EL2520
Control Theory and Practice

Welcome!

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The practical

- Course information and schedule
  https://www.kth.se/social/course/EL2520/

- All slides and exercises available on homepage after class
- Course book:English and Swedish versions; Kåren or Internet
- Course material and practicalities: STEX, Osquldas väg 10

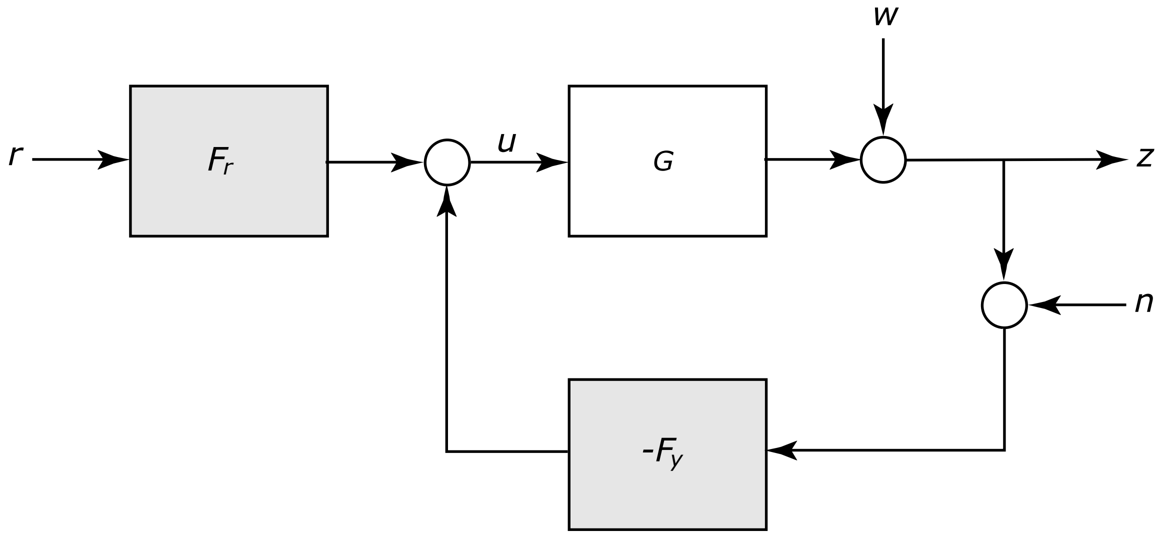
- Computer exercises: need kth.se account
- Register for labs via home page, from 22nd of March
- Lab access: details next week

- Expectations and feedback: email me at mikaelj@ee.kth.se

Course elements

- 14 lectures, Mikael Johansson
- 8 exercise sessions, Olle Trollberg
  - all exercises given in English
  - one Q&A session per week
- 4 computer exercise sessions, Håkan Terelius
  - groups of 2 students
  - written report on exercise 1 and 2, (short sheet on 3, 4)
- 1 project
  - groups of 4 students
  - two sessions in laboratory, written report
- Exam: 5 hours, written exam on 25th of May 2012, 09.00-14.00

Why feedback control?



- Make a system behave as desired
  - e.g. stabilize unstable system
- Reduce effects of disturbances and component variations
  - e.g. keep variables constant
- New freedom for designers
  - physical design vs. feedback control

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Why "Control Theory and Practice"?

- Solve complex control problems
  - Fundamental limitations
  - Sensitivity and robustness analysis
  - Systems with multiple inputs and outputs
  - Controller design using optimization
  - Systems with constraints
  - Computer-based control systems
- Understand dynamic systems! The research front
- Applications!

Course structure

1. Basic control revisited (4 lectures)
2. Modern control of multivariable linear systems (7 lectures)
3. Control of systems with constraints (2 lectures)

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Applications

Use control theory to analyze and modify system properties!

Applications in most engineering domains:

- Aerospace
- Cars and heavy vehicles
- Autonomous systems and robots
- Process industry
- Consumer products
- Communication systems
- Economics
- Biology
- …

