System planning VT13

Simulation of electricity markets using Monte Carlo methods

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Simulation of electricity markets using Monte Carlo methods

Why do a simulation?

- predict the long-term behavior of an electricity market by calculating indices such as EENS, LOLP and ETOC

- Actors in the electricity market might use the simulation to evaluate if an investment is beneficial or not
- The government might use the simulation to see what consequences a certain regulation have on electricity prices, environment, etc before realizing it

System planning VT13

- Lecture L15: **Monte Carlo**
- Content:

1. **Basics about Monte Carlo**
2. Simple sampling
3. Convergence criteria
4. Random number generation
5. Inverse transform method

Basics: –Why use Monte Carlo simulations?

- When it is too complicated to calculate expected values theoretically.

$$E[X] = \int_{-\infty}^{\infty} x f_X(x)\,dx$$

$$E[X] = \sum_{-\infty}^{\infty} x \cdot f_X(x),$$

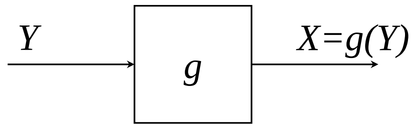
Unknown!

Use Monte Carlo if $f_X(x)$ is unknown or the integral is difficult to calculate!

Basics:
- Why use Monte Carlo to simulate the electricity market?

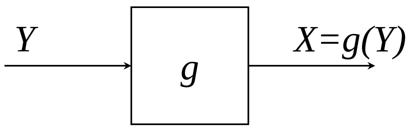
- Remember we are interested in system indices such as EENS, LOLP and ETOC! These are expected values of result variables
  $\Rightarrow$ Monte Carlo simulation
- The electricity market is complicated to predict! Sometimes we need more complicated models than probabilistic production cost simulation (PPS). For example if we want to consider:
  - Transmission limitations
  - Transmission losses
  - Correlations between stochastic variables

Basics:
–What is Monte Carlo simulation?



- The scenario parameters, *Y*, have **known** probability distributions (input).
- Mathematical model (known), *g,* of the system we want to simulate
- The result variables, *X*, have **unknown** probability distributions (output).

Basics:
–What is Monte Carlo simulation?



The purpose of the simulation is to study the probability distribution for X

- Reconstruct the whole distribution (estimate f_X)
- Estimate statistical measures such as the expected value, *E[X]*, and the variance, *Var[X]*

Basics:
–What is Monte Carlo simulation?

- Basic principal:
  - The expected value of a random variable can be determined by random observations of the variable.
- The expected value can be estimated:
  - Expected value = The mean value of an infinite number of observations of a random variable

Basics:
–What is Monte Carlo simulation?

- It is not possible to perform an infinite number of samples/observations. However, the more observations we use the "better" estimation of the expected value we get.
- *Simple sampling*: Estimation of the expected value by taking the mean value of a sufficient number of independent observations

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Simple sampling

- *Theorem 6.21*

If there are n independent observations, x1, ..., xn, of the random variable X then the mean of these observations, i.e.,

$$m_X = \frac{1}{n}\sum_{i=1}^{n} x_i$$

is an estimate of E[X].

Simple sampling

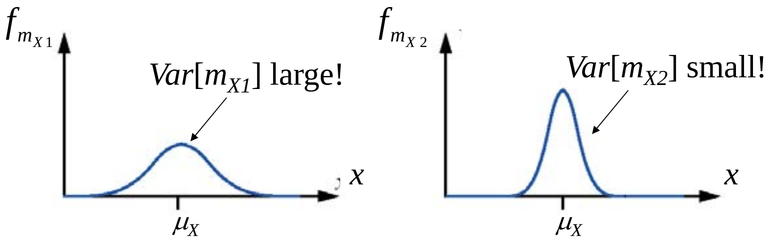
Thus:

- m_X is an estimation of $E[X]$
- m_X is a random variable (since it is a mean of observations of a random variable)
- The expected value of m_X is the same as the expected value of X:
  $E[m_X] = E[X]$

Simple sampling

- The variance of the estimate, $Var[m_X]$, is interesting because it states how much an estimate might deviate from the true value.

