KTH Electrical Engineering

Exam in EG2050 System Planning, 14 March 2013, 8:00–13:00, E31, E32, E35, E36, E51, E52

Allowed aids

In this exam you are allowed to use the following aids:

- Calculator without information relevant to the course.
- One **handwritten**, **single-sided** A4-page with **your own** notes (original, not a copy), which should be handed in together with the answer sheet.

PART I (MANDATORY)

Write all answers on the answer sheet provided. Motivations and calculations do not have to be presented.

Part I can yield 40 points in total. The examinee is guaranteed to pass if the score is at least 33 points. If the result in part I is at least 31 points, then there will be a possibility to complement for passing the exam with the grade E.

Problem 1 (4 p)

Answer the following theoretical questions by choosing *one* alternative, which you find correct.

a) (2 p) AB Elbolaget is a balance responsible retailer of electricity. The electricity trading on the market where AB Elbolaget is active has a trading period of one hour. AB Elbolaget has only one customer, namely AB Industri. Assume that the consumption of AB Industri is varying between 800 MW and 1 250 MW with an average of 1 000 MW during the hour. In which the following cases is AB Elbolaget fulfilling its balance responsibility without any imbalance: I) Then the generation of AB Elbolaget is varying between 800 MW and 1 250 MW with an average of 980 MW during the hour, II) Then AB Elbolaget buys 500 MWh from the local power exchange, ElKräng, and is producing 400 MW during the first half hour and 600 MW during the second half hour, III) Then AB Elbolaget buys 1 250 MWh from ElKräng.

1. None of the statements is true.
2. Only II is true.
3. I and II are true but not III.
4. II and III are true but not I.
5. All the statements are true.

b) (2 p) The following applies to a bilateral electricity market: I) Producers are free to sell to any other producer, retailer or consumer, II) All electricity trading has to be performed at a power pool, III) The consumers are free to buy from any producer or retailer.

1. None of the statements is true.
2. Only I is true.
3. Only II is true.
4. Only III is true.
5. I and III are true but not II.

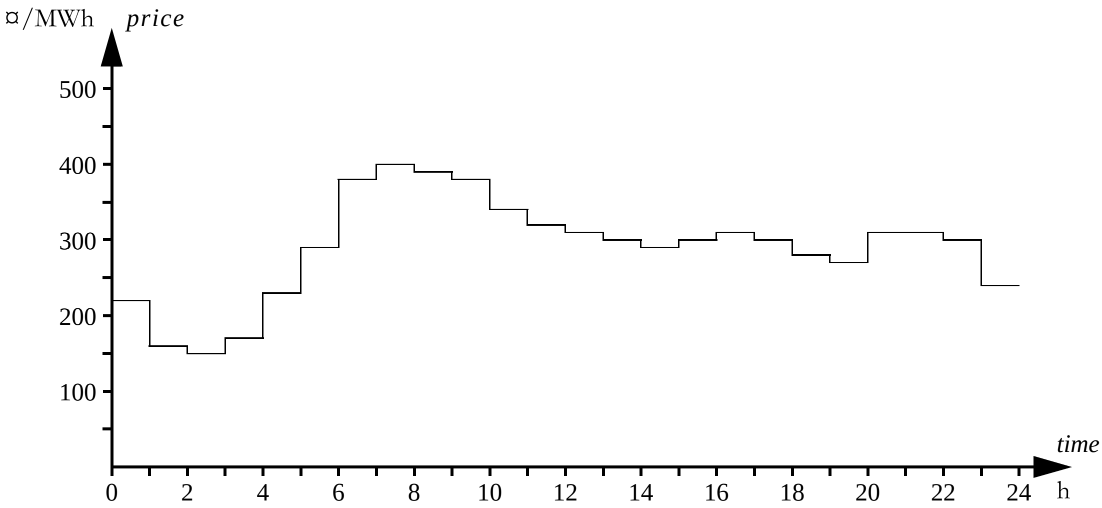
Problem 2 (6 p)

Assume that the electricity market in Land has perfect competition, all players have perfect information, and there are neither transmission nor capacity limitations. The hydro reservoirs of Land has a maximal storage capacity equal to 40 TWh. On 1 January the reservoirs holds in total 20 TWh and according to the long-term forecast for the electricity market (which as already mentioned is assumed to be faultless), the reservoirs should hold 25 TWh on 31 December. The inflow is 50 TWh between 1 January and 30 June, and 20 TWh between 1 July and 31 December. The variable operation cost in the hydro power is negligible.

a) (2 p) Assume that the electricity price is 360 ¤/MWh between 1 January and 30 June, whereas it is 380 ¤/MWh between 1 July and 31 December. What is the contents of the reservoirs on 30 June in the evening?

b) (1 p) How large is the total hydro power generation between 1 January and 30 June?