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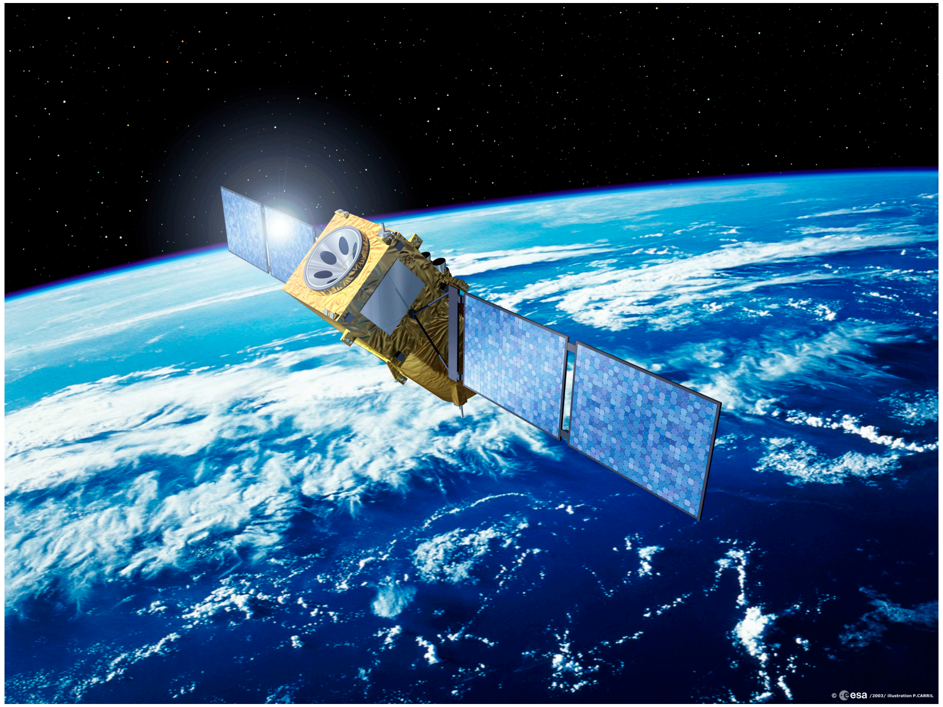
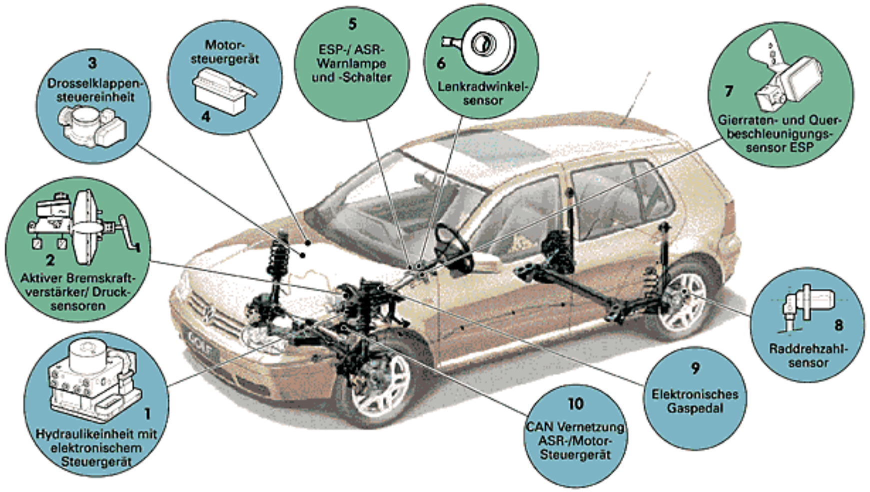
Aerospace



Vehicle control

Elektronisches Stabilitätsprogramm (ESP)



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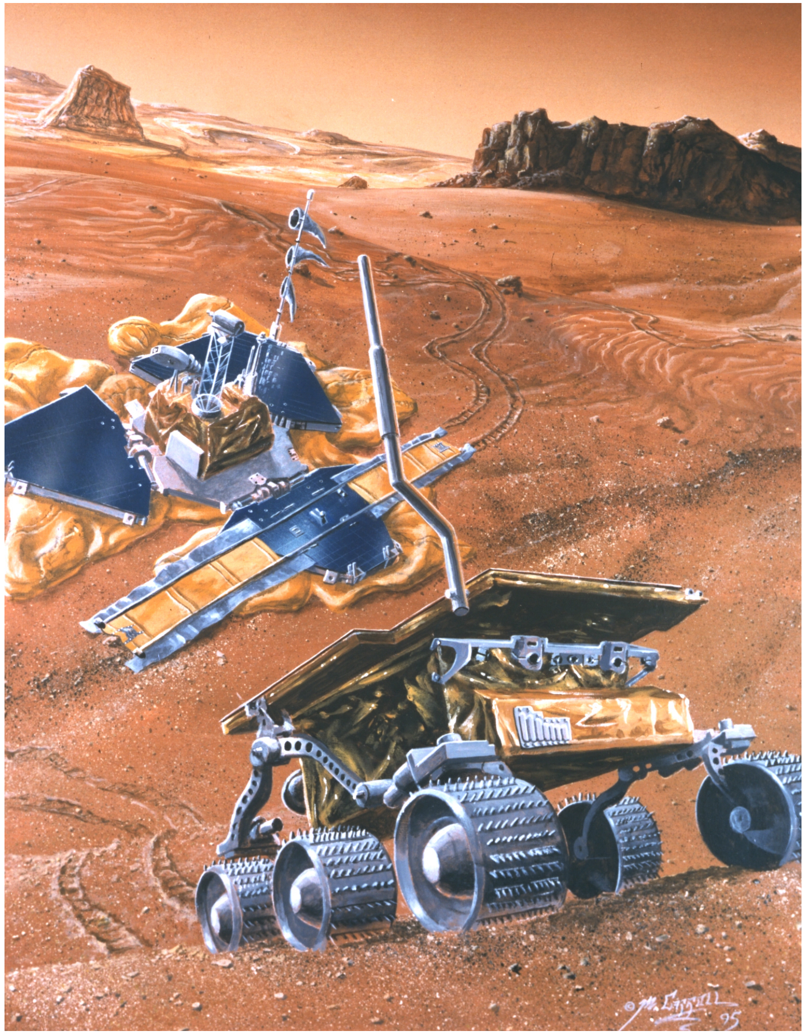
Autonomous systems and robotics



