KTH Electrical Engineering

Exam in EG2050 System Planning,
13 June 2012, 8:00–13:00, V34, V35

Allowed aids

In this exam you are allowed to use the following aids:

- Calculator without information relevant to the course.
- One **handwritten**, **single-sided** A4-page with **your own** notes (original, not a copy), which should be handed in together with the answer sheet.

PART I (MANDATORY)

Write all answers on the answer sheet provided. Motivations and calculations do not have to be presented.

Part I can yield 40 points in total. The examinee is guaranteed to pass if the score is at least 33 points. If the result in part I is at least 31 points, then there will be a possibility to complement for passing the exam with the grade E.

Problem 1 (4 p)

Answer the following theoretical questions by choosing *one* alternative, which you find correct.

a) (2 p) A balance responsible player is economically responsible that the system during each trading period (for example one hour) is supplied a much energy as consumed by the customers of the player. In practice, this responsibility is managed by I) The balance responsible player is obliged to buy financial instruments in the ahead market, II) The balance responsible player is obliged to buy and sell regulating power in the real-time market, III) The balance responsible player is obliged to buy and sell balance power in the post market.

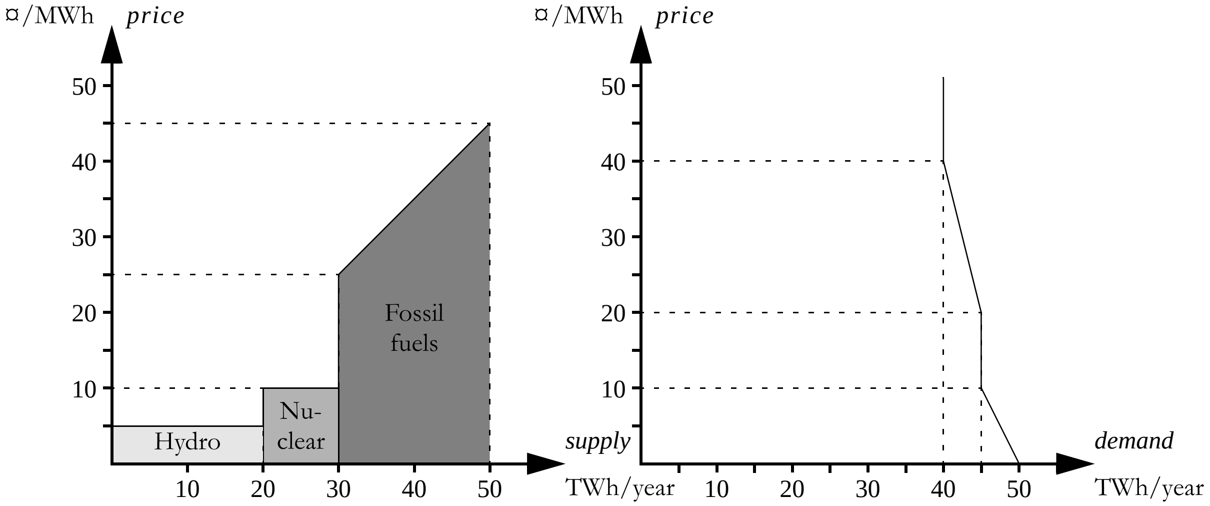
1. Only I is true.
2. Only II is true.
3. Only III is true.
4. I and II are true but not III.
5. II and III are true but not I.

b) (2 p) We use the notion "ahead trading" to describe all the trading which occurs before the hour of delivery (or any other trading period). In the ahead market it is possible to trade with the following contracts: I) Take-and-pay contracts, i.e., when the customer subscribes to a specific maximal power, and during the duration of the contract is allowed to buy any amount of energy per trading period as long as the maximal power is not exceeded, II) Firm power, i.e., when the seller is committed to deliver a specific amount of energy per trading period during the duration of the contract, III) Regulation power, i.e., when a player at request from the system operator is supplying more power to the system (up-regulation) or when a player at request from the system operator is supplying less power to the system (down-regulation).

1. None of the statements is true.
2. Only I is true.
3. Only II is true.
4. Only III is true.
5. I and II are true but not III.

Problem 2 (6 p)

a) (3 p) The figures below shows the supply and demand curves of a certain electricity market. What will the electricity price become in this electricity market if we assume perfect competition, perfect information and that there are neither transmission, reservoir nor capacity limitations?



b) (2 p) What would happen to the electricity price in this electricity market if there also was 2 TWh wind power available per year?

c) (1 p) Assume that AB Vattenkraft owns a hydro power plant and that there are no other hydro power plants in the same river. The electricity market where AB Vattenkraft is operating has perfect competition, perfect information and no transmission limitations. The installed capacity in the power plant of AB Vattenkraft is 1 000 MW and the reservoir can store water corresponding to 10 000 MWh. During the hour 10-11 AB Vattenkraft generates 800 MWh. At 11 the hydro reservoir is completely empty. What can be stated about the electricity price in this electricity market?

1. The electricity price hour 10-11 cannot be less than the electricity price hour 11-12.
2. The electricity price hour 10-11 cannot be greater than the electricity price hour 11-12..
3. There is no relation at all between the electricity prices during these two hours.

Problem 3 (6 p)

Consider a power system divided in five areas. Data for the primary control in the system are given in table 1. Data for the transmission lines between the countries are shown in table 2. Each transmission line is equipped with a protection system which after a short time delay disconnects the line if the power flow exceeds the maximal capacity of the line. The power flow on the HVDC line are not affected by the frequency of the system, but can only be controlled manually..

Table 1 Data for the primary control.

Area	Gain (available between 49.0 and 51.0 Hz) [MW/Hz]
A	3 000
B	3 000
C	1 000
D	1 000

Table 2 Data for the interconnections.

