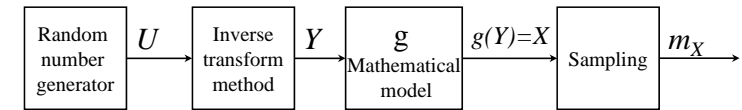# System planning VT13

- Lecture L16: **Monte Carlo**
- Content:
  1. **Sampling of electricity markets**
  2. Multi-area model
  3. Calculate!

Sampling of electricity markets

- **Objective**: To predict the behavior of the electricity market given a set of conditions (inputs).
- Can not do a physical experiment with the electricity market, perform a simulation by the help of a computer.
- Each sample is an observation on how the electricity market behaves given a certain *scenario*. There are an infinite number of scenarios.
- Note that both X and Y can be vectors of stochastic variables.
- X is a vector of the variables whose expected values we want to estimate. Often many stochastic variables affect X, these variables is given in a vector Y.
- What are X, E[X], Y and g when you sample an electricity market?

X: Vector of result variables

- X is a vector of stochastic variables. This vector is our output and contains the *result variables*.

- The result variables probability distributions are **unknown**.

- Typical result variables for simulation of electricity market:
  - TOC: Total Operation Cost.
  - LOLO: Loss Of Load Occasion, takes the value 1 if loss of load and 0 otherwise.
  - ENS: Energy Not Served

- What is E[X] that the Monte Carlo simulation estimates?

E[X]: System indices

- In many cases it is inconvenient to consider all possible outcomes of the result variables X. More appropriate to use key values, which summarizes the most important characteristics of an electricity market.

- Therefore we are satisfied to only study the system indices (the expected values of X):

  $$E[X]=\{X=g(Y)\}=E[g(Y)]$$

- Typical system indices for electricity market simulations:
  - ETOC = E[TOC], the expected total operation cost
  - LOLP = E[LOLO], the loss of load probability (risk of power deficit)
  - EENS = E[ENS], the expected energy not served

Y: Vector of scenario parameters

- Y is a vector of stochastic variables. These stochastic variables are our inputs and are called *scenario parameters*.

- The scenario parameters' probability distributions are **known**.

- The scenario parameters describe the conditions on the electricity market.

- Typical scenario parameters in for electricity market simulation:
  - Load
  - Maximal generation capacity for the different power plants

g: Electricity market model

- g is a mathematical model that shows how an electricity market reacts in a given scenario. This model is called the *electricity market model*.

- In this model there are some constants that are the same in <u>all</u> scenarios. These constants are called *model constants*.

- It is possible to apply any model – depend on the purpose.

- Common electricity market model: *Multi-area model*.

System planning VT13

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Multi-area model

- *The Multi-area model* is the most common electricity market model and the only electricity market model that is presented in this course.

- An electricity market model consists of two parts:
  1. Assumptions on how the actors in the electricity market react
  2. Model of the power system

- We will soon see that the multi-area model is an optimization problem that includes both part 1 and part 2.

Multi-area model

1. <u>Assumptions on how the actors in the electricity market reacts</u>

- We assume that the actors in the electricity market will act to maximize their profit by solving the optimization problem:

|	
maximize	*profit*
when considering	*the physical limits in the power system*

- The multi-area problem corresponds to a simplified short-term planning problem that the actors in the electricity market have to solve. The difference is that in the short-term planning problem more detailed models of the power plants and power system are used.

Multi-areamodellen

1. <u>Contin: Assumptions on how the actors in the electricity market react</u>

$$Y \longrightarrow \boxed{g} \longrightarrow X=g(Y)$$
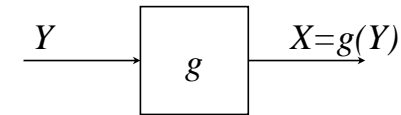
- For each observation of y_i (a certain load level, a certain maximal generation capacity, etc) each actor maximizes its profit. With the solution of the optimization problem an observation of x_i is obtained. By solving the optimization problem many times for different y_i we generate many observations on x_i and can thereby estimate the expected values: ETOC, LOLP, and EENS.

- For the actors to make optimal decisions they have to have perfect information and it has to be perfect competition. We assume that we have perfect information and perfect competition.

Multi-area model

2. <u>Model of the power system:</u>

- **Problem**: Monte Carlo simulation contains a great number scenarios and for each scenario an optimization problem need to be solved $\Rightarrow$ Very time consuming!!

- **Solution**: Only have as few as possible variables in the optimization problem $\Rightarrow$ do simplifications of the power system.