Process industry

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Consumer products

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Course structure

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How to learn more?

- Internet (e.g., links at www.ee.kth.se)
- Books (see course information, home page)
- Journals (IEEE Transactions on Automatic Control, Automatica, …)
- Courses (Nonlinear Control, Modelling of Dynamical Systems, …)
- Software (Matlab, …)

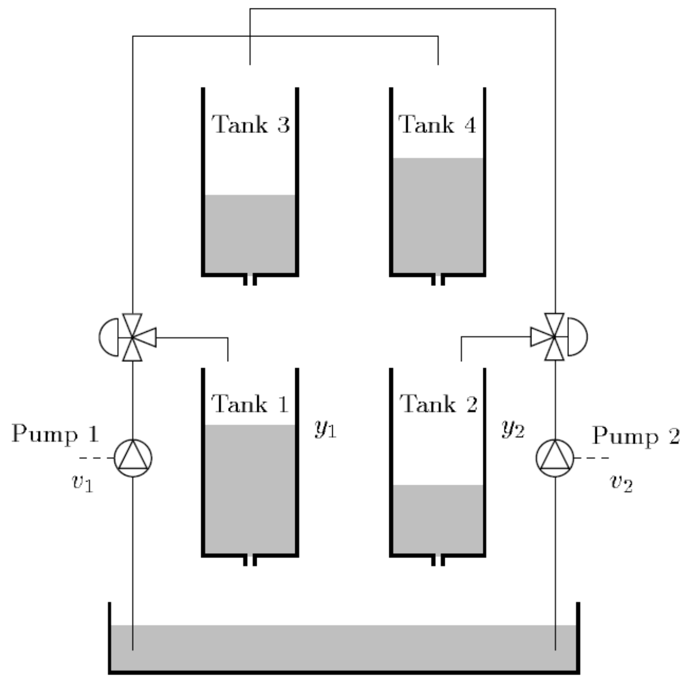
- Interact with us, and take the opportunity to learn!

Central component: multivariable systems

Key aspects:

- several inputs and outpus,
- dynamics coupled (single input affects many outputs)

Laboratory project example: quadruple tank system



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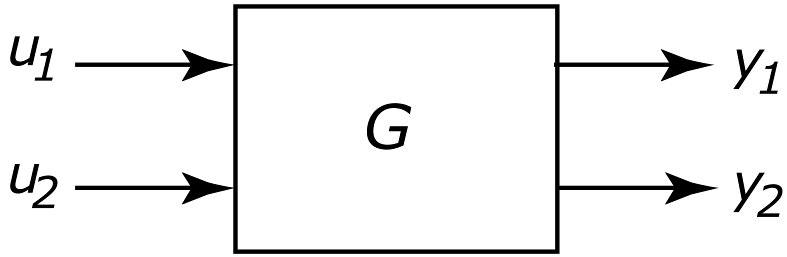
The need for multivariable control

Example. Consider a linear system with two inputs and two outputs

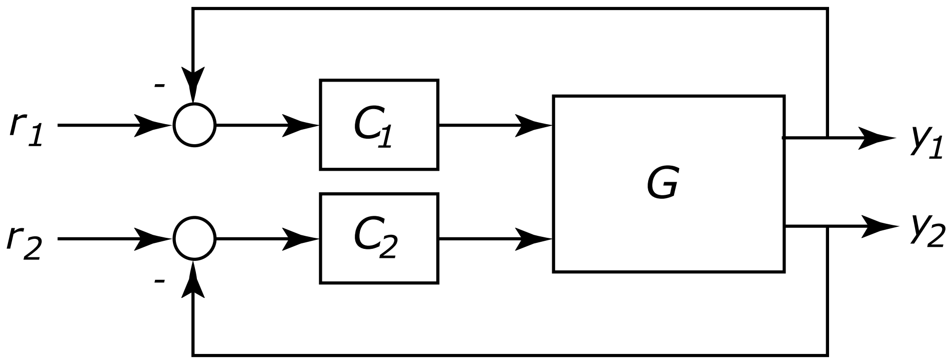


$$Y_1(s) = \frac{2}{s+1}U_1(s) + \frac{3}{s+2}U_2(s)$$
$$Y_2(s) = \frac{1}{s+1}U_1(s) + \frac{1}{s+1}U_2(s)$$

Note: inputs influence both outputs!