Connection	Type	Current transmission (at 10:15) [MW]	Maximal capacity [MW]
A ↔ B	Alternating current	1 000 MW from A to B	2 000
A ↔ C	Alternating current	600 MW from A to C	1 000
B ↔ D	Alternating current	1 000 MW from B to D	1 500
C ↔ D	Direct current (HVDC)	500 MW from C to D	600

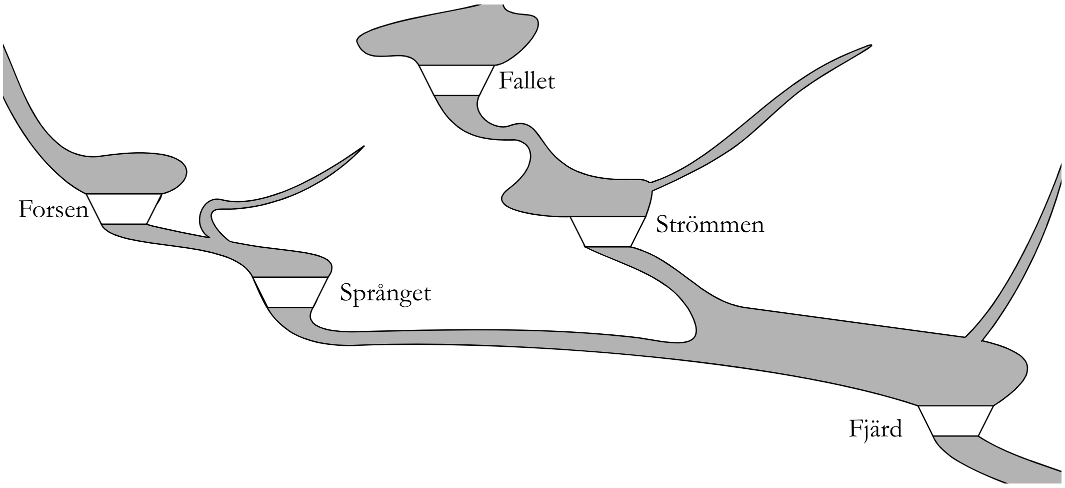
a) (1 p) At 10:00 there is balance between production and consumption in the system, no transmission lines are overloaded and the frequency in area A is 49.94 Hz. What is the frequency in area D at this occasion?

b) (2 p) At 10:15 there is balance between production and consumption in the system and the frequency is exactly equal to 50 Hz in area A. At this occasion 1 200 MW generation is lost in area B and shortly thereafter another 1 000 MW generation is disconnected in area B. How large is the transmission from area A to area B then the primary control has stabilised the frequency in the system after these two events? (Answer 0 MW if the connection is disconnected due to overloading.)

c) (1 p) What is the frequency in area A when the primary control has stabilised the frequency in the system after the events in part b?

d) (2 p) What is the frequency in area B when the primary control has stabilised the frequency in the system after the events in part b?

Problem 4 (12 p)



AB Vattenkraft owns five hydro power plant located as in the figure above. The following symbols have been introduced in a short-term planning problem for these hydro power plants:

Indices for the power plants: Forsen 1, Språnget 2, Fallet 3, Strömmen 4, Fjärd 5.

γ_i = expected future production equivalent for water stored in reservoir i, $i = 1, \ldots, 5$,
λ_t = expected electricity price hour t, $t = 1, \ldots, 24$,
λ_{25} = expected electricity price after the end of the planning period,
$M_{i,0}$ = contents of reservoir i at the beginning of the planning period, $i = 1, \ldots, 5$,
$M_{i,t}$ = contents of reservoir i at the end of hour t, $i = 1, \ldots, 5$, $t = 1, \ldots, 24$,
$\overline{M}_i$ = maximal contents of reservoir i, $i = 1, \ldots, 5$,
$\mu_{i,j}$ = marginal production equivalent in power plant i, segment j, $i = 1, \ldots, 5$, $j = 1, 2$.
$Q_{i,j,t}$ = discharge in power plant i, segment j, during hour t, $i = 1, \ldots, 5$, $j = 1, 2$, $t = 1, \ldots, 24$,
$\overline{Q}_{i,j}$ = maximal discharge in power plant i, segment j, $i = 1, \ldots, 5$, $j = 1, 2$,
$S_{i,t}$ = spillage from reservoir i during hour t, $i = 1, \ldots, 5$, $t = 1, \ldots, 24$,
$\overline{S}_i$ = maximal spillage from reservoir i, $i = 1, \ldots, 5$,
$V_{i,t}$ = local inflow to reservoir i during hour t, $i = 1, \ldots, 5$, $t = 1, \ldots, 24$.

a) (4 p) The best efficiency in the hydro power plant Forsen is obtained for the discharge 120 m^3/s and the electricity generation is then 48 MW. The maximal discharge in Forsen is 200 m^3/s and the relative efficiency is then 95%. Assume that we need a piecewise linear model of electricity generation as function of the discharge in Forsen. The model should have two segments and the breakpoint between them should be located at the best efficiency. Calculate the following parameters:

$\mu_{1,j}$ = marginal production equivalent in Forsen, segment j,
$\overline{Q}_{1,j}$ = maximal discharge in, segment j.

b) (5 p) Formulate the objective function if the purpose of the planning is to maximise the income of generated hydro power plus the value of stored water. Use the symbols defined above.

c) (2 p) The thermal power plant Flisinge is fuelled by biomass. The fuel costs 260 ¤/m^3 and has a density of 400 kg/m^3. The heat contents of the fuel is 5 MWh/ton and the efficiency of the power plant is 40%. How large is the variable generation cost in Flisinge?

d) (1 p) The following variables and parameters have been introduced in a short-term planning problem for a thermal power plant:

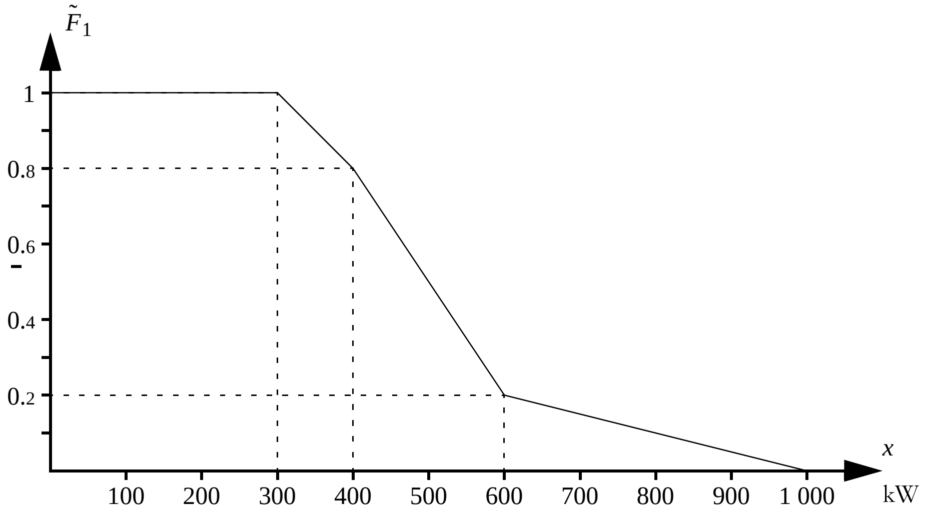
$\underline{G}$ = minimal generation when the power plant is committed,
G_t = generation in the power plant during hour *t*,
u_t = unit commitment in the power plant during hour *t* (1 if the power plant is committed during hour *t*, otherwise 0).