- **Method**:
  1. Simplify the power system by merging nodes in the system. This is done by dividing the power system into areas.
  2. The loads in each area is merged into one load and the load is assumed not to be price sensitive.
  3. The electricity network is only represented by the lines between the areas (the transmission lines). Transmission capacity and transmission losses between the areas are considered. The distribution lines within the area are not considered.

Multi-area model

2. <u>Contin: Model of the power system:</u>

Area 1 Area 2 Area 3
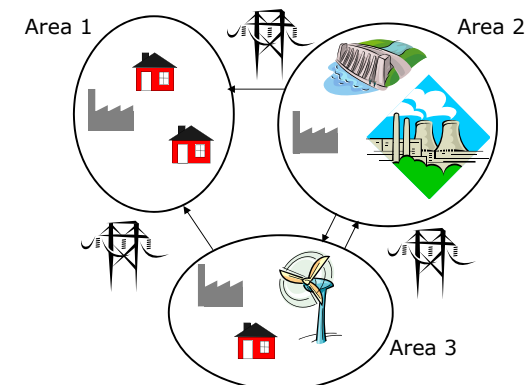
Multi-area model

2. <u>Contin: Model of the power system:</u>

- More simplifications:

    1. **Thermal power plants**:
        - Modeled by a certain available generation capacity and a cost function. Only consider the generation cost (NOT the start-up cost due to integer variables)

    2. **Non-dispatchable power plants**:
        - Power plants where the generation capacity depend on some factor beyond the human control, such as wind power plant and hydro plants without reservoirs
        - The generation cost is neglected and the available generation capacities in all non-dispatchable units are summed per area

---

Multi-area problem

- To maximize the profit is the same as to minimize the cost.

- The following optimization problem is solved for each scenario:

minimize   *generation cost +*
              penalty cost of load shedding

Subject to   *load balance in each area*

                limitations in generation and transmission capacity

---

Multi-area problem

- Optimization variables:
    G_g = generation in thermal power plant g
    W_n = non-dispatchable generation in area n
    $P_{n,m}$ = transmission from area n to area m
    U_n = unserved load in area n

    Parameters:
    P = set of interconnections (n,m)
    $P_{n \leftarrow m}$ = set of areas m capable of exporting to area n
    $P_{n \rightarrow m}$ = set of areas m capable of importing from area n

---

Multi-area problem

Parameters (contin.):
    C_{Gg} = cost function for generation in thermal power plant g
    C_{Un} = penalty cost for unserved load in area n
    D_n = load in area n
    $\overline{G_g}$ = maximal generation in thermal power plant g
    $\overline{W_n}$ = maximal non-dispatchable generation in area n
    G = set of thermal power plants g
    G_n = set of thermal power plants g located in area n
    $L_{n,m}$ = loss function for transmission from area n to area m
    N   = set of areas n
    $\overline{P}_{n,m}$ = Maximal transmission from area n to m

Multi-area problem

Objective function:

Note that the start-up costs for the thermal power plants are NOT considered!

$$\min \sum_{g \in G} C_{Gg}(G_g) + \sum_{n \in N} C_{Un}(U_n)$$

Total generation cost for the thermal power plants

Total penalty cost for disconnected load

Note that the generation cost is neglected for non-dispatchable power plants!

Multi-area problem

- Constraints for load balance in each area:

Note that the generation from the non-dispatchable power plants is merged for each area n

$$\sum_{g \in G_n} G_g + W_n + \sum_{m \in P_{n \leftarrow m}} \left(P_{m,n} - L_{m,n}(P_{m,n}) \right) = D_n + \sum_{m \in P_{n \rightarrow m}} P_{m,n} - U_n, \quad \forall n \in N$$

Generation in area n | Import to area n | Load i area n | Export from area n | Unserved load in area n

---

Multi-area problem

- Constraints for the optimization variables: Limitations in generation and transmission capacity:

$$0 \le G_g \le \overline{G}_g \qquad \forall\, g \in G$$

$$0 \le P_{n,m} \le \overline{P}_{n,m} \qquad \forall\, (n,m) \in P$$

$$0 \le U_n \le D_n \qquad \forall\, n \in N$$

$$0 \le W_n \le \overline{W}_n \qquad \forall\, n \in N$$

Multi-area problem

- Run the optimization. Then calculate:

$$TOC = \sum_{g \in G} C_{Gg}(G_g) + \sum_{n \in N} C_{Un}(U_n)$$

Delete if the penalty cost is made-up and not real compensations paid to consumers.

$$LOLO = \begin{cases} 0 & om \sum_{n \in N} U_n = 0 \\ 1 & om \sum_{n \in N} U_n > 0 \end{cases}$$

At least one area is disconnected.

