AUTOMATIC CONTROL

KTH

EL2745 Principles of Wireless Sensor Networks

Exam 08:00–13:00, June 8, 2012

Aid:

Lecture notes (slides) from the course, reading material, and textbook ("Principles of Embedded Networked System Design" by Pottie & Kaiser or similar text approved by course responsible). Mathematical handbook (e.g., "Beta Mathematics Handbook" by Råde & Westergren). The course compendium, other textbooks, handbooks, exercises, solutions, calculators etc. may **not** be used.

Observandum:

- Name and social security number (*personnummer*) on every page.
- Only one solution per page and write on one side per sheet.
- Each answer has to be motivated.
- Specify the total number of handed in pages on the cover.
- Each subproblem is marked with its maximum credit.

Grading:

Grade A: ≥ 43, Grade B: ≥ 38

Grade C: ≥ 33, Grade D: ≥ 28

Grade E: ≥ 23, Grade Fx: ≥ 21

Results:

The results will be available at STEX (*Studerandeexpeditionen*), Osquldas väg 10.

If you want your result emailed, please, state this and include your email address.

Responsible: Carlo Fischione, `carlofi@kth.se`

Good Luck!

1. **Quantization of sensor observations**

   Consider a sensor sampling a bandlimited continuous time signal $s(t)$ at time nT_s, where n is the sample number and T_s is the sampling interval. The signal is assumed to lie within a predefined range $[-A, A]$. The sensor samples $s(nT_s)$ are then quantized and a analog-to-digital (A/D) conversion follows. The A/D converter assigns amplitude values in this range to a set of integers. The quantization interval is the range of values assigned to the same integer. A B-bit converter produces one of the integers $\{0, 1, \ldots, 2^B - 1\}$ for each sampled input. The D/A converter then recovers the original signal by converting integers to amplitudes and assigning an amplitude equal to the value lying halfway in the quantization interval.

(a) [2p] Since values lying anywhere within a quantization interval are assigned the same value, the original amplitude value is recovered with errors. Characterize the distribution of the quantization error as function of the number of bits.

(b) [2p] Let be Δ the quantization interval. Let the quantization error be denoted by ϵ. Calculate the root mean square *rms* value of the quantization error.

(c) [2p] For a sinusoid signal $s(t)$ characterize the signal to noise ratio resulting after the quantization.

(d) [2p] How many bits would be required in the A/D converter to ensure that the maximum amplitude quantization error is less than 60 dB smaller than the signal's peak value?

(e) [2p] Suppose that we would like to have a 16-bit quantization. To what signal-to-noise ratio does this correspond?

2. **MAC**

Consider a simple ARQ scheme through a single transmission link of data rate R. In ARQ, the sender transmits a data packet across the link. Once the receiver receives the packet, it checks if data have been corrupted due to collisions with multiple transmissions or bad channel. If there is no error, an acknowledgement packet is sent to the sender to acknowledge the correct reception of the data packet. If there is an error, an ARQ packet is sent for asking a retransmission. The sender resends the packet immediately after it receives the ARQ packet. Assume the lengths of data and ARQ packets are L and L_{ARQ} respectively, and the propagation delay along the link is given as t_d. Neglect the turn-around time at the sender and the receiver. Suppose that the probability the data packet is corrupted during transmission is P_e, whereas ARQ packets are always correctly received.

(a) [2p] Determine the average number of transmissions required for a packet to be correctly received.

(b) [2p] Find the average delay a packet experiences. The delay is defined as the time interval between the start of the first packet transmission and the end of the correct packet reception. Note it does not include the transmission of the last acknowledgement packet.

(c) [2p] Now, suppose that the transmission of a packet is governed by a Slotted Aloha MAC and that the number of competing senders n is not exactly known. We assume that in each time slot each sender transmits with probability p. Argue how P_e defined above is related to p.

(d) [4p] In the Slotted Aloha MAC scenario defined above, let the probability that the slot can be used (i.e. the probability that exactly one station transmits) be $\Pr(\text{success}) = n \cdot p(1-p)^{n-1}$. If n is fixed, we can maximize the above expression and get the optimal p. Now assume we only know that $A \leq n \leq B$, with A and B being two known constants. What is the value of p that maximizes $\Pr(\text{success})$ for the worst $n \in [A, B]$?

3. **Routing**

Let the required energy to send a packet from node A to node B be $E(A, B) = d(A, B)^\alpha$, where $d(A, B)$ is the distance between node A and B and α is a system parameter with $\alpha > 2$. Assume that you are allowed to place a number of equidistant routing nodes between source node S and destination node T. Here, routing nodes serve as intermediate nodes to route packets from S to T. For instance, if S and T would use routing nodes A and B, the message would be sent from S to A, from A to B and finally from B to T.

(a) [1p] Characterize the expression of the total energy consumption to route packets from node A to T.

(b) [2p] Based on the total energy derived above, give the ideal number of routing nodes to send a message from S to T with minimum energy consumption and how much energy would be consumed in such an optimal case.

(c) [2p] Assume now an energy model that determines the energy required to send a message from A to B as $E(A, B) = d(A, B)^\alpha + c$, with $c > 0$. Argue why this energy model is more realistic the the one described in the text of the exercise.