How should a linear constraint be formulated in order to describe the relation between $\underline{G}$, G_t and u_t for hour *t*?

1. $G_{g,t} - \underline{G}_g \cdot u_{g,t} \le 0.$
2. $G_{g,t} - \underline{G}_g \cdot u_{g,t} = 0.$
3. $G_{g,t} - \underline{G}_g \cdot u_{g,t} \ge 0.$

Problem 5 (12 p)

Ekibuga is a town in East Africa. The town is not connected to a national grid, but has a local system of its own. The local grid is supplied by a hydro power plant. The hydro power plant does not have a reservoir, but the water flow is always sufficient to generate the installed capacity (900 kW) and the risk for outages in the power plant is negligible.



a) (3 p) How large is the expected generation in the hydro power plant?

b) (3 p) Assume that there in addition to the hydro power plant is also a wind power plant in Ekibuga. A model of the available generation capacity is given in table 3. What is the *LOLP* of this system?

Hint: The convolution formula for a multi-state model reads

$$\tilde{F}_g(x) = \sum_{i=1}^{N_g} p_{g,i} \tilde{F}_{g-1}(x - x_{g,i}).$$

c) (2 p) Use the inverse transform method to randomise a value of the total load, *D*. Start with the random number 0.15 from a *U*(0,1)-distribution. What is the corresponding complementary random number, *D**.

Table 3 Model of the wind power plant in problem 5b.

Available generation capacity [kW]	Probability [%]
0	40
50	40
100	20

d) (3 p) A Monte Carlo simulation of Ekibuga used a more detailed model of the wind power farm than the one given in table 3. Moreover, the transmission losses in the power system have been considered. The results of 1 000 analysed scenarios is shown in table 4. Which estimate of *LOLP* is obtained from these results?

Table 4 Results from a Monte Carlo simulation of the power system in Ekibuga.

Stratum, h	Stratum weight, ω_h	Number of scenarios, n_h	Results, $\sum_{i=1}^{n_h} x_{i,h}$, (where $x_{i,h}$ is the observed value of *LOLO* in scenario i, stratum h)
1	0.85	50	0
2	0.06	900	150
3	0.09	50	50

e) (1 p) The expectation value $E[X]$ is to be determined using a control variate. Let x_i denote the i:th observation of X and let z_i denote the i:th observation of the control variate, Z. The total number of observations is n. How is the estimate m_X calculated?

1. $m_X = \frac{1}{n}\sum_{i=1}^{n} x_i + E[Z]$.
2. $m_X = \frac{1}{n}\sum_{i=1}^{n} (x_i - z_i) + E[Z]$.
3. $m_X = \frac{1}{n}\sum_{i=1}^{n} (x_i - z_i) - E[Z]$.

PART II (FOR HIGHER GRADES)

All introduced symbols must be defined. Solutions should include sufficient detail that the argument and calculations can be easily followed.

The answer to each problem must begin on a new sheet, but answers to different parts of the same problem (a, b, c, etc.) can be written on the same sheet. The fields *Namn* (Name), *Blad nr* (Sheet number) and *Uppgift nr* (Problem number) must be filled out on every sheet.

Part II gives a total of 60 points, but this part will only be marked if the candidate has obtained at least 33 points in part I. Then the results of parts I and II and the bonus points will be added together to determine the examination grade (A, B, C, D, E).

Problem 6 (10 p)

The electricity market in Land is divided in three price areas: North, Central and South. Data for the generation and consumption are given in table 5 and data for the transmission capacity between the price areas are found in table 6. The variable operation costs are assumed to be linear within the intervals, i.e., the production is zero if the price is on the lower price level and the production is maximal at the higher price level.

Elbolaget AB has decided to improve its environmental profile by investing in a really large wind farm. The investment cost is the same regardless of where in Land the wind farm is built. However, in northern Land it is possible to find better sites for the wind farm; the annual generation should then be 0.6 TWh, which can be compared to 0.5 TWh if the wind farm is built in any of the two other price areas. Where should Elbolaget AB build the wind farm to get the best possible yield from the investment?

Table 5 Data for generation and consumption on the electricity market in Land.

Power source	Production capability [TWh/year]			Variable costs [¤/MWh]
	North	Central	South	
Hydro power	60	10		5
Nuclear power		30	30	80–120
Fossil fuels	10	15	15	350–650
Consumption	23	65.5	58.5	

Table 6 Data for transmission between the price areas in Land.

Interconnection	Capacity [TWh/year]
North ↔ Central	40
Central ↔ South	20

Problem 7 (10 p)

The power system in Rike is divided in two price areas. There is a lot of hydro power in the northern part of the system, but most of the load is located in the southern part. The primary control of Rike is divided in a normal operation reserve and a disturbance reserve. The normal operation reserve is available in the frequency range 49.9–50.1 Hz and has a total gain of 3 000 MW/Hz, where 2 500 MW/Hz is provided by power plants in northern Rike. The disturbance reserve is available in the frequency range 49.5–49.9 Hz and has a total gain of 2 500 MW/Hz, where 2 000 MW/Hz is provided by power plants in northern Rike.

At a certain occasion the frequency in the system is 49.92 Hz and the total transmission from northern to southern Rike is 3 100 MW. The control room at Riksnät (who is the system operator in Rike) has decided to activate up-regulation bids in order to increase the frequency in the system and relieve primary control reserves. Moreover, Riksnät wants to free some transmission capacity between northern and southern Rike. The available up-regulation bids are shown in table 7. Which bids should Riksnät activate in order to minimise the costs if the objective is to increase the frequency to at least 50.0 Hz and reduce the transmission from northern to southern Rike below 2 950 MW?

Table 7 Up-regulation bids in the real-time balancing market in Rike.