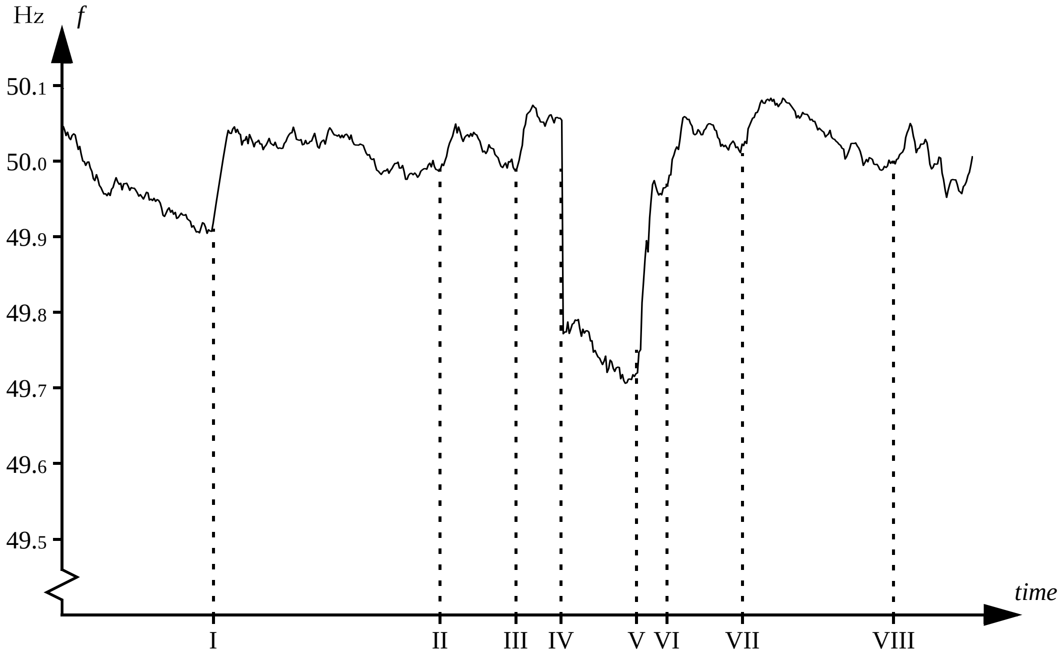
c) (3 p) The figure below shows the electricity price in a certain electricity market during one day. Assume that there is perfect competition in this electricity market and that all players have access to perfect information. How much will be generated during this day in a power plant with the variable operation cost 350 ¤/MWh and the installed capacity 100 MW?



Problem 3 (6 p)

The primary control in Land is divided in a normal operation reserve and a disturbance reserve. The normal operation reserve has the gain 2 000 MW/Hz and is intended to manage normal variations in for example load and wind power generation. The disturbance reserve has the gain 1 500 MW/Hz and is intended to manage outages in larger power plants. The normal operation reserve is available in the frequency range 49.9–50.1 Hz and the disturbance reserve is available in the frequency range 49.5–49.9 Hz.

The figure below shows the frequency in the power system of Land during a certain hour. At eight occasions during this hour (indicated by the Roman numbers I to VIII in the time scale) there are larger frequency changes, which are not due to normal variations in load and wind power generation. Three of the events causing these frequency changes are described below. State at which time (I–VIII) each event occurs.



a) (2 p) The system looses 500 MW generation due to a shortcut in a transformer.

b) (2 p) The system operator activates an up-regulation bid and a power plant is therefore increasing its generation by 250 MW.

c) (2 p) The transmission on an HVDC line to another synchronous grid is reduced linearly during a five minute period from an export of 500 MW until the line does not transfer any power at all. At the same time as the export on this interconnection is reduced, the generation of the system is reduced by 100 MW once per minute.

