11 Networked Control Systems

EXERCISE 11.1 Matrix Exponential

Let A be an $n \times n$ real or complex matrix. The exponential of A, denoted by e^A or $\exp(A)$, is the $n \times n$ matrix. Find e^A using two different way, where

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} .$$

EXERCISE 11.2 Stability

Given a bi-dimensional state space system

$$X_{t+1} = \Phi X_t ,$$

1. show how to compute the eigenvalues of Φ.
2. make some comments on the relationship between the eigenvalues of Φ and the stability.

EXERCISE 11.3 Modeling

Model the dynamics of a coordinated turn (circle movement) using Cartesian and polar velocity. Here we assume that the turn rate ω is piecewise constant.

EXERCISE 11.4 Linearized Discretization

In some cases, of which tracking with constant turn rate is one example, the state space model can be discretized exactly by solving sampling formula

$$x(t+T) = x(t) + \int_t^{t+T} a(x(\tau))d\tau ,$$

analytically. The solution can be written as

$$x(t+T) = f(x(t)) .$$

Using this method, discretize the models in Ex:11.3.

EXERCISE 11.5 Modeling of the Temperature Control

Assume that in winter, you'd like to keep the temperature in the room warm automatically by controlling a house heating system. Let T_i, T_o and T_r denote the temperature inside, outside and radiator. Thus the process model can be simplified as

$$\begin{aligned} \dot{T}_i =& \alpha_1(T_r - T_i) + \alpha_2(T_o - T_i) \\ \dot{T}_r =& \alpha_3(u - T_r) . \end{aligned}$$

1. Model the dynamics in standard state space form. Here assume that the outside temperature is aroud zero, $T_o = 0$.