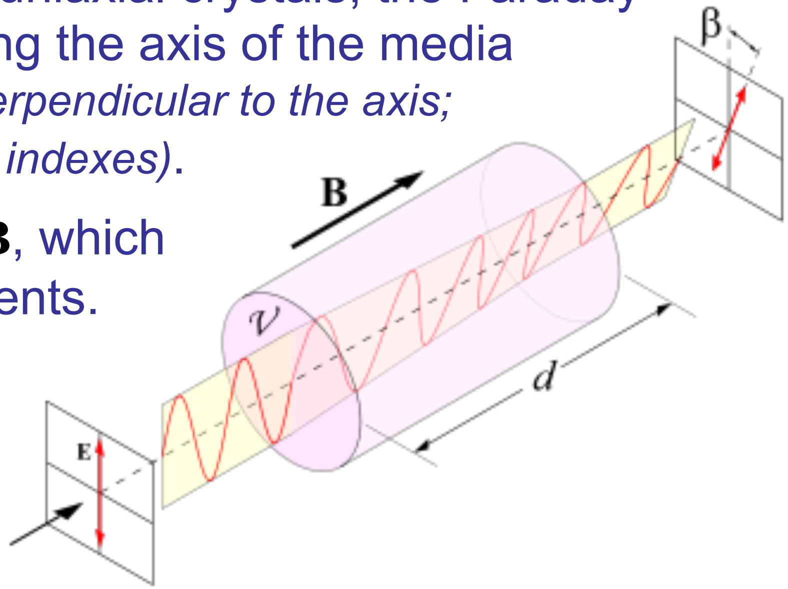
c) Place this polarization on the Poincare sphere when the transverse plane is represented by $\mathbf{e}^1=\mathbf{e}_x$ and $\mathbf{e}^2=\mathbf{e}_y$

d) If the crystal is a quarter wave plate at the frequency f, how long/thick is the plate?

e) What is the polarization of the first three harmonic components when leaving the crystal?

Example 2: Faraday effect

In magnetoactive media a type of birefringence can be found that is called the Faraday rotation. But unlike the birefringence in uniaxial crystals, the Faraday rotation appears when the wave propagates along the axis of the media *(Remember: in a crystal the propagation have to be perpendicular to the axis; then the transverse plane have two different refractive indexes)*.



The effect originates from the Lorentz force $q\mathbf{v}\times\mathbf{B}$, which gives the dielectric tensor off-diagonal components.

$$K = \begin{bmatrix} S & -iD & 0 \\ iD & S & 0 \\ 0 & 0 & P \end{bmatrix}$$

a) Derive the dispersion relations and the transverse eigenvectors when $\mathbf{k}=k\mathbf{e}_z$

b) Show that the eigenvectors are orthogonal (in the Euclidian metric), i.e. they form an orthonormal basis for the space of polarized transverse waves.

c) Express $\mathbf{e}_x$ and $\mathbf{e}_y$ in terms of the eigenvector.

d) With initial condition $\mathbf{E}=E\mathbf{e}_x$, how does the media change the polarization after travelling a distance L?

e) When $D=|\mathbf{B}|$, where $|\mathbf{B}|$ is the static magnetic field strength, and S is independent of $|\mathbf{B}|$, then what is the relation between $|\mathbf{B}|$ and the change in polarization?