Problem 4 (12 p)

Stads energi AB owns a thermal power plant with four blocks. Moreover, the company owns a wind farm. Assume that the company has formulated their short-term planning problem as a MILP problem and that the following symbols have been introduced:

Indices for the power plants: Block I - 1, Block II - 2, Block III - 3, Block IV - 4.

β_{Gg} = variable operation cost in power plant g, $g = 1, \dots, 4$,
C_g^+ = start-up cost in power plant g, $g = 1, \dots, 4$,
$G_{g,t}$ = generation in power plant g, hour t, $g = 1, \dots, 4$, $t = 1, \dots, 24$,
$\bar{G}_g$ = installed capacity in power plant g, $g = 1, \dots, 4$,
$\underline{G}_g$ = minimal generation when power plant g is committed, $g = 1, \dots, 4$,
λ_t = expected electricity price at ElKräng hour t, $t = 1, \dots, 24$,
$s_{g,t}^+$ = start-up variable for power plant g, hour t, $g = 1, \dots, 4$, $t = 1, \dots, 24$,
$u_{g,0}$ = unit commitment of power plant g at the beginning of the planning period, $g = 1, \dots, 4$,
$u_{g,t}$ = unit commitment of power plant g, hour t, $g = 1, \dots, 4$, $t = 1, \dots, 24$,
W_t = expected wind power generation in hour t, $t = 1, \dots, 24$.

a) (3 p) Which of the symbols above represent optimisation variables and parameters respectively?

b) (4 p) Stads energi AB sells power to the local power exchange ElKräng.. Formulate the objective function if the aim of the planning problem is to maximise the income of sold electricity minus the costs of the thermal power plant. Use the symbols defined above.

c) (4 p) Formulate the constraint that sets the relation between $u_{g,t}$, $u_{g,t-1}$ and $s_{g,t}^+$ for hour t. Notice that the constraint must be formulated without using any other optimisation variables than those defined above!

d) (1 p) The reservoir of the hydro power plant Ån holds 2 000 HE at 10:00. The local inflow as well as discharge and spillage from the power plant upstream amounts to 150 m^3/s between 10:00 and 11:00. During the same time, 160 HE are discharged from Ån. How much will the reservoir of Ån hold at 11:00? Notice that the answer should be given in m^3!

Problem 5 (12 p)

Akabuga is town in East Africa. The town is not connected to a national grid, but a group of local businessmen are considering to start a local power company, Akabuga Electricity Company Ltd. (AECL). One of the options that are studied for AECL is to supply the system with a wind power plant and four diesel generator sets. A simplified model of the wind power plant is given in table 1. The diesel generator sets would have a capacity of 200 kW each, availability 90% and operation cost 0.5 ¤/kWh. The figure below shows a duration curve for the load in Akabuga (solid line) as well as an approximation of the same duration curve (dashed line).

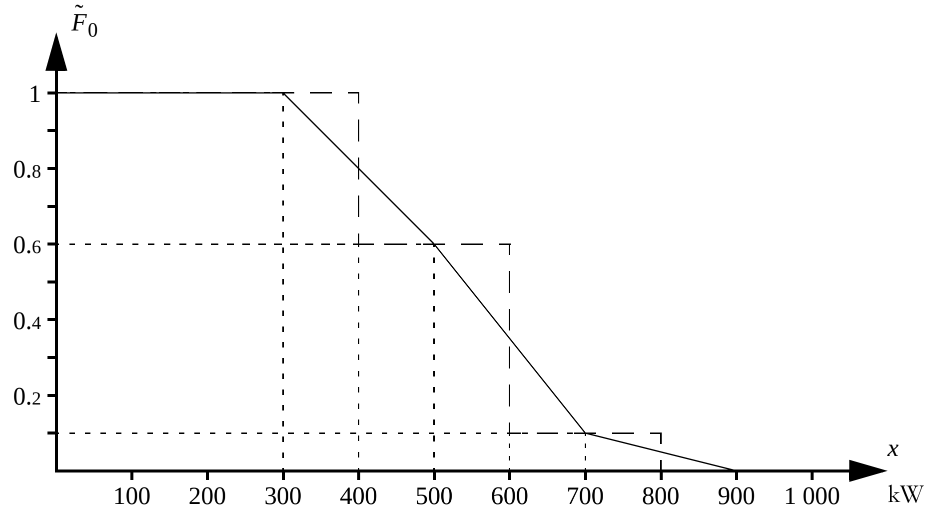


Table 1 Model of the wind power plant in problem 5.