The need for multivariable control

Simple approach: pair inputs and outputs, use SISO control



Use u_1 to control y_1 and u_2 to control y_2. PI-control

$$U_i(s) = \frac{K_i(s+1)}{s}(R_i(s) - Y_i(s)) \quad i = 1,2$$

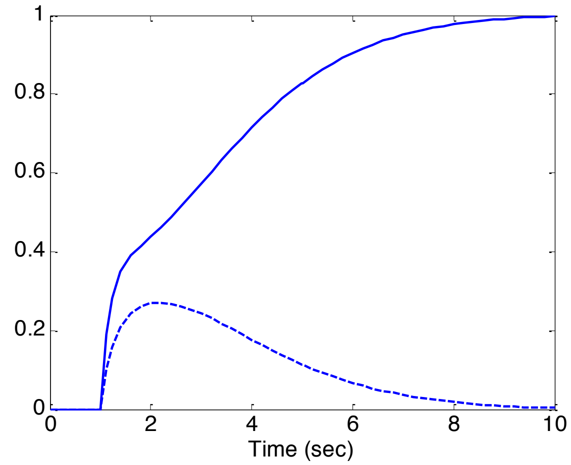
gives transfer functions

$$G_{r_1 \to y_1} = \frac{2K_1}{2+2K_1} \quad G_{r_2 \to y_2} = \frac{K_2}{s+K_2}$$

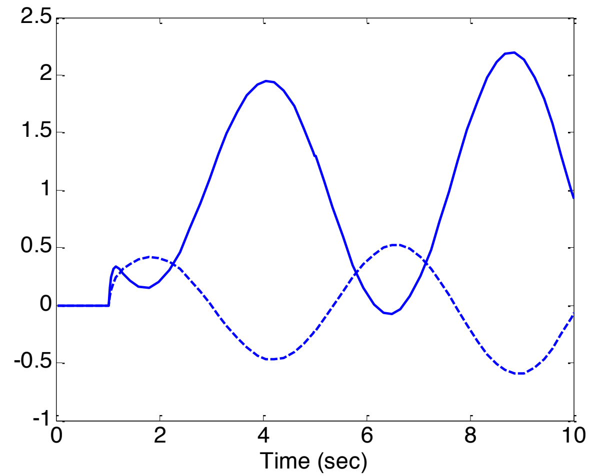
Stable for all (strictly) positive values of K_1, K_2!

The need for multivariable control

Response to step-change in r_1 when $K_1=1$, $K_2=2$…

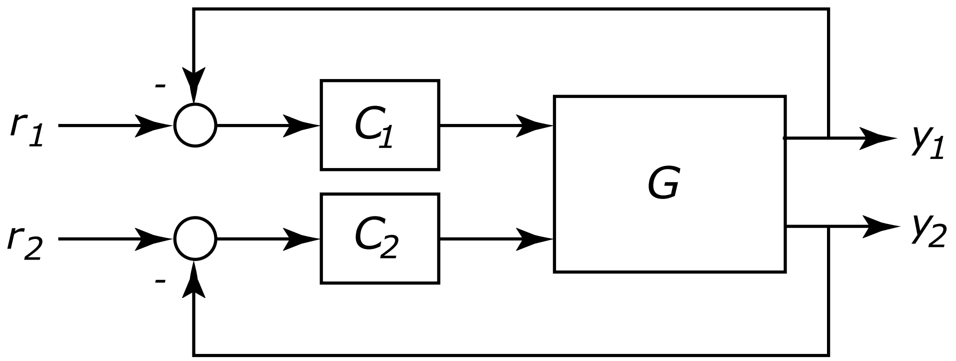


…and when $K_1=4$, $K_2=8$



Multivariable system unstable, even if SISO analysis indicates stability!

What is happening?



$$Y_1(s) = \frac{2}{s+1}U_1(s) + \frac{3}{s+2}U_2(s)$$
$$Y_2(s) = \frac{1}{s+1}U_1(s) + \frac{1}{s+1}U_2(s)$$

Interactions in the system makes the control loops coupled!

Multivariable analysis

$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix} \left(\begin{bmatrix} R_1 \\ R_2 \end{bmatrix} - \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} \right) \Rightarrow Y = (I + GC)^{-1} GC\, R$$

Elements of closed-loop transfer matrix very different from SISO analysis!