Remember: $\mu_X = E[m_X] = E[X]$ ← *This is what we want!*



Here m_{X1} is ***likely*** to be less accurate than m_{X2}.

Simple sampling

- Want a sufficient accurate estimate. Use the estimate's variance.

- Theorem 6.22 :

The variance of the estimate from simple sampling is:

$$Var[m_X] = \frac{Var[X]}{n}$$

=> The more observations/samples we use the more likely is it that we get a more accurate result

Simple sampling

Example 6.20

Problem:

- Tossing a coin
- Calculate the probability distribution for the average outcome of tossing the coin 1,2,10,100 and 1000 times.

Solution:

- For a complete solution see the compendium.

Simple sampling

- Let C_i be the result of tossing a coin:
  Heads => C_i=1
  Tails => C_i=0

  – H_n: the average outcome of n throws is

$$H_n = \frac{1}{n}\sum_{i=1}^{n} c_i$$

  – H_n is a random variable since it is the sum of random observations
  – We want to study the probability function of H_n

Simple sampling

- Remember from before:

$$E[m_X] = E[X]$$

where m_x is the estimation of E[X]

- Here we instead have:

$$E[H_n] = E[C]$$

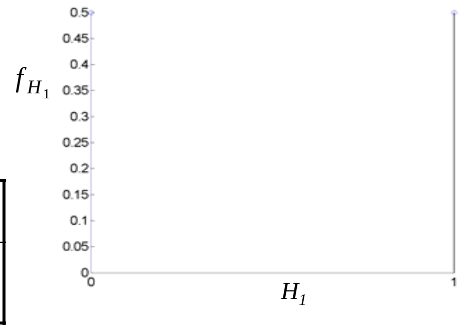
where H_n is the estimation of E[C]
In this case E[C] is simple to calculate theoretically:

$$E[C] = \{C\,discrete\} = \underbrace{0.5\cdot 0 + 0.5\cdot 1}_{\text{since } E[X]=\sum_x x\cdot f_X(x)} = 0.5$$

Simple sampling

- One trial:

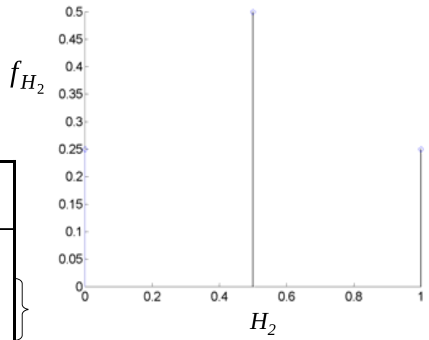
C_1	Probability [%]	H_1
0	50	0
1	50	1



Simple sampling

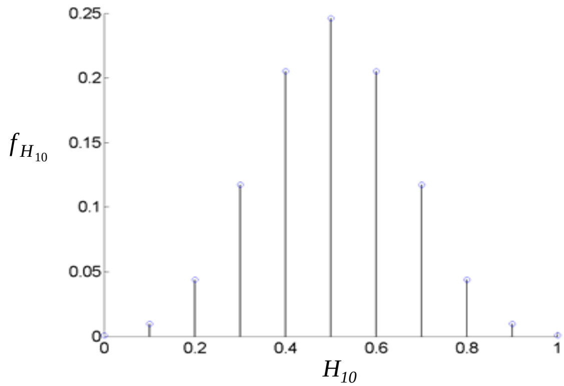
Two trials:

C_1	C_2	Probability [%]	H_2
0	0	25	0
0	1	25	0.5
1	0	25	0.5
1	1	25	1



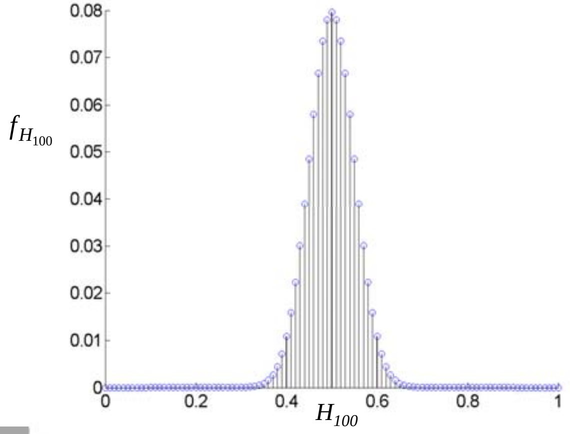
Simple sampling

- Ten trials:



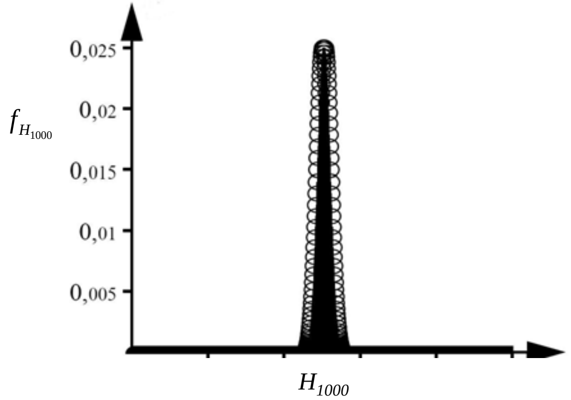
Simple sampling

- Hundred trials:



Simple sampling

- Thousand trials:



Simple sampling

- To conclude: More samples increase the likelihood that our estimation is close to the true expected value (more accurate).
- BUT: The more samples we take, the longer simulation times we get!
- How many samples should we take??
  - We need a convergence criteria to say when to end the simulation!

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Convergence criteria

- Two different methods:
  1. Predefined number of samples
     (intuition or calculated)
  2. Study the coefficient of variation, a

Convergence criteria

1. Predetermined number of samples
   (intuition or calculated)

- See **Exampel 6.21** in the compendium on how to calculate the number of samples n

Convergence criteria

2. Study the coefficient of variation, a

$$a = \frac{\sqrt{Var[m_X]}}{m_X}$$

Var[m_X] is not known during the simulation but it can be estimated using Theorem 6.22:

$$Var[m_X] = \frac{Var[X]}{n} \qquad s_X^2 = \frac{\sum_{i=1}^{n}(x_i - m_X)^2}{n}$$

Convergence criteria

2. Contin.: Study the coefficient of variation, a

How is this done?

Step 1. Choose a number of samples, n.

Step 2. Calculate Var[X] according to:

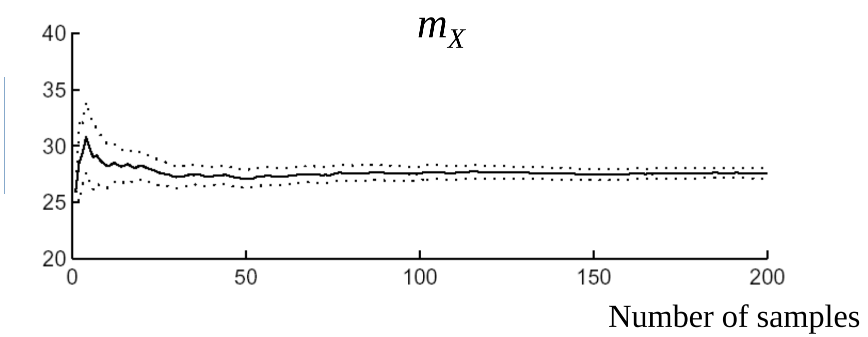
$$s_X^2 = \frac{\sum_{i=1}^{n}(x_i - m_X)^2}{n} \quad \Rightarrow \text{calculate a, see previous slide}$$

Step 3. Test if a < ρ? (ρ=tolerance limit)

Yes ⇒Simulation done.

No ⇒ Generate some more samples and add those to the existing samples. Go to step 2.

