AUTOMATIC CONTROL

KTH

EL2745 Principles of Wireless Sensor Networks

Exam 08:00–13:00, March 14, 2013

Aid:

Lecture notes (slides) from the course, reading material, and textbook ("Principles of Embedded Networked System Design" by Pottie & Kaiser or similar text approved by course responsible); Mathematical handbook (e.g., "Beta Mathematics Handbook" by Råde & Westergren) and pocket calculators are approved. The course compendium, your notes to the exercise lectures, other textbooks, handbooks, exercises, solutions, smartphones, tablets, etc. may **not** be used.

Observandum:

- Name and social security number (*personnummer*) on every page.
- Only one solution per page and write on one side per sheet.
- Each answer must be motivated.
- Specify the total number of handed in pages on the cover.
- Each subproblem is marked with its maximum credit.

Grading:

Grade A: $\geq$ 43, Grade B: $\geq$ 38

Grade C: $\geq$ 33, Grade D: $\geq$ 28

Grade E: $\geq$ 23, Grade Fx: $\geq$ 21

Results:

The results will be available at STEX (*Studerandeexpeditionen*), Osquldas väg 10 between one and two weeks from the exam.

If you want your result emailed, please, state this and include your email address.

Responsible: Carlo Fischione, `carlofi@kth.se`

Good Luck!

1. **BPSK modulation over fading channels**

When a node transmits a modulated signal with a given level of power over the wireless channel, the channel may introduce attenuations and additive noises which result in detecting bits in error at the receiver.

(a) [2p] Describe the binary phase shift keying (BPSK) modulation format. It is supported by the IEEE 802.15.4 standard?

(b) [2p] Suppose that the wireless channel has no attenuations. However, the received signal is affected by an Additive White Gaussian noise (AWGN). Compute the probability of error for BPSK. Illustrate the computations in the detail and motivate the result.

(c) [3p] Suppose that the wireless channel introduces a Rayleigh fading. Compute the average probability of error for a Rayleigh fading channel for BPSK given the error probability of the AWGN channel model. Illustrate the computations in the detail and motivate the result.

(d) [3p] Now assume that the wireless channel introduces a Lognormal fading. Compute the average probability of error for a Lognormal fading channel for BPSK given the error probability of the AWGN channel model. Illustrate the computations in the detail and motivate the result. [Hint: use the Stirling approximation,

$$f(\theta) = f(\mu)+(\theta-\mu)\frac{f(\mu+h)-f(\mu-h)}{2h}+\frac{1}{2}(\theta-\mu)^2\frac{f(\mu+h)-2f(\mu)+f(\mu-h)}{h^2}+\cdots,$$

where $h = \sqrt{3}\sigma$]

2. **Energy harvesting in WSNs**

A small solar battery can be used to harvest energy for sensor nodes. Assume the solar battery produces energy good enough for communication purposes. However, the energy production is subject to the wether conditions. This means that if the weather is bad, the nodes cannot work. In what follows, suppose that the time is divided into slots of duration Ts.

(a) [2p] Let p be the probability of energy generation in the good weather conditions in every slot T, and let q be the probability of energy generation in bad weather conditions. Further, let k be the transition probability from the good to the bad weather state, and let r be the transition probability from the bad state to the good weather state. Propose a Gilbert-Elliot model to characterize the energy generation of solar batteries.

(b) [2p] Calculate the stationary probabilities of being in each state of the Gilbert-Elliot model derived in the previous item.

(c) [3p] Set the probabilities $p = 0.9$, $q = 0.3$, $k = 0.1$, and $r = 0.2$. Calculate the steady state rate of energy generation in each slot.

(d) [3p] Define the harvesting burst as the number of slots spent in the good weather state before switching to the bad weather state. Calculate the distribution and the average of the harvesting burst.

3. **Neighbor discovery protocol in WSNs**

   A WSN has been deployed in a field. Each one of the WSN is equipped with an identifier (ID). First step in the initializing of the WSN is for the nodes to discover the identities of their neighbors. This is done by a synchronous ALOHA-like neighbor discovery algorithm, where time is slotted. At the beginning of each slot a node decides to send a probing message (PROB) that contains its ID with probability p or listen to the channel with probability $1 - p$. A collision happens when two nodes simultaneously transmit on the same slot. In such an occurrence, PROB is assumed to be lost.

(a) [2p] Assume that a node has N nodes within its communication range. Formulate the probability that such a node discovers a particular neighbor in a given time slot.

(b) [2p] Find the optimum transmission probability $p^\star$ such that the discovery probability obtained in the previous item is maximized. Calculate the corresponding discovery probability using $p^\star$.

(c) [3p] Consider the neighbor discovery process as divided into frames, where each frame consists of one or more time slots. Let W_i denote the length of frame $i, 0 \leq i \leq N - 1$. The frame starts when the node i is discovered and ends when node $i + 1$ is discovered. Calculate the expectation of W_i, $\mathbf{E}[W_i]$, for $i = 0, 1, \ldots$ [Hint: W_i is a random variable with geometric distribution and probability depending on the number of discovered neighbors].