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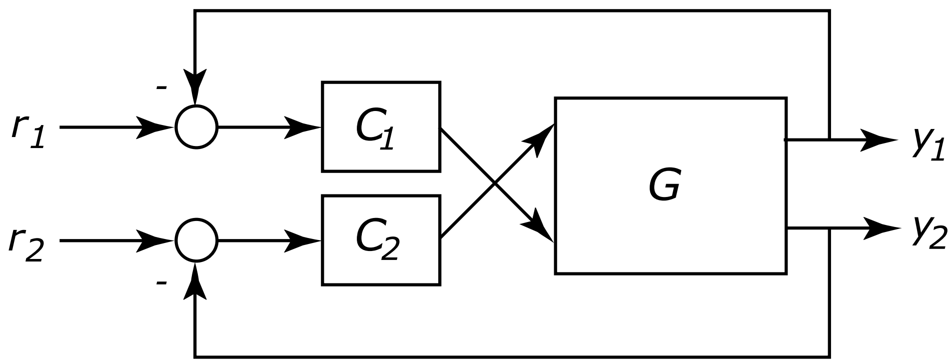
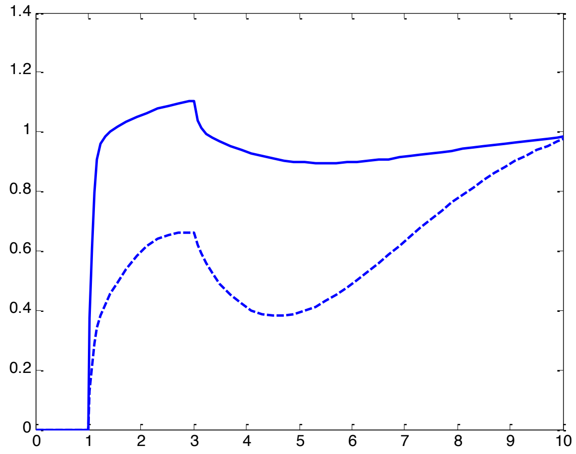
The need for multivariable control

What if we pair signals “the other way around”? (with new controllers)





A system with strong and complex interactions!

- Can no longer stabilize system with positive gains
  (and system becomes unstable when if one of these controllers fail)

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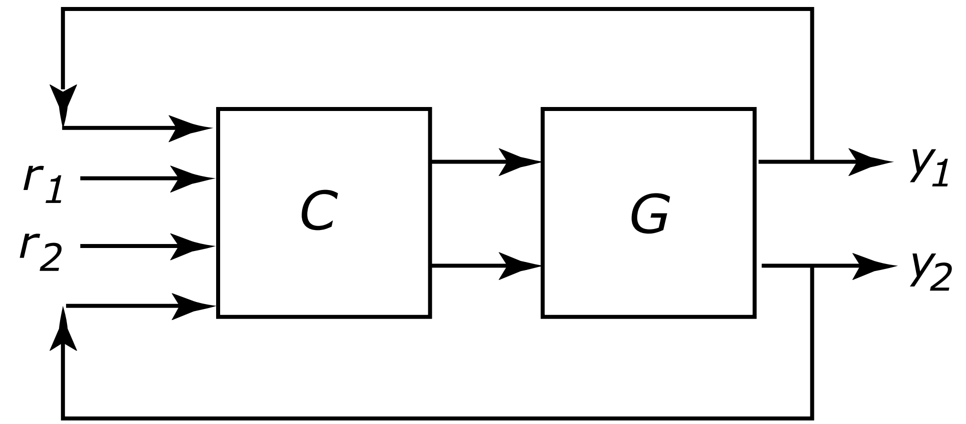
What to do?



Learn the material in this course!

- Understand dynamics of multivariable systems
- Analyze when interactions are harmful, or constraining.
- Design multivariable controllers
  - that use Y_1, Y_2, R_1 and R_2 simultaneously to compute U_1 and U_2
  - optimal controllers, or combinations of simple SISO controllers

(the course contains more than this…)

Part I – Basic Control Revisited

Lecture 1: Signal norms, system gains, and input-output stability
Lecture 2: Analysis and specification of closed loop performance
Lecture 3: Robustness of closed loop system to model uncertainties
Lecture 4: Fundamental limitations

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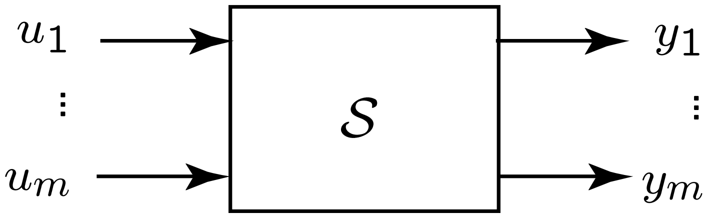
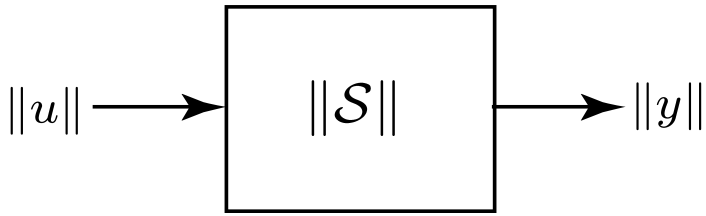
Systems as ”mapping of signals”





Key components:

- Signal norms: measure ”size” of signals
- System gains: measure the system’s amplification
- Frequency responses

Admits natural extensions from scalar to multivariable systems!