Convergence criteria



—— Estimation of m_X as a function of the number of samples used
········ +- one standard deviation

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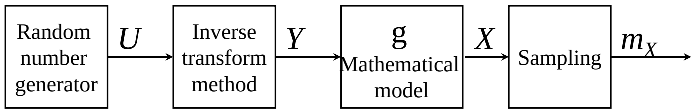
The principal of simple sampling



- *Y*: Indata, random variables that have known probability distributions (scenario parameters).
- g: Mathematical model of the system we want to simulate.
- *X*: output, random variables that have unknown probability distributions (result variables).
- m_X is an estimation of E[X].
- U is a random variable that is uniformed distributed in the interval [0 1], hence U(0,1)-distributed.

Random number generation

- We want random numbers that are U(0,1):
  - Use Matlabs **rand**.
- But how does Matlabs **rand** work?
  - **rand** is a radnom number generator that generates pseudorandom number.
  - A pseudorandom number is not a “real” random number but it is as good as it gets.
  - Given a seed (a certain number) the random number generator generates a long sequence of random numbers before repeating itself.
  - A good pseudorandom number generator produces a sequence which closely mimics the properties of a *U*(0, 1)-distribution and where the correlations between the random numbers are negligible.

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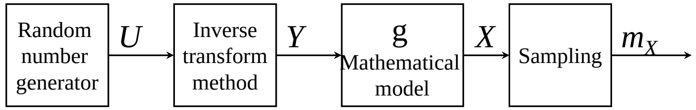
The principle for simple sampling



- g: Mathematical model of the system we want to simulate.
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- m_X is an estimatation of E[X].
- U is a random variable that is uniformed distributed in the interval [0 1], hence U(0,1)-distributed.

Inverse transform method

- It is not likely that the scenario parameters Y are U(0, 1)-distributed
- Hence we want to translate our random number U to a random number from Y:s probability distribution!

  ⇒Use the inverse transform method:

Theorem E.1.:

> If a random variable U is U(0, 1)-distributed then the random variable $Y=F_Y^{-1}(U)$ has the distribution function $F_Y(y)$.

Inverse transform method

- Remember the definition for the distribution function:

$F_Y(y) = P(Y \leq y) \quad \Rightarrow \quad F_Y(y)$ take values between 0 och 1.

Define:

$$Y = F_Y^{-1}(U) \quad \Rightarrow \quad F_Y(y) = F_Y(F_Y^{-1}(U)) = U$$

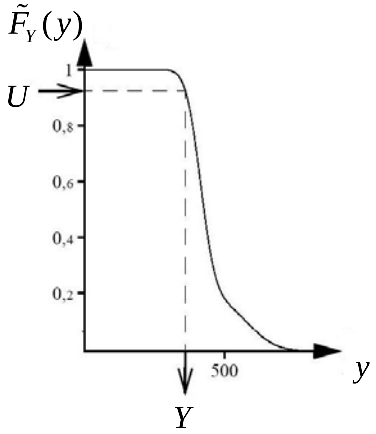
Note that we can use $\tilde{F}_Y(y)$ instead of $F_Y(y)$

$$\tilde{F}_Y(y) = P(Y > y) = 1 - P(Y \leq y) = 1 - F_Y(y)$$

The duration curve is more interesting in our application to the electricity market.

Inverse transform method

- Easiest to understand from a figure:



Inverse transform method

- Often Y is normally distributed with the distribution function $\Phi(y)$
- BUT $\Phi^{-1}(y)$ does not exist!
  $\Rightarrow$ Use the approximation of $\Phi^{-1}(y)$

Explained in the compendium in Theorem E.2.

The most important from today:

- The inverse tranform method
- The principal for simple sample

Next time:

- Remaining: Apply Monte Carlo simulations to the electricity market

  Objective

  Predict the long-term behavior of an electricity market

  Who has an interest to do this?

  – Actors in the electricity market might use the simulation to evaluate if an investment is beneficial or not
  – The government might use the simulation to see what consequences a certain regulation have on electricity prices, environment, etc before realizing it