(d) [3p] Calculate the average time needed for discovering $N = 4$ neighbors given that each slot is of length 81 ms.

4. **Distributed detection/estimation**

A set of N nodes is randomly deployed on a field. Every node makes observations on an unknown parameter $x \in [0, 2]$. The observations are corrupted by an additive noise

$$y_k = x + n_k, \qquad k = 1, 2, \ldots, N,$$

where n_k is the noise, which is modeled as a random variable independent of other noises and identically distributed. In particular, the noise is uniformly distributed over $[0, 2]$, with a probability distribution function (pdf)

$$p(n_k) = \frac{1}{2}, \qquad \text{if } n_k \in [0, 2].$$

To get an accurate estimate of the parameter x, each node reports its observations to a fusion centre. However, due to message losses and medium access control protocol, each node is allowed to transmit a message composed only by one bit. Thus, each node reports a message $m_k(y_k) \in \{0, 1\}$ to the fusion center. The bit of the message is chosen as follows

$$m_k(y_k) = \begin{cases} 1, & \text{if } y_k \geq 2 \\ 0, & \text{if } y_k < 2. \end{cases}$$

(a) [3p] Compute the expectation $\mathbf{E}(m_k)$ and variance of the one-bit message $\mathbf{E}(m_k - \mathbf{E}(m_k))^2$ for node k.

(b) [2p] Show that $\mathbf{E}(m_k - \mathbf{E}(m_k))^2$ is bounded above and compute the upper bound. Motivate why it is important to have an upper bound.

(c) [3p] Suppose that the fusion center uses a final fusion function f and estimator $\hat{x}$ to decide upon the parameter given by

$$\hat{x} := f(m_1, \ldots, m_N) = \frac{2}{N} \sum_{k=1}^{N} m_k.$$

Find $\mathbf{E}(\hat{x})$ and $\mathbf{E}(\hat{x} - x)^2$. What is the meaning of these two expectations? Is the fusion function giving a biased or unbiased estimator?

(d) [2p] What is the minimum number of nodes N to deploy so that the variance of estimate $\hat{x}$ of the previous item is less than ϵ?

5. **Networked Control System** Consider the Networked Control System (NCS) in

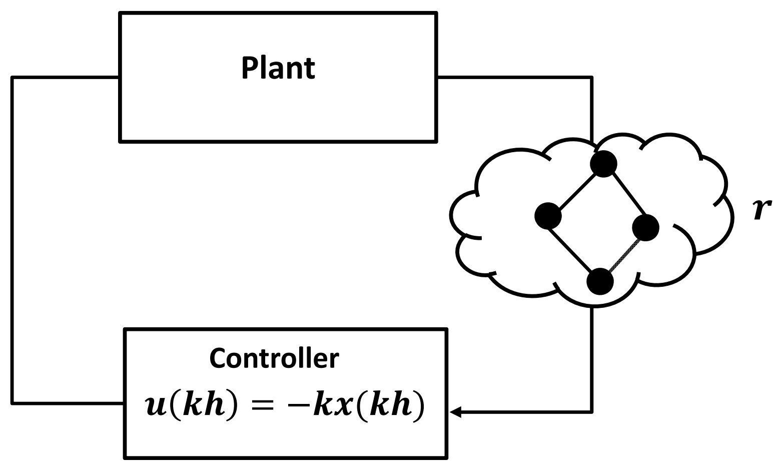

Figure 1: Wireless Sensor Networked Control System.

Fig. 1. The system consists of a continuous plant

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{1a}$$
$$y(t) = Cx(t)\,, \tag{1b}$$

where $A = 0$, $B = 1$, $C = 1$. The system is sampled with sampling time $h = 1$, and the discrete controller is given by

$$u(kh) = -x(kh), \qquad k = 0, 1, 2, \ldots\,,$$

where K is a constant.

(a) [1p] Suppose that the WSNCS has a medium access control (MAC) that introduces a constant delay $\tau \leq h$. Derive a sampled system corresponding to Eq.(1) with a zero-order-hold.

(b) [1p] Argue which MAC category introduces a constant delay and how the MAC of IEEE 802.15.4 can be adapted to ensure that the delay is satisfied.

(c) [2p] Under the same assumption above that the WSNCS introduces a delay $\tau \leq h$, give an augmented state-space description of the closed loop system so to account for such a delay.

(d) [2p] Under the same assumption above that the WSNCS introduces a delay $\tau \leq h$, characterize the conditions for which the closed loop system becomes unstable.

(e) [4p] Now, suppose that the WSNCS does not induce any delay, but unfortunately introduces packet losses with probability p. Let $r = 1 - p$ be the probability of successful packet reception. Give and discuss sufficient conditions for which the closed loop system is stable. When these conditions would not be satisfied, discuss what can be done at the network level or at the controller level so to still ensure closed loop stability. Which MAC category introduces packet losses and why there are such losses?