EL2520
Control Theory and Practice

Welcome!

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The practical

- Course information and schedule
  https://www.kth.se/social/course/EL2520/

- All slides and exercises available on homepage after class
- Course book:English and Swedish versions; Kåren or Internet
- Course material and practicalities: STEX, Osquldas väg 10

- Computer exercises: need kth.se account
- Register for labs via home page, from 22nd of March
- Lab access: details next week

- Expectations and feedback: email me at mikaelj@ee.kth.se

Course elements

- 14 lectures, Mikael Johansson

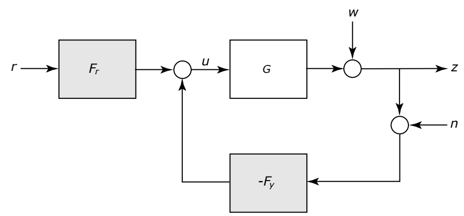
- 8 exercise sessions, Olle Trollberg
  - all exercises given in English
  - one Q&A session per week

- 4 computer exercise sessions, Håkan Terelius
  - groups of 2 students
  - written report on exercise 1 and 2, (short sheet on 3, 4)

- 1 project
  - groups of 4 students
  - two sessions in laboratory, written report

- Exam: 5 hours, written exam on 25th of May 2012, 09.00-14.00

Why feedback control?



- Make a system behave as desired
  - e.g. stabilize unstable system
- Reduce effects of disturbances and component variations
  - e.g. keep variables constant
- New freedom for designers
  - physical design vs. feedback control

Why "Control Theory and Practice"?

- Solve complex control problems
  - Fundamental limitations
  - Sensitivity and robustness analysis
  - Systems with multiple inputs and outputs
  - Controller design using optimization
  - Systems with constraints
  - Computer-based control systems
- Understand dynamic systems! The research front
- Applications!

Course structure

1. Basic control revisited (4 lectures)
2. Modern control of multivariable linear systems (7 lectures)
3. Control of systems with constraints (2 lectures)

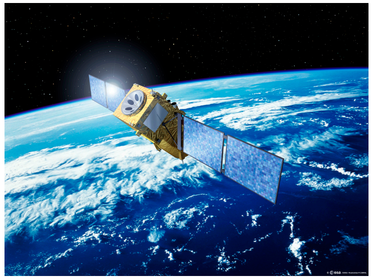
Applications

Use control theory to analyze and modify system properties!

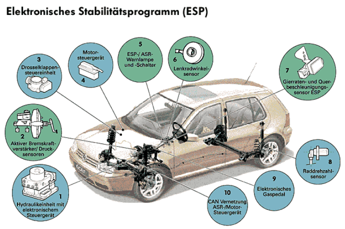
Applications in most engineering domains:

- Aerospace
- Cars and heavy vehicles
- Autonomous systems and robots
- Process industry
- Consumer products
- Communication systems
- Economics
- Biology
- …

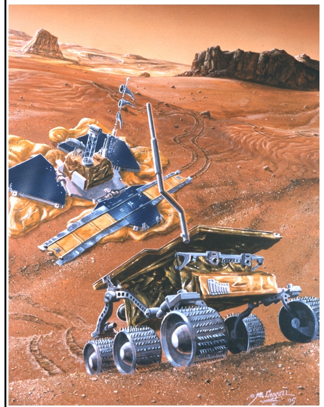
Aerospace



Vehicle control



Autonomous systems and robotics



Process industry

Consumer products

Course structure

1. Basic control revisited (4 lectures)
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How to learn more?

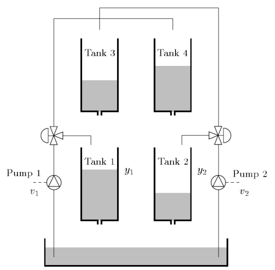
- Internet (e.g., links at www.ee.kth.se)
- Books (see course information, home page)
- Journals (IEEE Transactions on Automatic Control, Automatica, …)
- Courses (Nonlinear Control, Modelling of Dynamical Systems, …)
- Software (Matlab, …)

- Interact with us, and take the opportunity to learn!