Multi-area model

- What are the scenario parameters and the result variables in the multi-area model when it is applied in a Monte Carlo simulation?

Scenario parameters (Y):
– The parameters that decide the conditions!
– Maximal available generation capacity in the power plants: $\overline{G}, \overline{W}$
– Available transmission capacity: $\overline{P}$
– Load: D

Result variables (X):
– Total operation cost: TOC
– Loss of load occasion: $LOLO$

System planning VT13

- Lecture L16: **Monte Carlo**
- Content:
  1. Sampling of electricity markets
  2. Multi-area model
  3. **Calculate!**

Calculate!

- Example 6.1: Multi-area model

  Do you want to pass home assignment 21)?

  Listen carefully…

Multi-area model

Example 6.1:

- In Mji there is a diesel generator set, having a capacity of 250 kWh/h. Due to poor maintenance, failures are quite frequent in the diesel generator set. The operation cost is 10 ¤/kWh.

- There is also a wind power plant at Mlima, which is located about 15 km from Mji. The installed capacity of the wind power plant is 200 kW and the operation cost is negligible.

- Just next to Mlima, there is a village, Kijiji, which also is connected to the power. The load in Mji and Kijiji can be considered normally distributed and independent of each other.

- Mlima and Kijiji are connected to Mji via a transmission line. It can be assumed that the losses on the line are 2% of the injected power and that the line is never subject to failures.

- **Formulate a two-area model of the power system. The objective of the study is to analyze ETOC and LOLP. What are Y and E[X]?**

Calculate!

- Example 6.22: Simple sampling of electricity markets

  This example will return for every variance reduction technique!

Sampling of electricity markets

Example 6.22:

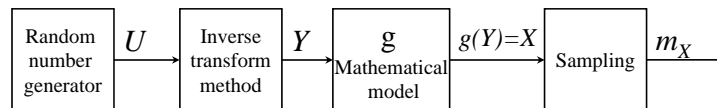Small village with their own grid. Data according to:

- Load is normally distributed with the mean 180 kW and standard deviation 40 kW.
- Three power plants:
  - Hydro plant (no reservoir), installed capacity 150 kW, operation cost 0 SEK/kWh
  - Diesel generator 100 kW, operation cost 1 SEK/kWh, never operated at less than 40 kW. If it is necessary a water heater can be used to consume any surplus generation.
  - Diesel generator 50 kW, operation cost 2 SEK/kWh
  - All power plants are 100% available
- **Calculate ETOC, using Monte Carlo (10 obs. Are enough for the estimation) and exact.**

Sampling of electricity markets

Solution using Monte Carlo:

Random number generator $\xrightarrow{U}$ Inverse transform method $\xrightarrow{Y}$ g Mathematical model $\xrightarrow{g(Y)=X}$ Sampling $\xrightarrow{m_X}$

Model constants:

Available capacity since all power plants are 100 % available they will be the same for all scenarios.

Scenario parameters Y:

Load, D, since it is stochastic with a known probability distribution. We know it is normal distributed with mean 180 and standard deviation 40.

Result variables X:

TOC since it is the expected value ETOC=E[TOC] we want

Mathematical model, g:

Assume all power plants are operated so that the operation cost is minimized.
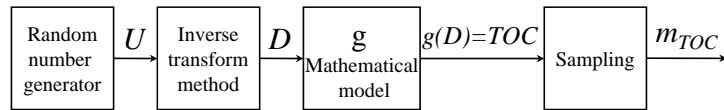
Sampling of electricity markets

- Table shows how the power plants are operated at different load levels, hence the function $g(D)=TOC.$

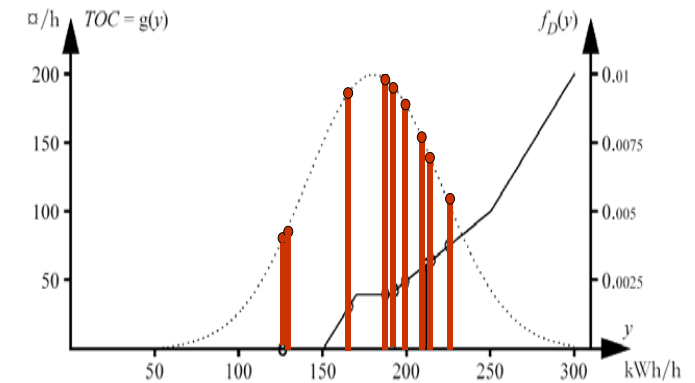| D [kW]	TOC [SEK/h]	Kommentar
0-150	0	Hydro plant is running
150-170	2*(D-150)	Hydro plant is running on max, the small generator follow the load
170-190	40	Hydro plant is running on max, the large generator is on 40 kW, surplus to heating
190-250	1*(D-150)	Hydro plant is running on max, the large generator follow the load
250-300	100+2*(D-250)	Hydro plant is running on max, the large generator is on max, the small generator follow the load
>300	200	All power plants are on max, load have to be disconnected