(d) [3p] Consider the energy model introduced in last item. Show that there exists an optimal number n of equidistant intermediate routing nodes between S and D that minimizes the overall energy consumption when using these intermediate nodes to route a packet from S to T. [Assume n as a continuous variable for simplicity].

(e) [2p] Derive a closed-form expression on how much energy will be consumed when using this optimal number n of relay nodes. [Assume n as a continuous variable for simplicity].

4. **Distributed detection, MAC, and routing**

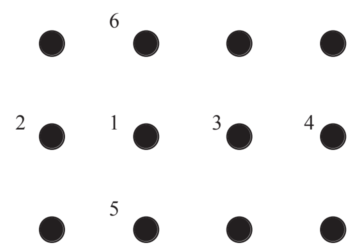

Figure 1: A grid of sensor nodes.

Sensor nodes are laid out on a square grid of spacing d, as depicted in Figure 1. Ever sensor wants to detect a common source.

(a) [2p] Suppose that the source signal has a Gaussian distribution with zero mean and with variance σ_S^2. Moreover, every sensor measures such a signal with an additive Gaussian noise of zero average and variance σ_n^2. If the measured signal is positive, the sensor decides for hypothesis H_0, otherwise the sensor decides for hypothesis H_1. Based on the measured signal, characterize the probability of false alarm and the probability of miss detection per every sensor.

(b) [3p] Now, suppose that the source signal is constant and has a power S. Such a signal power is received at every sensor with an attenuation given by r_i^2, where r_i is the distance between the source and sensor i. Sensor node 1 is malfunctioning, producing noise variance $10\sigma_n^2$. The two best nodes in terms of SNR will cooperate to provide estimates of the source. Characterize the region of source locations over which node (1) will be among the two best nodes.

(c) [3p] The grid depicted in Figure 1 is also used for relaying. Assume it costs two times the energy of a hop among nearest neighbors (separated by distance d) to hop diagonally across the square (e.g. node 2 to 5) and eight times the energy to go a distance of 2d in one hop (e.g. node 2 to 3). Let p be the packet loss probability. Characterize p for which it is better to consider using two diagonal hops to move around the malfunctioning node.

(d) [2p] Under the same assumption of the previous item, suppose that there is an ARQ protocol, but the delay constraints are such that we can only tolerate three retransmission attempts. Let 0.99 be the probability of having up to three retransmissions. Assuming packing dropping events are independent, characterize the constraint that probability of packet losses per transmission should satifsy.

5. **Networked Control System**

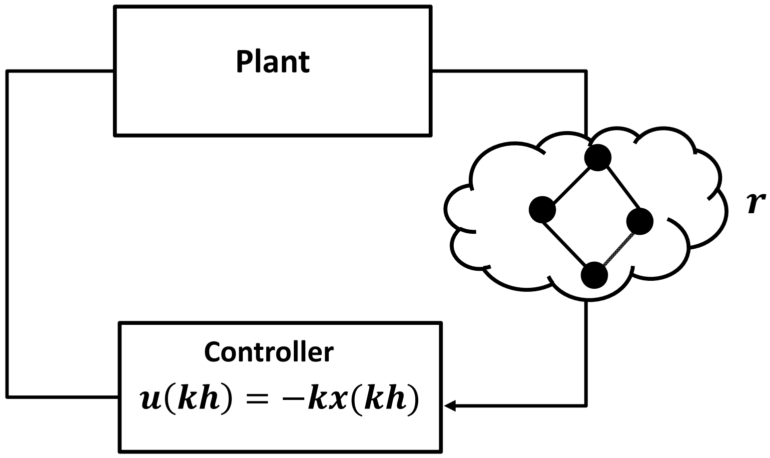

Figure 2: Closed loop system over a WSN.

Consider the Networked Control System (NCS) in Fig. 2. The system consists of a continuous plant

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{1a}$$
$$y(t) = Cx(t)\,, \tag{1b}$$

where $A = a$, $B = 1$, $C = 1$. The system is sampled with sampling time h, and the discrete controller is given by

$$u(kh) = -Kx(kh), \qquad k = 0, 1, 2, \ldots\,,$$

where K is a constant.

(a) [2p] Suppose that the sensor network has a medium access control and routing protocols that introduce a delay $\tau \leq h$. Derive a sampled system corresponding to Eq.(1) with a zero-order-hold.

(b) [2p] Under the same assumption above that the sensor network introduces a delay $\tau \leq h$, give an augmented state-space description of the closed loop system so to account for such a delay.

(c) [3p] Under the same assumption above that the sensor network introduces a delay $\tau \leq h$, characterize the conditions for which the closed loop system becomes unstable [Hint: no need of computing numbers, equations will be enough]

(d) [3p] Now, suppose that the network does not induce any delay, but unfortunately introduces packet losses with probability p. Let $r = 1 - p$ be the probability of successful packet reception. Give and discuss sufficient conditions for which the closed loop system is stable. If these conditions are not satisfied, discuss what can be done at the network level or at the controller level so to still ensure closed loop stability.