Bid	Power [MW]	Price area	Price [¤/MWh]
1	40	North	400
2	50	North	410
3	100	South	415
4	50	North	425
5	50	North	450
6	40	South	500
7	80	South	520
8	50	North	550

Problem 8 (20 p)

In the power exchange ElKräng, players in the electricity market can sell and purchase electricity for each hour during the next day. The two most important bids are sell bids and purchase bids. A sell bid is valid for a specific hour t, $t = 1, \ldots, 24$, and comprises a certain maximal volume, $\bar{r}_{i,t}$, $i = 1, \ldots, N_t$,[1] $t = 1, \ldots, 24$, as well as a requested price, $\beta_{Ri,t}$, $i = 1, \ldots, N_t$, $t = 1, \ldots, 24$. A purchase bid is valid for a specific hour t, $t = 1, \ldots, 24$, and comprises a certain maximal volume, $\bar{p}_{j,t}$, $j = 1, \ldots, M_t$,[2] $t = 1, \ldots, 24$, as well as a value of purchased electricity, $\beta_{Rj,t}$, $j = 1, \ldots, M_t$, $t = 1, \ldots, 24$. Notice that these bids do not have to be accepted as a whole; if somebody submits a 100 MW sell bid, it is possible that the player will only get to sell 50 MW.

Bids to ElKräng must be submitted at last 12:00 noon. Then ElKräng compiles the bids and decides which bids that will be accepted as well as the electricity price for each hour (all accepted bids for one hour receive the same electricity price). This is done by solving an optimisation problem, where the objective function is to maximise the value of the accepted purchase bids minus the requested price for the accepted sell bids.

a) (8 p) Formulate the planning problem of ElKräng as an LP problem. Use the notation introduced above for the parameters (it is however permitted to add further symbols if you consider it necessary).

NOTICE! The following is required to get full score for this problem:

- The symbols for the optimisation variables must be clearly defined.
- The optimisation problem should be formulated so that it is easy to determine what the objective function is, which constraints there are and which limits there are.
- The possible values for all indices should be clearly stated for each equation.

b) (2 p) How can ElKräng compute the hourly electricity prices when the planning problem from part a has been solved?

c) (10 p) ElKräng is also planning to introduce block bids. These bids will be a class of sell bids, which are valid for several consecutive hours and which can only be accepted as a whole. Sellers can choose between five types of block bids, compromising different time periods (see table 8). How must the planning problem from part a be reformulated in order to consider these block bids? Do not forget to define all new variables and parameters that you introduce!

Hint: Introduce a binary variable, $u_{k,s}$, $k = 1, \ldots, L_s$,[3] $s = 1, \ldots, 5$, which is equal to one if a certain block bid is accepted and equal to zeros if the bid is rejected.

Table 8 Different types of block bids at ElKräng.

Type, s	Hours, t
1	1–24
2	1–6
3	7–12
4	13–18
5	19–24

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[1] Where N_t is the number of sell bids that have been submitted for hour t.

[2] Where M_t is the number of purchase bids that have been submitted for hour t.

[3] Where L_s is the number of block bids of type s.

Problem 9 (20 p)

Mji region is not connected to the national grid in Nchi, but has a regional 33-kV transmission system comprising the urban areas Mji and Kijiji, as well as a number of smaller villages. The regional grid is supplied by seven diesel generator sets in Mji and two more diesel generator sets in Kijiji. Each diesel generator set in Mji has a capacity of 500 kW, 90% availability and the variable operation cost 10 ¤/kWh. The diesel generator sets in Kijiji also have 500 kW capacity each and the same variable costs, but the availability is slightly worse (85%).

a) (10 p) The figure below shows the duration curve of the total load in Mji region. Table 9 shows the most important points of the equivalent load duration curve after adding first the seven diesel gensets in Mji and then the two diesel gensets in Kijiji.

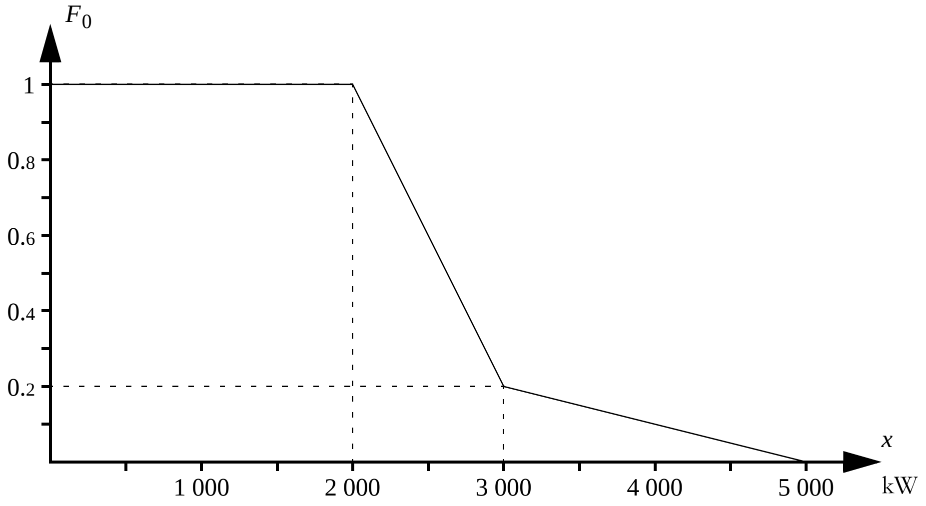

Table 9 Some results from a probabilistic production cost simulation of the power system in Mji region.

x	$\tilde{F}_9(x)$	x	$\tilde{F}_9(x)$	x	$\tilde{F}_9(x)$
2 000	1.0000	4 000	0.1781	6 000	0.0041
2 500	0.8618	4 500	0.1046	6 500	0.0007
3 000	0.5672	5 000	0.0505	7 000	0.0001
3 500	0.3156	5 500	0.0173	7 500	0.0000

Close to Kijiji is Kijoto, where it would be possible to build a hydro power plant. Such an investment would result in fixed costs of 60 M¤/year. The hydro power plant would have a capacity of 1 000 kW, 100% availability and negligible variable costs. Is it profitable to build the hydro power plant if only the expected operation cost of the system is considered (i.e., neglect the costs of disconnected load)?

b) (10 p) The losses in the power system of Mji region are comparatively high; therefore, reinforcements of some of the 33 kV lines are being considered. In order to analyse which reinforcements that are profitable, a multi-area model has been created. This model will then be used for Monte Carlo simulations of different variants of the power system (with and without the hydro power plant in Kijoto, with and without reinforced lines, etc.) It has been decided to use stratified sampling for these simulations. Using a strata tree, the following three strata have been defined:

- *Stratum 1:* All scenarios where the total load is lower than the available generation capacity minus the maximal losses in the grid ($D_{tot} \le \overline{H} + \overline{G}_{tot} - \overline{L}$).
- *Stratum 2:* All scenarios where the total load is lower than the available generation capacity except whose scenarios that belong to stratum 1 ($\overline{H} + \overline{G}_{tot} - \overline{L} < D_{tot} \le$

$\overline{H} + \overline{G}_{tot}$).

