AUTOMATIC CONTROL

KTH

EL2745 Principles of Wireless Sensor Networks

Exam 08:00–13:00, August 20, 2012

Aid:

Lecture notes (slides) from the course, reading material, and textbook ("Principles of Embedded Networked System Design" by Pottie & Kaiser or similar text approved by course responsible). Mathematical handbook (e.g., "Beta Mathematics Handbook" by Råde & Westergren). The course compendium, other textbooks, handbooks, exercises, solutions, calculators etc. may **not** be used.

Observandum:

- Name and social security number (*personnummer*) on every page.
- Only one solution per page and write on one side per sheet.
- Each answer has to be motivated.
- Specify the total number of handed in pages on the cover.
- Each subproblem is marked with its maximum credit.

Grading:

Grade A: $\geq$ 43, Grade B: $\geq$ 38

Grade C: $\geq$ 33, Grade D: $\geq$ 28

Grade E: $\geq$ 23, Grade Fx: $\geq$ 21

Results:

The results will be available at STEX (*Studerandeexpeditionen*), Osquldas väg 10.

If you want your result emailed, please, state this and include your email address.

Responsible: Carlo Fischione, `carlofi@kth.se`

Good Luck!

1. **Slotted CSMA protocol in WSNs**

   Assume a network of N sensors within the communication range of each other and transmitting over a single channel. Access mechanism is as follows: When a node wants to send a packet, it sets a counter with a uniformly drawn random number between $[1, M]$, where M is defined as the contention window. The time is divided into fixed intervals of length t_s. After each t_s seconds, the node counts down until the counter expires. Then the sensor senses the channel and if it is idle it sends the packet at the beginning of the next slot. Consider that the packet transmission time is much larger than t_s so each contention round will finish by only one packet transmission that is either successful or collided.

(a) [2p] Assume that every node has a packet to transmit and we only consider one contention round. Explain in words under what scenario sensor i can win the contention round (successfully send its packet)? What are the conditions for which a particular slot m can happen to be a collision slot? [hint: collision happens when at least two nodes transmit simultaneously.]

(b) [4p] Define P_s as the probability of having a successful transmission after the contention round with M maximum window size and N contenders. Also denote $p_s(m)$ as the probability of success at slot m. Find $p_s(m)$ and P_s.

(c) [4p] Let P_c be the probability of collision after contention round and $p_c(m)$ be the probability of collision at slot m. Propose an analytical model to calculate P_c and $p_c(m)$.

2. **IEEE 802.15.4 and Duty-cycled WSN**

   The IEEE 802.15.4 standard can implement a synchronous duty-cycle medium access control (MAC) and a asynchronous duty-cycle MAC. In the synchronous duty-cycled WSNs, nodes are synchronized to sleep to save energy. Asynchronous duty-cycled WSN have a more complex mechanism: nodes are in dormant state to save energy and they wakeup at random once in a range of $[0, T]$ seconds. Here, when a sender node wants to send a packet to a particular destination (single hop unicast transmission), it needs to send a long preamble message (for length T) to hit the receiver whenever it wakes up. When the receiver wakes up and sees the preamble, it sends an ACK packet with sequence number -1 immediately to the sender. The ACK indicates that it is available to receive. After receiving the acknowledgment, sender transmits the data packet. If the packet is received correctly by the receiver, it will send an ACK packet with sequence number > 0, otherwise it will send another ACK packet with sequence number -1. The same scenario repeats until the packet gets through.

(a) [2p] Describe in less than one page the medium access control (MAC) protocol of the IEEE 802.15.4.

(b) [3p] Based on the description of the IEEE 802.15.4, the MAC works in two modalities: beacon enabled and non-beacon enabled (also called beaconless). Describe in less than a page which duty-cycling mechanism can be implemented in which modality of the standard.

(c) [5p] Now consider a single channel of data rate R and assume the lengths of data and ACK packets are L and L_{ack}, respectively, and the propagation delay along the link is t_d s. Neglect the turn-around time at the sender and the receiver. Suppose that the probability the data packet is dropped is P_e and ACK packets are always correctly received. The average delay a packet experiences is defined as the time interval between the moment the sender starts sending preambles and the end of the correct data packet reception (note that it does not include the transmission of the last ACK packet). Suppose $1 - P_e$ is the link PRR. What is the average delay a packet experiences from source to destination?

3. **Routing**

Consider a WSN with a single sink and multiple senders. One can find the minimum spanning tree (MST) minimizing the expected number of transmissions for each node. Let ETX[i] be the expected number of transmissions required for node i to send a packet to the sink. An example of such a WSN is depicted in Figure 1, where node 1 is the sink and success probabilities (link PRR's) are depicted on the edges.

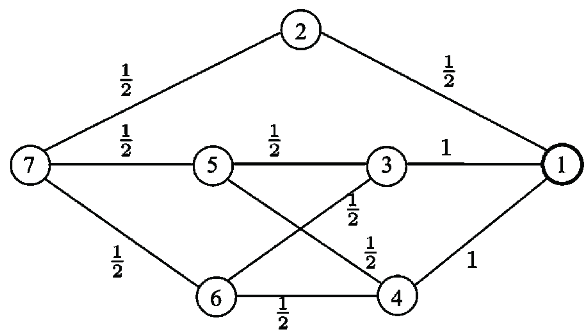

Figure 1: Initial network graph with link success probabilities. The destination (sink) is node 1.