Central component: multivariable systems

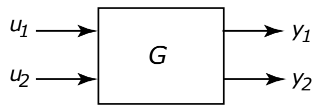
Key aspects:
- several inputs and outpus,
- dynamics coupled (single input affects many outputs)

Laboratory project example: quadruple tank system



The need for multivariable control

Example. Consider a linear system with two inputs and two outputs

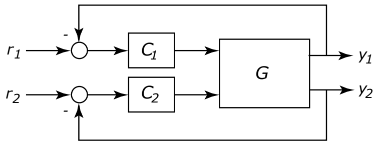


$$Y_1(s) = \frac{2}{s+1}U_1(s) + \frac{3}{s+2}U_2(s)$$
$$Y_2(s) = \frac{1}{s+1}U_1(s) + \frac{1}{s+1}U_2(s)$$

Note: inputs influence both outputs!

The need for multivariable control

Simple approach: pair inputs and outputs, use SISO control



Use u_1 to control y_1 and u_2 to control y_2. PI-control

$$U_i(s) = \frac{K_i(s+1)}{s}(R_i(s) - Y_i(s)) \quad i = 1, 2$$

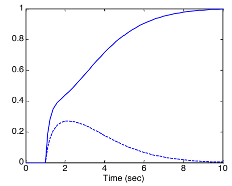
gives transfer functions

$$G_{r_1 \to y_1} = \frac{2K_1}{2+2K_1} \quad G_{r_2 \to y_2} = \frac{K_2}{s+K_2}$$

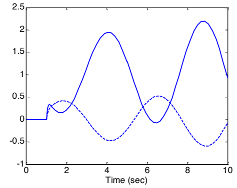
Stable for all (strictly) positive values of K_1, K_2!

The need for multivariable control

Response to step-change in r_1 when $K_1=1$, $K_2=2$…

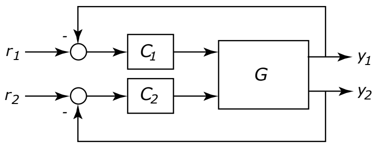


…and when $K_1=4$, $K_2=8$



Multivariable system unstable, even if SISO analysis indicates stability!

What is happening?



$$Y_1(s) = \frac{2}{s+1}U_1(s) + \frac{3}{s+2}U_2(s)$$
$$Y_2(s) = \frac{1}{s+1}U_1(s) + \frac{1}{s+1}U_2(s)$$

Interactions in the system makes the control loops coupled!

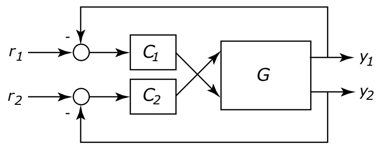
Multivariable analysis

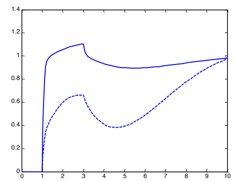
$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix} \left(\begin{bmatrix} R_1 \\ R_2 \end{bmatrix} - \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} \right) \Rightarrow Y = (I+GC)^{-1}GC\,R$$

Elements of closed-loop transfer matrix very different from SISO analysis!

The need for multivariable control

What if we pair signals “the other way around”? (with new controllers)

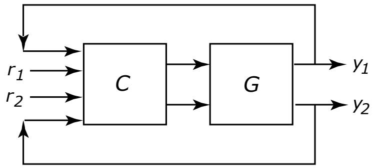




A system with strong and complex interactions!

- Can no longer stabilize system with positive gains (and system becomes unstable when if one of these controllers fail)

What to do?



Learn the material in this course!