- *Stratum 3:* All scenarios where the total load is higher than available generation capacity ($\overline{H} + \overline{G}_{tot} < D_{tot}$).

However, this stratification will only be efficient if there is a reasonable estimate of the maximal losses in the grid, $\overline{L}$. Unfortunately, it is difficult to compute this quantity exactly for a system with seven areas; therefore, $\overline{L}$ will be estimated from some randomly generated scenarios. Then, additional scenarios are generated for each stratum. An example of this simulation strategy is given in tables 10 and 11. Which estimate of the expected total operation cost is obtained for each of the three strata in this Monte Carlo simulation?[4]

Hint: The scenarios used to estimate $\overline{L}$ can also be used to estimate expected operation cost!

Table 10 Scenarios to estimate the maximal losses in Mji region.

Scenario	$\overline{H}$ [kW]	$\overline{G}_{tot}$ [kW]	D_{tot} [kW]	L [kW]	TOC [¤/h]
1	1 000	4 000	4 649	604	40 000
2	1 000	4 500	2 820	321	21 410
3	1 000	4 500	2 040	282	13 216
4	1 000	3 000	2 957	492	24 492
5	1 000	4 500	2 203	278	14 818
6	1 000	4 500	3 909	723	36 316
7	1 000	4 500	2 437	324	17 615
8	1 000	4 500	2 947	482	24 289

Table 11 Scenarios to estimate the expected operation cost in Mji region.

Scenario	$\overline{H}$ [kW]	$\overline{G}_{tot}$ [kW]	D_{tot} [kW]	L [kW]	TOC [¤/h]
1	1 000	3 000	2 469	259	17 282
2	1 000	4 500	2 818	460	22 789
3	1 000	3 500	2 470	309	17 794
4	1 000	3 500	2 458	435	18 933
5	1 000	4 000	2 700	384	20 841
6	1 000	3 500	2 959	538	24 975
7	1 000	4 500	4 702	583	42 848
8	1 000	3 500	3 543	464	30 066
9	1 000	4 500	4 592	658	42 497
10	1 000	4 000	4 509	679	40 000
11	1 000	4 500	4 775	746	45 000
12	1 000	4 500	4 614	677	42 910
13	1 000	3 500	4 789	650	35 000
14	1 000	3 500	4 840	620	35 000
15	1 000	3 000	4 097	689	30 000
16	1 000	3 500	4 700	711	35 000
17	1 000	3 500	4 686	623	35 000
18	1 000	3 000	45 31	716	30 000

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[4] To compute the final expected operation cost it would be necessary to have the stratum weights, but that computation is too complex for this problem.

KTH Electrical Engineering

Answer sheet for part I

Name: ....

Personal number: ....

Problem 1

a) Alternative .... is correct.

b) Alternative .... is correct.

Problem 2

a) .... ¤/MWh **b)** .... ¤/MWh

c) Alternative .... is correct.

Problem 3

a) .... Hz **b)** .... MW

c) .... Hz **d)** .... Hz

Problem 4

a) μ_1 .... MWh/HE μ_2 .... MWh/HE

$\overline{Q}_1$ .... TE $\overline{Q}_2$ .... TE

b) ....

....

c) .... ¤/MWh

d) Alternative .... is correct.

Problem 5

a) .... kWh/h **b)** .... %

c) D .... kW $D*$ .... kW

d) .... %

e) Alternative .... is correct.

Problem 1

a) 3, **b)** 5.

Problem 2

a) The electricity price is determined by the intersection of the supply and demand curves. The intersection can be found graphically by drawing both curves in the same figure. An alternative method of solution is to assume an electricity price, λ, between 25 and 40 ¤/MWh. The supply at these price levels can be written as 30 (hydro & nuclear) + ($\lambda - 25$) (fossil fuels) and the demand can be written $45 - (\lambda - 20)/4$. These two expressions should be equal, which results in the electricity price $\lambda = 36$ ¤/MWh.

b) The additional 2 TWh wind power changes the supply curve in the interval between 25 and 40 ¤/MWh to $32 + \lambda - 25$, which results in the electricity price $\lambda = 34.4$ ¤/MWh.

c) 1.

Problem 3

a) All areas are connected by AC lines; thus, they constitute a synchronous grid. The frequency in area D is thus the same as in area A, i.e., 49.94 Hz.

b) Half of the gain of the system is in areas A and C: The primary control in these two areas will therefore increase their generation by 1 100 MW (half the outage). the power flow between area C and D only is controlled manually, which means that the increased generation must be transmitted on the line between area A and area B. This line does however not have sufficient unused capacity and will therefore be disconnected. Hence, the transmission is 0 MW.

c) Areas A and C constitute a separate synchronous grid after the disconnection of the line between A and B. The total gain in this synchronous grid is 4 000 MW/Hz and there is now a surplus of 1 000 MW (the power that earlier was going to area B), which results in a frequency increase $\Delta f = \Delta G/R = 1\,000/4\,000 = 0.25$ Hz, i.e., the new frequency is $50 + 0.25 = 50.25$ Hz.

d) Areas B and D constitute a separate synchronous grid after the disconnection of the line between A and B. The total gain in this synchronous grid is 4 000 MW/Hz and there is now a deficit of 2 200 MW (the outages in the power plants in area B) + 1 000 MW (the power that earlier was imported from area A), which results in a frequency decrease $\Delta f = \Delta G/R = 3\,200/4\,000 = 0.80$ Hz, i.e., the new frequency is $50 - 0.80 = 49.20$ Hz.