Sampling of electricity markets

| Random number generator | U → | Inverse transform method | D → | g Mathematical model | $g(D)=TOC$ → | Sampling | m_{TOC} → |

1. Assume 10 observations of TOC are enough to estimate ETOC.
2. Start with generate 10 random numbers from U(0,1) and transform these according to Appendix E to random numbers for the load from a N(180,40). We now have 10 observations of D: d_i, i=1,…,10.

Sampling of electricity markets

Sampling of electricity markets

| Random number generator | U → | Inverse transform method | D → | g Mathematical model | $g(D)=TOC$ → | Sampling | m_{TOC} → |

1. Assume 10 observations of TOC are enough to estimate ETOC.
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3. Use g that are given in the table to calculate 10 observations on TOC: $g(d_i)=TOC_i$, i=1,…,10.

Sampling of electricity markets

- Table shows how the power plants are operated at different load levels, hence the function $g(D)=TOC$.

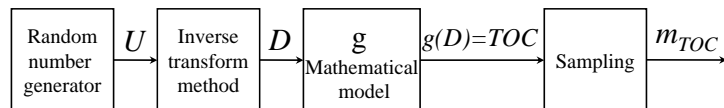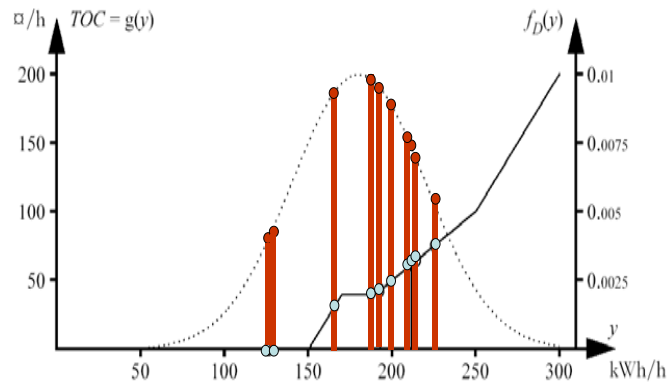| D [kW]	TOC [SEK/h]	Kommentar
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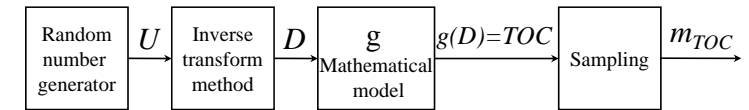# Sampling of electricity markets

Sampling of electricity markets

1. Assume 10 observations of TOC are enough to estimate ETOC.
2. Start with generate 10 random numbers from U(0,1) and transform these according to Appendix E to random numbers for the load from a N(180,40). We now have 10 observations of D: d_i, i=1,…,10.
3. Use g that are given in the table to calculate 10 observations on TOC: $g(d_i)=TOC_i$, i=1,…,10.
4. We want ETOC=E[TOC]. We estimate ETOC by taking the average value of our 10 observations:

$$m_{TOC} = \frac{1}{10}\sum_{i=1}^{10} TOC_i$$

Sampling of electricity markets

- Estimation of ETOC:

$$m_{TOC} = \frac{1}{10}\sum_{i=1}^{10} TOC_i = 42.61 \text{ SEK/h}$$

Solution when ETOC is calculated exactely:
- For this small example it is possible to calculate ETOC exact:

$$ETOC = \int_{-\infty}^{\infty} TOC(x) f_D(x) dx = \ldots = 39.66 \text{ SEK/h}$$

The integral is solved numerically!

Calculate!

- Exercise 6.13: Simple sampling of electricity markets

Sampling of electricity markets

- Exercise 6.13:

In a Monte Carlo simulation of an multi-area model, 10 000 scenarios have been generated.

In 12 of these, the total unserved power, ΣUn, was larger than zero. What is the *LOLP* estimate from this simulation?

The most important from today:

- Function g = Electricity market model = multi-area model

- Multi-area model: Formulated as a optimization problem considering assumptions on how the actors in the electricity market reacts and a model of the power system

Next time:

- Objective: Want to make our estimate of E[X] more accurate without having to generate more samples/observations!

- Method: Choose our samples in a smarter way by using variance reduction techniques:
  - Complementary Random Numbers
  - Control Variates
  - Stratified Sampling
  - Or combined them…