Available generation capacity [kW]	Probability [%]
0	25
100	35
200	30
300	10

Table 2 Results from a probabilistic production cost simulation of the planned power system in Akabuga.

	x = 0	x = 300	x = 400	x = 500	x = 600	x = 700	x = 800	x = 900	x = 1 000	x = 1 100
$\int_x^\infty \tilde{F}_1(\xi)d\xi$	715	415	316	222	140	76	35	12	3	1
$\int_x^\infty \tilde{F}_2(\xi)d\xi$	735	435	336	241	157	91	45	19	7	2
$\int_x^\infty \tilde{F}_3(\xi)d\xi$	755	455	356	261	175	106	56	26	10	3
$\int_x^\infty \tilde{F}_4(\xi)d\xi$	775	475	376	280	193	121	68	34	15	6
$\int_x^\infty \tilde{F}_5(\xi)d\xi$	795	495	396	300	212	137	81	43	20	9

a) (1 p) Use the approximate duration curve of the load to compute the equivalent load duration curve including outages in the wind power plant, $\tilde{F}_1(x)$, for the interval $400 \le x < 500$.

b) (1 p) Use the approximate duration curve of the load to compute the equivalent load duration curve including outages in the wind power plant, $\tilde{F}_1(x)$, for the interval $500 \le x < 600$.

c) (1 p) Use the approximate duration curve of the load to compute the equivalent load duration curve including outages in the wind power plant, $\tilde{F}_1(x)$, for the interval $600 \le x < 700$.

d) (3 p) Table 2 shows some results from a probabilistic production cost simulation of Akabuga. Use these results to compute the expected operation cost per hour.

e) (2 p) Generate a value of the load using the inverse transform method and the random number 0.2 from a $U(0, 1)$-distribution. (Use the exact duration curve in this problem!)

f) (4 p) The planned power system of Akabuga has also been simulated using Monte Carlo methods. The model used in the Monte Carlo simulation is slightly more advanced than the model used in probabilistic production cost simulation; for example, it considers that some consumers are price sensitive. To obtain an accurate result from the Monte Carlo simulation, it has been decided to use stratified sampling.

The results of the fifteen first scenarios of the Monte Carlo-simulation are compiled in table 3. Which estimates of *ETOC* and *LOLP* are obtained from these results?

Table 3 Results from a Monte Carlo simulation of the planned power system in Akabuga.

Stratum	Stratum weight	Observations of *TOC* [¤/h]	Observations of *LOLO*
1	0.900	185, 175, 240, 230, 370	0, 0, 0, 0, 0
2	0.025	290, 290, 300, 390, 290	0, 0, 1, 0, 0
3	0.075	300, 400, 300, 300, 200	1, 1, 1, 1, 1

PART II (FOR HIGHER GRADES)

All introduced symbols must be defined. Solutions should include sufficient detail that the argument and calculations can be easily followed.

The answer to each problem must begin on a new sheet, but answers to different parts of the same problem (a, b, c, etc.) can be written on the same sheet. The fields *Namn* (Name), *Blad nr* (Sheet number) and *Uppgift nr* (Problem number) must be filled out on every sheet.

Part II gives a total of 60 points, but this part will only be marked if the candidate has obtained at least 33 points in part I. Then the results of parts I and II and the bonus points will be added together to determine the examination grade (A, B, C, D, E).