Problem 4

a) The following data are given in the problem text:

$$\begin{aligned}\bar{Q} &= \text{maximal discharge in Forsen} = 200,\\ \hat{Q} &= \text{discharge in Forsen at best efficiency} = 120,\\ \hat{H} &= \text{generation in Forsen at best efficiency} = 48,\\ \eta(\bar{Q}) &= \text{relative efficiency at maximal discharge in Forsen} = 0.95.\end{aligned}$$

To calculate the marginal production equivalents, we need the generation at maximal discharge, which can be calculated using the formula $H = \gamma_{max} \cdot \eta(Q) \cdot Q$. First however, we must calculate the maximal production equivalent, which is obtained at best efficiency:

$$\gamma_{max} = \text{maximal production equivalent in Forsen} = 48/120 = 0.4 \text{ MWh/HE}.$$

The generation we need is now given by

$$\bar{H}_i = \text{maximal generation in Forsen} = 0.4 \cdot 0.95 \cdot 200 = 76 \text{ MW}.$$

The marginal production equivalents can now be calculated according to

$$\mu_1 = \frac{\hat{H}}{\hat{Q}}$$

and

$$\mu_2 = \frac{\bar{H} - \hat{H}}{\bar{Q} - \hat{Q}},$$

which results in the following linear models of the power plant:

$$\mu_{1,j} = \text{marginal production equivalent in Forsen, segment } j =$$

$$= \begin{cases} 0.40 & j = 1, \\ 0.35 & j = 2, \end{cases}$$

$$\bar{Q}_{1,j} = \text{maximal discharge in Forsen, segment } j = \begin{cases} 120 & j = 1, \\ 80 & j = 2. \end{cases}$$

b) maximise

$$\sum_{t=1}^{24} \lambda_t \sum_{i=1}^{5} \sum_{j=1}^{2} \mu_{i,j} Q_{i,j,t} + \lambda_{25}(\gamma_1 + \gamma_2 + \gamma_5)M_{1,24} + \lambda_{25}(\gamma_2 + \gamma_5)M_{2,24}$$
$$+ \lambda_{25}(\gamma_3 + \gamma_4 + \gamma_5)M_{3,24} + \lambda_{25}(\gamma_4 + \gamma_5)M_{4,24} + \lambda_{25}\gamma_5 M_{5,24}.$$

c) The heat contents of one m^3 fuel is 0.4 ton/m^3 · 5 MWh/ton = 2 MWh/m^3. Since the efficiency is 40% the electricity generation is $0.4 \cdot 2 = 0.8$ MWh/m^3. If the fuel costs 260 ¤/m^3 then the variable generation cost is $260/0.8 = 325$ ¤/MWh.

d) 3.

Problem 5

a) The expected generation is calculated as

$$EG_1 = EENS_0 - EENS_1 = \int_0^{\infty} \tilde{F}_0(x)dx - \int_{900}^{\infty} \tilde{F}_1(x)dx.$$

Since the hydro power plant has 100% availability, we get that $\tilde{F}_1(x) = \tilde{F}_0(x)$, which means that we obtain the following result:

$$EG_1 = \int_0^{\infty} \tilde{F}_0(x)dx - \int_{900}^{\infty} \tilde{F}_0(x)dx = \int_0^{900} \tilde{F}_0(x)dx =$$
$$= 300 \cdot 1 + 100 \cdot (1 + 0.8)/2 + 200 \cdot (0.8 + 0.2)/2 + 300 \cdot (0.2 + 0.05)/2 = 527.5 \text{ kWh/h}.$$

b) $LOLP = \tilde{F}_2(1\,000) = 0.2\tilde{F}_1(1\,000) + 0.4\tilde{F}_1(950) + 0.4\tilde{F}_1(900) =$
$= 0.2 \cdot 0 + 0.4 \cdot 0.025 + 0.4 \cdot 0.05 = 3.0\%$.

c) The inverse transform method states that $D = F_D^{-1}(U)$, where U is a $U(0, 1)$-distributed random number. Since it is the duration curve that is given in the problem, we may as well use the transform $D = \tilde{F}_D^{-1}(0.15)$ = {use the figure in the problem text} = 700 kW. The complementary random number is obtained by transforming 1 $U^* = 1 - U$; hence, we get $D^* = \tilde{F}_D^{-1}(0.85)$ = 375 kW.

d) The estimate of the expectation value of a single stratum is given by

$$m_{Xh} = \frac{1}{n_h} \sum_{i=1}^{n_h} x_{i,h},$$

which yields $m_{X1} = 0$, $m_{X2} = 1/6$ and $m_{X1} = 1$. The results are weighted according to

$$m_X = \sum_{h=1}^{3} \omega_h m_{Xh},$$

which results in the $LOLP$ estimate $0.85 \cdot 0 + 0.06 \cdot 1/6 + 0.09 \cdot 1 = 10\%$.

e) 2.

Problem 6

För att avgöra vilken placering som ger den största intäkten måste vi beräkna elpriset i de tre prisområdena. Vi börjar med att göra detta utan vindkraftparken. Om man bortser från transmissionsbegränsningarna så skulle elpriset då bli 477,50 ¤/MWh, eftersom vattenkraft och kärnkraft kan leverara maximalt 147 TWh/år; alltså behövs 17 TWh/år från fossila bränslen, vilket betyder att man utnyttjar 17/40 av prisintervallet för fossila bränslen. Elpriset skulle således bli 477,5 ¤/MWh. Vid detta elpris är elproduktionen i prisområde Norr 64,25 TWh/år medan konsumtionen är 23 TWh/år; således skulle Norr behöva exportera 41,25 TWh/år, men det är inte möjligt eftersom exportkapaciteten till prisområde Mitt endast är 40 TWh/år. Vi kan alltså dra slutsatsen att transmissionskapaciteten mellan Norr och Mitt kommer att vara fullt utnyttjad.

Den sammanlagda elförbrukningen i prisområdena Mitt och Syd är 124 TWh/år. Import från Norr, vattenkraft och kärnkraft kan leverara sammanlagt 110 TWh/år och därmed måste de andra två kraftslagen tillsammans producera 14 TWh, vilket betyder att man utnyttjar 14/30 av prisintervallet för fossila bränslen. Elpriset skulle således bli 490 ¤/MWh. Vid detta elpris skulle elproduktionen i prisområde Mitt plus importen ge en tillförsel på 87 TWh/år medan elförbrukningen är 65,5 TWh. Alltså skulle man behöva exportera 21,5 TWh/år, men förbindelsens kapacitet är endast 20 TWh/år. Även denna förbindelse kommer med andra ord att vara fullt utnyttjad.

I och med att förbindelserna mellan är fullt utnyttjade kommer man att få olika elpriser i de tre prisområdena. Det lägsta priset kommer att vara i Norr (annars skulle man inte exportera något därifrån) och det högsta priset kommer att vara i Syd (annars skulle man inte importera något dit). Frågan är om det högre priset i Syd gör det lönsamt att bygga vindkraftparken där, trots att vindläget är sämre.