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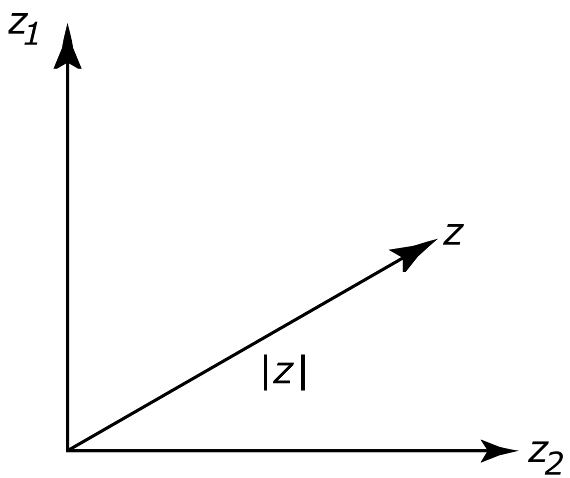
Vector norms

Vector norms measure the “size” of vectors.

– common choice: Euclidian norm (also known as 2-norm)



$$|z|^2 = \sum_i z_i^2 = z^T z$$

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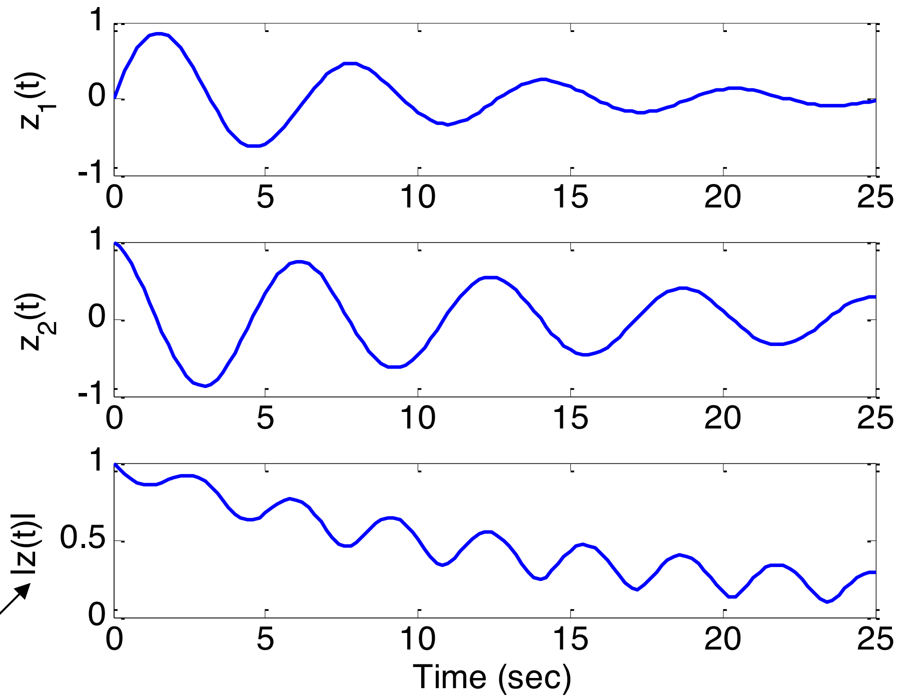
Signal norms

Signals are functions of time

- signal norms measure size across both space and time.



$\Rightarrow ||z||$

summing up over channels

summing up over time

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Signal norms

The *peak-norm*, or L_∞-norm, of a signal is defined as

$$\|z\|_\infty = \sup_{t\geq 0} |z(t)|$$

A signal is *bounded* if its peak-norm is bounded ($\|z\|_\infty < \infty$)

The *energy-norm*, or L_2-norm, of a signal is defined as

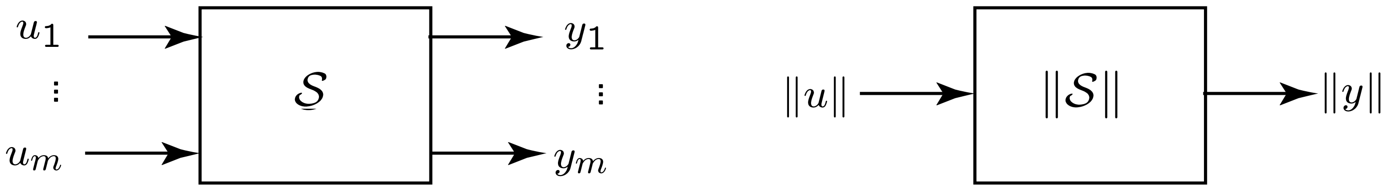
$$\|z\|_2 = \sqrt{\int_{-\infty}^{\infty} |z(t)|^2 \, dt}$$