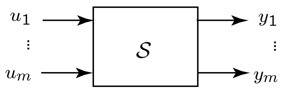
- Understand dynamics of multivariable systems
- Analyze when interactions are harmful, or constraining.
- Design multivariable controllers
  - that use Y_1, Y_2, R_1 and R_2 simultaneously to compute U_1 and U_2
  - optimal controllers, or combinations of simple SISO controllers

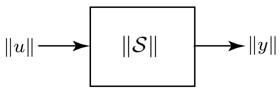
(the course contains more than this…)

Part I – Basic Control Revisited

Lecture 1: Signal norms, system gains, and input-output stability
Lecture 2: Analysis and specification of closed loop performance
Lecture 3: Robustness of closed loop system to model uncertainties
Lecture 4: Fundamental limitations

Systems as "mapping of signals"





Key components:

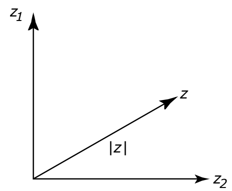
- Signal norms: measure "size" of signals
- System gains: measure the system's amplification
- Frequency responses

Admits natural extensions from scalar to multivariable systems!

Vector norms

Vector norms measure the "size" of vectors.

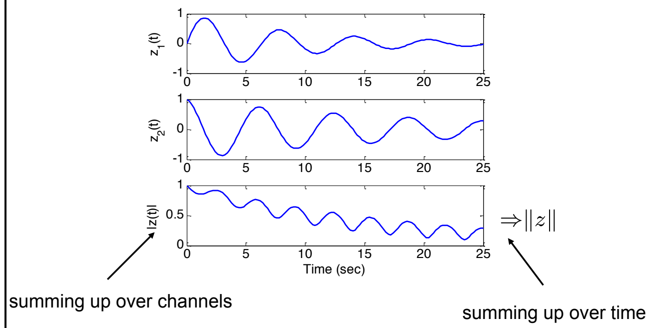
- common choice: Euclidian norm (also known as 2-norm)



$$|z|^2 = \sum_i z_i^2 = z^T z$$

Signal norms

Signals are functions of time
- signal norms measure size across both space and time.



Signal norms

The *peak-norm*, or L_∞-norm, of a signal is defined as

$$\|z\|_\infty = \sup_{t \geq 0} |z(t)|$$

A signal is *bounded* if its peak-norm is bounded ($\|z\|_\infty < \infty$)

The *energy-norm*, or L_2-norm, of a signal is defined as

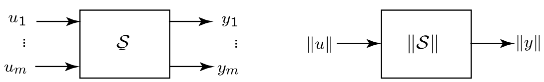
$$\|z\|_2 = \sqrt{\int_{-\infty}^{\infty} |z(t)|^2 \, dt}$$

A signal is *finite-energy* if $\|z\|_2 < \infty$

Note: bounded signals may have infinite energy (and vice versa)
we will only work with the 2-norm in this course

The energy-gain of a system

Measures "energy amplification" of system



The amplification for a specific signal $u \neq 0$ is given by

$$\frac{\|y\|_2}{\|u\|_2} = \frac{\|\mathcal{S}u\|_2}{\|u\|_2}$$

The (energy) gain is the maximal amplification (over all finite-energy signals)

$$\|\mathcal{S}\| = \sup_{0 < \|u\|_2 < \infty} \frac{\|\mathcal{S}u\|_2}{\|u\|_2}$$

Finite gains and stability

Theorem. The linear time-invariant system $y = Gu$ maps any signal u of finite energy into a signal y of finite energy if and only if the transfer function $G(s)$ is stable.

Energy gains of scalar linear systems

Stable scalar linear time-invariant system $\mathcal{S}: Y(s) = G(s)U(s)$

Assume that $|G(i\omega)| \leq K$ with equality for $\omega = \omega^\star$

Then, Parseval's theorem yields

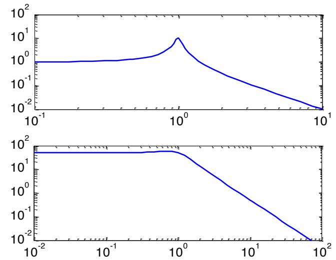
$$\begin{aligned} \|y\|_2^2 &= \frac{1}{2\pi}\int_{-\infty}^{\infty} |Y(i\omega)|^2 d\omega = \\ &= \frac{1}{2\pi}\int_{-\infty}^{\infty} |G(i\omega)|^2 |U(i\omega)|^2 d\omega \leq K^2 \|u\|_2^2 \end{aligned}$$

Since equality holds for $u(t) = \sin(\omega^\star t)$, we have

$$\|\mathcal{S}\| = \sup_\omega |G(i\omega)| = \|G\|_\infty$$

Quiz: energy gains and Bode diagrams

Quiz: the Bode diagrams below represent two different linear time-invariant systems. Which one has the largest energy-gain?