(a) [2p] Given success probability p_i between nodes i and j calculate the expected number of transmissions to deliver a packet from i to j.

(b) [3p] Calculate ETX for each node $i = 2, \ldots, 7$ to reach the sink. Find the routing tree based on the ETX metric.

(c) [4p] Define the cost of each edge (d_{ij}) as the expected number of packet transmissions for sending a packet from i to j. Run Bellman-Ford algorithm for the graph (the shortest distance from $i = 1$ to any node) and achieve the shortest path routing and the minimum distance vector. What do you observe?

4. **Sensor node localization**

GPS uses a sensor network constellation of 24 satellites and their ground stations as reference points to calculate positions accurate to a matter of meters. Suppose we find our distance measurements from three satellites to be 18 000, 19 000, and 20 000 km respectively. Collectively this places the location at either of the two points where the 20 000 km sphere cuts through the circle that is the intersection of the 18 000 and 19 000 km spheres. Thus by ranging from three satellites we can narrow our position to just two points in space. To decide which one is our true location we could make a fourth measurement. However, usually one of the two points is a ridiculous answer (either too far from Earth or moving at an impossible velocity) and can be rejected without a measurement.

(a) [2p] Apply the above principle of location in a two-dimensional space. Assume that points A, B, and C are reference points with known locations, respectively at $(x1, y1)$, $(x2, y2)$, and $(x3, y3)$, and that the unknown position is 3.0 meters from point A, 4.0 meters from point B, and 5.0 meters from point C. Suppose that accurate measurements are available. Then the three measurements can be used to uniquely determine the position. Let $(x1, y1) = (0, 3.0)$, $(x2, y2) = (4.0, 0)$, $(x3, y3) = (4.0, 3.0)$. Find the position.

(b) [3p] Consider again the previous item. Assume that all measurements include a single timing offset that corresponds to an error of 0.5 m. In other words, the position is observed to be 3.5 m from point A, 4.5 m from point B, and 5.5 m from point C. Develop a generic procedure to find the true position.

(c) [5p] Apply the above principle of location in a three-dimensional space. Assume that the three satellites are located respectively at $(x1, y1, z1)$, $(x2, y2, z2)$, and $(x3, y3, z3)$, and that the distance between us and the three satellites are respectively $d1, d2, d3$. The following nonlinear system of equations needs to be solved,

$$\begin{aligned} (x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2 &= d_1^2 \\ (x - x_2)^2 + (y - y_2)^2 + (z - z_2)^2 &= d_2^2 \\ (x - x_3)^2 + (y - y_3)^2 + (z - z_3)^2 &= d_3^2 \end{aligned} \tag{1}$$

Obviously linearization is desirable in this case. Assume that the reference point is $(0, 0, 0)$. Prove that the resulting system after linearizing (1) is

$$2 \begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & y_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x_1^2 + y_1^2 + z_1^2 - d_1^2 \\ x_2^2 + y_2^2 + z_2^2 - d_2^2 \\ x_3^2 + y_3^2 + z_3^2 - d_3^2 \end{bmatrix}$$

5. **Stability of Networked Control Systems with Network-induced Delay**

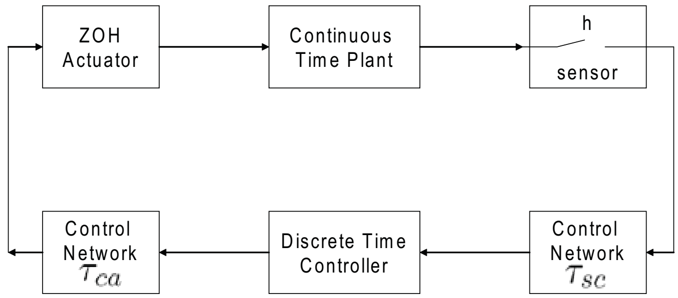

Figure 2: Networked Control System with communication delay.

Consider the Networked Control Systems (NCS) in Figure 2, where the sensor attached to the plant transmits the sampled state over a network where the receiver is a sensor connected to the controller. The system consists of a continuous plant

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{2a}$$

$$y(t) = Cx(t)\,, \tag{2b}$$

and a discrete controller

$$u(kh) = -Kx(kh), \qquad k = 0, 1, 2, \dots\,,$$

where $A \in \mathbb{R}$, $B \in \mathbb{R}$, $C \in \mathbb{R}$.

(a) [3p] Derive a sampled system corresponding to Eq. (2) with a zero-order-hold under the assumptions that $\tau_{sc} + \tau_{ca} \leq h$.

(b) [3p] Under the same assumption above, give an augmented state-space description of the closed loop system so to account for such a delay.

(c) [4p] Let $A = 0$, $B = I$. Illustrate the stability properties of the system as function of the network delays τ_{sc} and τ_{ca} under the same assumptions above that $\tau_{sc}+\tau_{ca} \leq h$ and that $h = 1/K$.