A signal is *finite-energy* if $\|z\|_2 < \infty$

Note: bounded signals may have infinite energy (and vice versa)
we will only work with the 2-norm in this course

The energy-gain of a system

Measures "energy amplification" of system



The amplification for a specific signal $u \neq 0$ is given by

$$\frac{\|y\|_2}{\|u\|_2} = \frac{\|\mathcal{S}u\|_2}{\|u\|_2}$$

The (energy) gain is the maximal amplification (over all finite-energy signals)

$$\|\mathcal{S}\| = \sup_{0<\|u\|_2<\infty} \frac{\|\mathcal{S}u\|_2}{\|u\|_2}$$

Finite gains and stability

Theorem. The linear time-invariant system $y = Gu$ maps any signal u of finite energy into a signal y of finite energy if and only if the transfer function $G(s)$ is stable.

Energy gains of scalar linear systems

Stable scalar linear time-invariant system $\mathcal{S} : Y(s) = G(s)U(s)$

Assume that $|G(i\omega)| \leq K$ with equality for $\omega = \omega^\star$

Then, Parseval's theorem yields

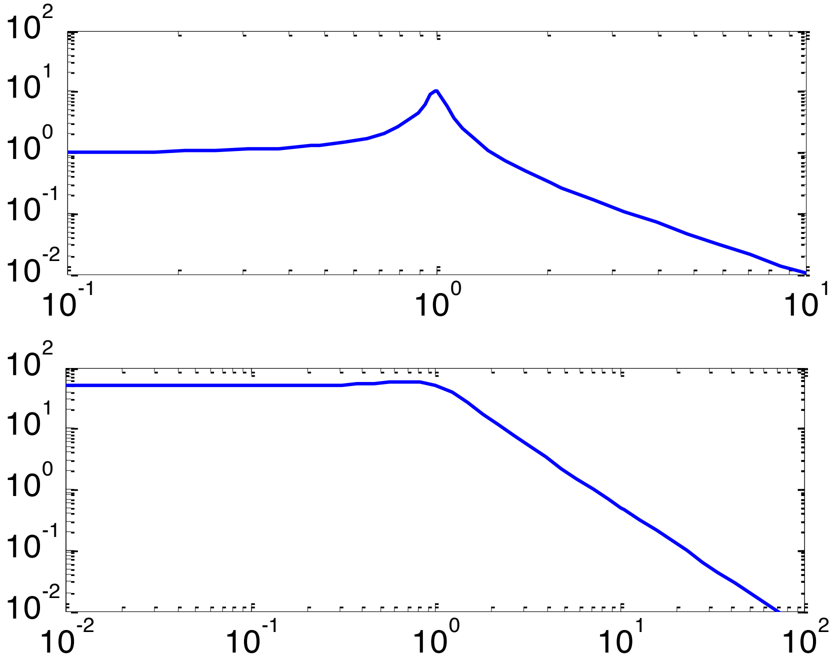
$$\|y\|_2^2 = \frac{1}{2\pi}\int_{-\infty}^{\infty} |Y(i\omega)|^2 d\omega = \\ = \frac{1}{2\pi}\int_{-\infty}^{\infty} |G(i\omega)|^2 |U(i\omega)|^2 d\omega \leq K^2 \|u\|_2^2$$

Since equality holds for $u(t) = \sin(\omega^\star t)$, we have

$$\|\mathcal{S}\| = \sup_{\omega} |G(i\omega)| = \|G\|_\infty$$

Quiz: energy gains and Bode diagrams

Quiz: the Bode diagrams below represent two different linear time-invariant systems. Which one has the largest energy-gain?



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Example: gain of nonlinear system

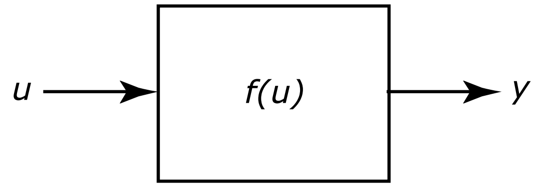
Static nonlinear system

$$\mathcal{S} : y(t) = f(u(t))$$

where

$$|f(x)| \leq K|x|$$

with equality for $x = x^\star$



Since

$$\|y\|_2^2 = \int_{-\infty}^{\infty} |f(u(t))|^2 \, dt \leq \int_{-\infty}^{\infty} K^2 |u(t)|^2 \, dt = K^2 \|u\|_2^2$$

the energy gain is

$$\|\mathcal{S}\| = \sup_u \frac{\|y\|_2}{\|u\|_2} = K$$

Example: gains of static linear systems

Consider the static linear system $y = Au$ with gain

$$\|A\| = \sup_{u \neq 0} \frac{|Au|}{|u|}$$

Since

$$\|A\|_2^2 = \sup_{u \neq 0} \frac{|Au|^2}{|u|^2} = \sup_{u \neq 0} \frac{u^T A^T A u}{u^T u} = \lambda_{\max}(A^T A)$$

the gain is the square root of the maximal eigenvalue of $A^T A$.