Efterfrågan i prisområde Norr blir 23 (lokal efterfrågan) + 40 (export) = 63 TWh/år. Vindkraftparken och vattenkraften leverar tillsammans 60,6 TWh, vilket betyder att 2,4 TWh måste produceras med fossila bränslen. Därmed utnyttjas 24% av prisintervallet för fossila bränslen, vilket ger elpriset 422 ¤/MWh. Intäkten från vindkraftparken skulle då bli 422 · 0,6 = 253,2 M¤/år.

I prisområde Syd erhålls totalt 50,5 TWh/år från import, kärnkraft och vindkraft. Resterande 8 TWh måste produceras med fossila bränslen. Därmed utnyttjas 8/15 av prisintervallet för fossila bränslen, vilket ger elpriset 510 ¤/MWh. Intäkten från vindkraftparken skulle då bli 510 · 0,5 = 255 M¤/år.

Slutsatsen blir alltså att det är bäst att bygga vindkraftparken i prisområde Syd, trots att den årliga elproduktionen skulle bli lägre.

Problem 7

För att höja frekvensen med minst 0,08 Hz krävs en uppreglering $\Delta G = R{\cdot}\Delta f = 3\,000 \cdot 0{,}08 =$ 240 MW, vilket man skulle kunna erhålla genom att aktivera de fyra billigaste buden. Frågan är då vad som händer med transmissionen från norr till söder. Då frekvensen ökar med 0,08 Hz minskar primärregleringen i norra Rike elproduktionen med $\Delta G = R{\cdot}\Delta f = 2\,500 \cdot 0{,}08 = 200$ MW, samtidigt som man aktiverar 140 MW uppreglering (bud 1, 2 och 4). Sammantaget innebär detta att man får ett underskott på 60 MW i norra Rike, vilket innebär att exporten söderut måste minska från 3 100 MW till 3 040 MW. Detta är dock mer än vad som är tillåtet. Fördelningen av primärregleringskapaciteten kan man inte göra något åt och således måste man se till att mer uppreglering genomförs i södra Rike. Bud 6 ger ytterligare 40 MW i södra Rike, men det är inte tillräckligt. Lösningen måste därför bli att aktivera både bud 6 och 7, vilket ger en total uppreglering på 220 MW i södxa Rike. Eftersom vi behöver minst 240 MW för att höja frekvensen till minst 50,0 Hz så måste även det lägsta budet i norra Rike aktiveras.

Riksnät ska alltså aktivera bud 1, 3, 6 och 7, vilket resulterar i en frekvens som är något högre än 50,0 Hz och en överföring på ungefär 2 923 MW.

Problem 8

a) I ord kan planeringsproblemet formuleras som

maximera	*värdet av antagna köpbud – begärt pris för säljbuden*,
med hänsyn till	*lastbalans*,
	maximal volym för samtliga bud.

Parametrar

Parametrarna är definierade i uppgiftslydelsen.

Optimeringsvariabler

$p_{j,t}$ = antagen volym av köpbud j, timme t, $j = 1, \ldots, M_t$, $t = 1, \ldots, 24$,
$r_{i,t}$ = antagen volym av säljbud i, timme t, $i = 1, \ldots, N_t$, $t = 1, \ldots, 24$.

Målfunktion

$$\text{maximera} \quad \sum_{t=1}^{24} \left(\sum_{j=1}^{M_t} \beta_{j,t} p_{j,t} - \sum_{i=1}^{N_t} \beta_{i,t} r_{i,t} \right).$$

Bivillkor

$$\sum_{j=1}^{M_t} p_{j,t} = \sum_{i=1}^{N_t} r_{i,t}, \qquad t = 1, \ldots, 24.$$

Variabelgränser

$$0 \le p_{j,t} \le \bar{p}_{j,t}, \qquad j = 1, \ldots, M_t, t = 1, \ldots, 24,$$

$$0 \le r_{i,t} \le \bar{r}_{i,t}, \qquad i = 1, \ldots, N_t, t = 1, \ldots, 24.$$

b) Elpriset kan erhållas direkt ur lösningen till optimeringsproblemet från a-uppgiften, närmare bestämt genom dualvariablerna till lastbalansvillkoret för motsvarande timme.

c) Följande nya parametrar behövs:

$\bar{b}_{k,s}$ = volym i blockbud k , typ s, $k = 1, \ldots, L_s$, $s = 1, \ldots, 5$,
$\beta_{k,s}$ = begärt pris för blockbud k , typ s, $k = 1, \ldots, L_s$, $s = 1, \ldots, 5$,
L_s = antal blockbud av typ s, $s = 1, \ldots, 5$.

Vi inför även nya optimeringsvariabler i enlighet med tipset:

$u_{k,s}$ = tillslag på blockbud , typ s, $k = 1, \ldots, L_s$, $s = 1, \ldots, 5$.

Kostnaden för antagna blockbud måste tas med i målfunktionen. Här måste man tänka på vilka timmar ett blockbud gäller, vilket ger oss följande uppdaterade målfunktion:

$$\text{maximera} \quad \sum_{t=1}^{24}\left(\sum_{j=1}^{M_t} \beta_{j,t}p_{j,t} - \sum_{i=1}^{N_t} \beta_{i,t}r_{i,t} - \sum_{k=1}^{L_1} \beta_{k,1}\bar{b}_{k,1}u_{k,1}\right) - \sum_{t=1}^{6}\sum_{k=1}^{L_2} \beta_{k,2}\bar{b}_{k,2}u_{k,2}$$
$$- \sum_{t=7}^{12}\sum_{k=1}^{L_3} \beta_{k,3}\bar{b}_{k,3}u_{k,3} - \sum_{t=13}^{18}\sum_{k=1}^{L_4} \beta_{k,4}\bar{b}_{k,4}u_{k,4} - \sum_{t=19}^{24}\sum_{k=1}^{L_5} \beta_{k,5}\bar{b}_{k,5}u_{k,5}.$$