Example: gain of nonlinear system

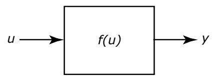
Static nonlinear system

$\mathcal{S}: y(t) = f(u(t))$

where

$|f(x)| \leq K|x|$

with equality for $x = x^\star$



Since

$$\|y\|_2^2 = \int_{-\infty}^{\infty} |f(u(t))|^2 \, dt \leq \int_{-\infty}^{\infty} K^2 |u(t)|^2 \, dt = K^2 \|u\|_2^2$$

the energy gain is

$$\|\mathcal{S}\| = \sup_u \frac{\|y\|_2}{\|u\|_2} = K$$

Example: gains of static linear systems

Consider the static linear system $y = Au$ with gain

$$\|A\| = \sup_{u\neq 0} \frac{|Au|}{|u|}$$

Since

$$\|A\|_2^2 = \sup_{u\neq 0} \frac{|Au|^2}{|u|^2} = \sup_{u\neq 0} \frac{u^T A^T A u}{u^T u} = \lambda_{\max}(A^T A)$$

the gain is the square root of the maximal eigenvalue of $A^T A$.

(the square roots of eig($A^T A$) are called the *singular values* of A)

Quiz: a flavour of multivariable systems

Quiz: What is the gain of the following (static) systems

a) $y = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} u$

b) $y = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} u$

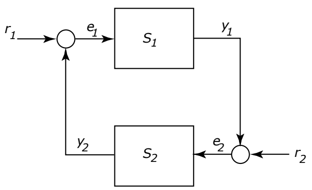
Which are the corresponding "worst-case" input vectors?
(vectors u with |u|=1 that give the maximum value of |y|)

Input-output stability

Definition. A system $\mathcal{S}$ is *input-output stable* if $\|\mathcal{S}\| < \infty$

Small gain theorem

Theorem. Consider the interconnection



If $\mathcal{S}_1$ and $\mathcal{S}_2$ are input-output stable and

$$\|\mathcal{S}_1\| \cdot \|\mathcal{S}_2\| < 1$$

Then, the closed-loop system with r_1, r_2 as inputs and e_1, e_2, y_1, y_2 as outputs is input-output stable.

Proof sketch

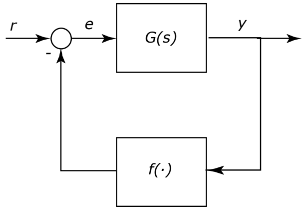
$$e_1 = r_1 + \mathcal{S}_2(r_2 + y_1)$$
$$y_1 = \mathcal{S}_1 e_1$$
$$\|e_1\| \leq \|r_1\| + \|\mathcal{S}_2\|(\|r_2\| + \|\mathcal{S}_1\| \cdot \|e_1\|)$$
$$\|e_1\| \leq \frac{\|r_1\| + \|\mathcal{S}_2\| \cdot \|r_2\|}{1 - \|\mathcal{S}_2\| \cdot \|\mathcal{S}_1\|}$$

Hence, the gain from r_1, r_2 to e_1 is finite.
A similar argument proves that the gain from r_1, r_2 to e_2 is finite

Note: for linear system, it is sufficient that $\|\mathcal{S}_1 \mathcal{S}_2\| \leq 1$

Quiz: a nonlinear interconnection

Is the feedback interconnection



with $G(s) = \dfrac{0.4}{s+1}$ and $|f(y)| \leq 2|y|$ input-output stable?

Conclusions

- Systems as mappings of signals
- Norms
  - Vector norms: measure size of vector “across channels”
  - Signal norms: measure size of signal across time and space
- Gains
  - The amplification of signal (in terms of the appropriate norm)
  - For stable linear systems, gain is infinity norm of frequency function
- Input-output stability and the small gain theorem

Next lecture: The closed-loop system (Chapter 6)