(the square roots of eig($A^T A$) are called the *singular values* of A)

Quiz: a flavour of multivariable systems

Quiz: What is the gain of the following (static) systems

a) $$y = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} u$$

b) $$y = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} u$$

Which are the corresponding "worst-case" input vectors?
(vectors u with |u|=1 that give the maximum value of |y|)

Input-output stability

Definition. A system $\mathcal{S}$ is *input-output stable* if $\|\mathcal{S}\| < \infty$

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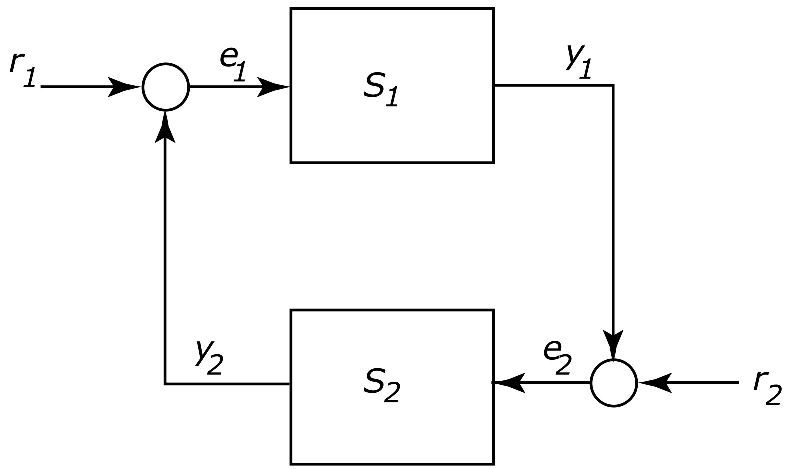
Small gain theorem

Theorem. Consider the interconnection



If $\mathcal{S}_1$ and $\mathcal{S}_2$ are input-output stable and

$$\|\mathcal{S}_1\| \cdot \|\mathcal{S}_2\| < 1$$

Then, the closed-loop system with r_1, r_2 as inputs and e_1, e_2, y_1, y_2 as outputs is input-output stable.

Proof sketch

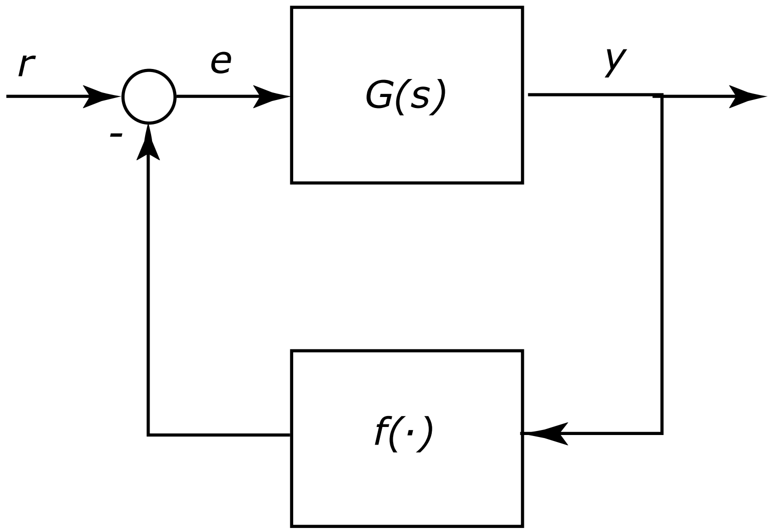
$$e_1 = r_1 + \mathcal{S}_2(r_2 + y_1)$$
$$y_1 = \mathcal{S}_1 e_1$$
$$\|e_1\| \leq \|r_1\| + \|\mathcal{S}_2\|(\|r_2\| + \|\mathcal{S}_1\| \cdot \|e_1\|)$$
$$\|e_1\| \leq \frac{\|r_1\| + \|\mathcal{S}_2\| \cdot \|r_2\|}{1 - \|\mathcal{S}_2\| \cdot \|\mathcal{S}_1\|}$$

Hence, the gain from r_1, r_2 to e_1 is finite.
A similar argument proves that the gain from r_1, r_2 to e_2 is finite

Note: for linear system, it is sufficient that $\|\mathcal{S}_1 \mathcal{S}_2\| \leq 1$

Quiz: a nonlinear interconnection

Is the feedback interconnection



with $G(s) = \dfrac{0.4}{s+1}$ and $|f(y)| \leq 2|y|$ input-output stable?

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Conclusions

- Systems as mappings of signals
- Norms
  - Vector norms: measure size of vector “across channels”
  - Signal norms: measure size of signal across time and space
- Gains
  - The amplification of signal (in terms of the appropriate norm)
  - For stable linear systems, gain is infinity norm of frequency function
- Input-output stability and the small gain theorem

Next lecture: The closed-loop system (Chapter 6)