Antagna blockbud måste också räknas in lastbalanserna. Även här måste man tänka på vilka timmar ett blockbud gäller, vilket ger oss följande uppdaterade bivillkor:

$$\sum_{j=1}^{M_t} p_{j,t} = \sum_{i=1}^{N_t} r_{i,t} + \sum_{k=1}^{L_1} \bar{b}_{k,1}u_{k,1} + \sum_{k=1}^{L_2} \bar{b}_{k,3}u_{k,3}, \qquad t = 1, \ldots, 6,$$

$$\sum_{j=1}^{M_t} p_{j,t} = \sum_{i=1}^{N_t} r_{i,t} + \sum_{k=1}^{L_1} \bar{b}_{k,1}u_{k,1} + \sum_{k=1}^{L_3} \bar{b}_{k,4}u_{k,4}, \qquad t = 7, \ldots, 12,$$

$$\sum_{j=1}^{M_t} p_{j,t} = \sum_{i=1}^{N_t} r_{i,t} + \sum_{k=1}^{L_1} \bar{b}_{k,1}u_{k,1} + \sum_{k=1}^{L_4} \bar{b}_{k,2}u_{k,2}, \qquad t = 13, \ldots, 18,$$

$$\sum_{j=1}^{M_t} p_{j,t} = \sum_{i=1}^{N_t} r_{i,t} + \sum_{k=1}^{L_1} \bar{b}_{k,1}u_{k,1} + \sum_{k=1}^{L_5} \bar{b}_{k,2}u_{k,2}, \qquad t = 19, \ldots, 24.$$

Slutligen måste vi ange variabelgränserna för de nya optimeringsvariablerna:

$$u_{k,s} \in \{0, 1\}, \qquad k = 1, \ldots, L_s, s = 1, \ldots, 5.$$

Problem 9

a)

KOMMENTAR: *Förklaring av styckvis linjär varaktighetskurva och brytpunkter…*

Vi börjar med att beräkna *ETOC* i systemet med endast dieselgeneratorer:

$$EENS_0 = \int_0^{\infty} \tilde{F}_0(x)dx = 2\,000 \cdot 1 + 1\,000 \cdot (1 + 0{,}2)/2 + 2\,000 \cdot 0{,}2/2 = 2\,800 \text{ kWh/h}.$$

$$EENS_9 = \int_{4\,500}^{\infty} \tilde{F}_9(x)dx = 500 \cdot ((0{,}1046 + 0{,}0505)/2 + (0{,}0505 + 0{,}0173)/2 + (0{,}0173 + 0{,}0041)/2$$
$$+ (0{,}0041 + 0{,}0007)/2 + (0{,}0007 + 0{,}0001)/2 + 0{,}0001/2) = 62{,}5 \text{ kWh/h}.$$

$$ETOC = 10EG = 10(EENS_0 - EENS_9) = 27\,375 \text{ ¤/h}.$$

Då vi simulerar systemet med vattenkraftverket ska detta lägga in först, eftersom det har lägre driftkostnad än dieselgeneratorerna. I och med att vattenkraftverket antas vara 100% tillförlitligt får vi i denna simulering att $\tilde{F}_1(x) = \tilde{F}_0(x)$. Vattenkraftverket påverkar därför inte varaktighetskurvan för den ekvivalenta lasten utan endast den installerade effekten i systemet; vi får med andra ord att $\tilde{F}_{10}(x)$ då man simulerar systemet med vattenkraftverket är lika med $\tilde{F}_9(x)$ då man simulerar systemet med endast dieselgeneratorer. *ETOC* i systemet med vattenkraftverket ges därmed av följande beräkningar:

$$EENS_1 = \int_0^{\infty} \tilde{F}_1(x)dx = 1\,000 \cdot 1 + 1\,000 \cdot (1 + 0{,}2)/2 + 2\,000 \cdot 0{,}2/2 = 1\,800 \text{ kWh/h}.$$

$$EENS_{10} = \int_{5\,500}^{\infty} \tilde{F}_9(x)dx = 500 \cdot ((0{,}0173 + 0{,}0041)/2 + (0{,}0041 + 0{,}0007)/2 + (0{,}0007 + 0{,}0001)/2$$
$$+ 0{,}0001/2) = 6{,}775 \text{ kWh/h}.$$

$$ETOC = 10EG = 10(EENS_1 - EENS_{10}) \approx 17\,932{,}5 \text{ ¤/h}.$$

På ett år skulle vattenkraftverket alltså förväntas sänka driftkostnaden med 8 760 · (27 375 – 17 932) ≈ 82,7 M¤. Eftersom besparingen är större än investeringskostnaden är kraftverket lönsamt.

b) Poängen med den valda stratifieringen är att alla scenarier där man inte på förhand kan säga om den tillgängliga produktionskapaciteten är tillräcklig ej ska ingå i stratum 2. Det är därför viktigt att man inte underskattar $\bar{L}$, eftersom man då skulle kunna få effektbristscenarier även i stratum 1. Vi kan därför inte skatta $\bar{L}$ som medelvärdet av förlusterna i scenarierna från table 10, utan vi bör välja vår skattning som de högsta observerade förlusterna plus en viss säkerhetsmarginal (det är ju extremt osannolikt att vi bland åtta slumpmässigt genererade scenarier skulle råka få med det scenario som verkligen resulterar i de högsta förlusterna). I det här fallet är de största förlusterna 723 kW och man kan då t.ex. välja att avrunda detta uppåt till 1 000 kW.

Vi kan nu skatta den förväntade driftskostnaden per stratum utifrån de totalt 26 scenarier vi har tillgängliga. (Notera att resultatet kan ändras lite beroende på vilken skattning av $\bar{L}$ som används.) Alla scenarierna i table 10 tillhör stratum 1 utom scenario 1, som tillhör stratum 2. Scenario 1–6 i table 11 tillhör stratum 1, scenario 7–12 tillhör stratum 3 och scenario 13–18 tillhör stratum 3. Detta ger oss följande skattningar:

$$m_{TOC1} = (21\,410 + 13\,216 + 24\,492 + 14\,818 + 36\,316 + 17\,615 + 24\,289 + 17\,282$$
$$+ 22\,789 + 17\,794 + 18\,933 + 20\,841 + 24\,975)/15 \approx 18\,318 \text{ ¤/h}.$$

$$m_{TOC2} = (40\,000 + 42\,848 + 30\,066 + 42\,497 + 40\,000 + 45\,000 + 42\,910)/7$$
$$\approx 40\,474 \text{ ¤/h}.$$

$$m_{TOC2} = (35\,000 + 35\,000 + 30\,000 + 35\,000 + 35\,000 + 30\,000\,)/6 \approx 33\,333 \text{ ¤/h}.$$