Problem 6 (10 p)

The company AB Industriel plans to build a new 1 000 MW nuclear power plant in Land. These plans have prompted the environment organisation Grönfrid (who are well-known opponents of nuclear power) to publish a critical report. In the summary of this report one can read the following line of argument:

> "AB Industriel has entered into a contract with Kärnkraftbyggarna AB to build a new nuclear power plant in Strålinge. According to official information from the companies, the nuclear power plant will be built for a fixed price of 30 000 million ¤. A depreciation time of 15 years and the required rate of return 5% would result in an annual cost of capital around 3 000 million ¤. This should be compared to the average price in ElKräng, which in the recent years has been around 300 ¤/MWh and the variable operation cost, which is about 100 ¤/MWh for the this kind of nuclear power plant. Thus, an annual generation of 8 TWh would give the net income 1.6 billions ¤ per year. In addition to the problems with nuclear waste and the risk for a nuclear meltdown, another reactor in Strålinge is a financial loss that is not even profitable with state subsidies concerning the needed grid expansion and the state taking responsibility for the costs in case of a nuclear accident."

The president of AB Industriel has briefly commented the Grönfrid report with the words "First, I want to emphasise that we of course follow all environmental legislation and the requirement placed upon us the by the Radiation Safety Authority of Land. Concerning profitability, AB Industriel is owned by large players in trade and industry of Land, and we would not make an investment this size if we did not believe in it."

Use the statistics on the next page and make your own analysis of whether the nuclear power plant in Strålinge is profitable or not.

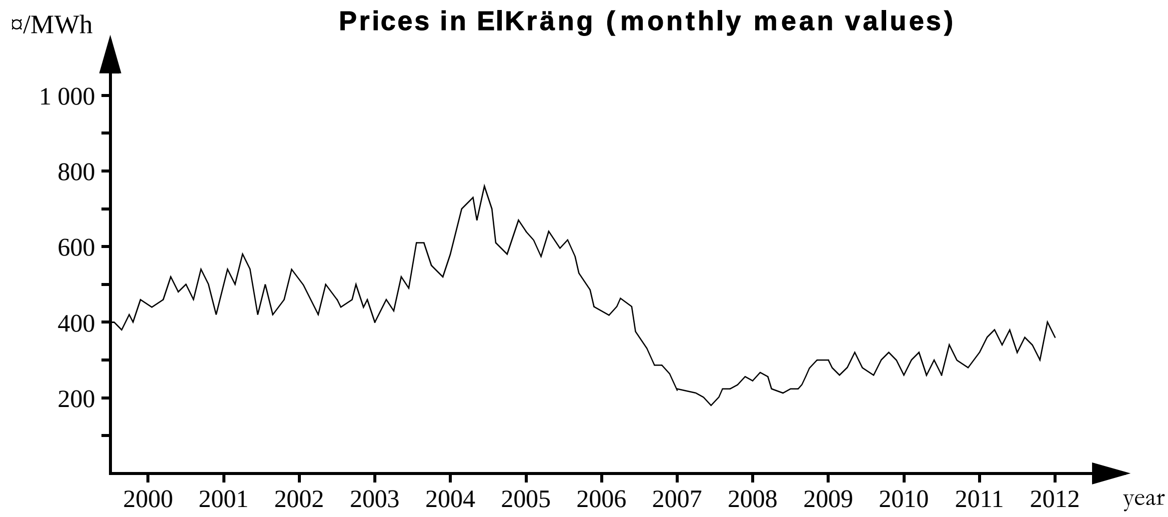
NOTICE! The answers to this problem will be evaluated not on the conclusion concerning the profitability, but on how well you can reason around and evaluate the statements above.

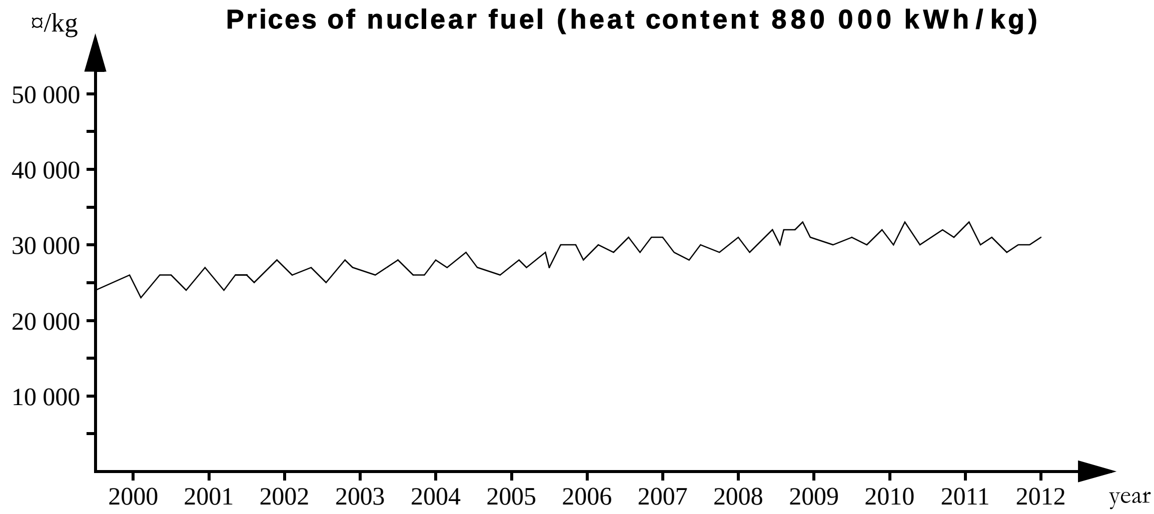
Hint: Annual investment costs can be computed using the annuity formula:

$$AC = I \cdot \frac{r/100}{1-(1+r/100)^{-n}},$$

where

AC = annual cost [¤],
I = investment cost [¤],
r = required rate of return [%],
n = depreciation time [years].





Problem 7 (10 p)

Consider a power system composed of the three countries Aland, Beland and Celand. Data for the transmission lines between the countries are shown in table 5. Each transmission line is equipped with a protection system which after a short time delay disconnects the line if the power flow exceeds the maximal capacity of the line. The power flow on the HVDC line are not affected by the frequency of the system, but can only be controlled manually.

The primary control in the three countries is divided in a normal operation reserve and a disturbance reserve, where the latter is designed to manage outages in larger power plants. The distribution of the gain for the disturbance reserve is shown in table 4. The table also shows the dimensioning fault for each area, i.e., the largest single outage that can occur (provided that the system does not simultaneously loose the generation in several power plants). The requirement on the power system is that it should manage a dimensioning fault occurring in one of the countries when the frequency is 49.9 Hz, without the frequency dropping below 49.5 Hz and without overloading any transmission lines.

Verify that the system in the situation described in table 5 fulfils the requirements if a dimensioning fault occurs. If this is not the case, what quick actions can be taken to ensure that the requirements are fulfilled?

Hint: Notice that the system does not need to manage that the dimensioning fault occurs at the same time in two countries or all three countries; it is sufficient that the system can manage the dimensioning fault in *one* country.

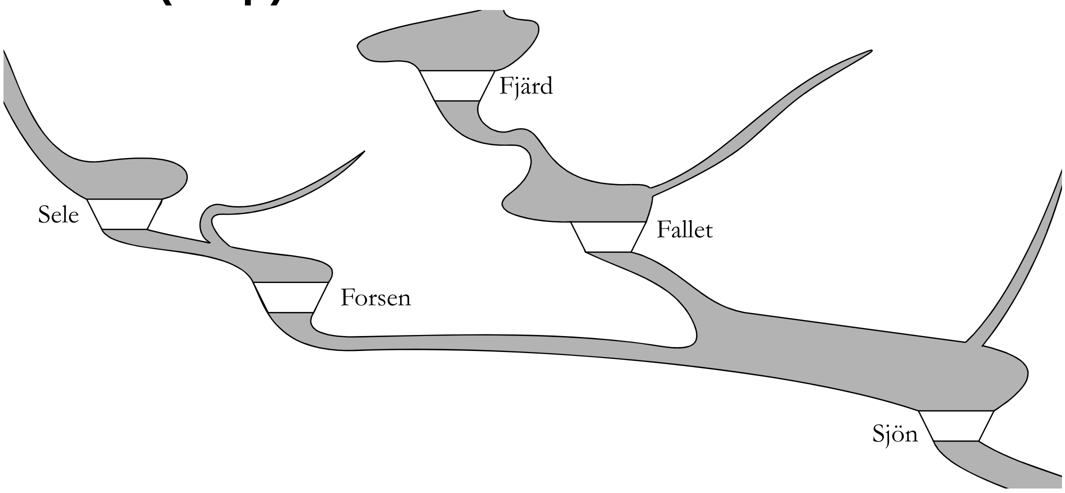
Table 4 Distribution of gain.

Country	Disturbance reserve (available between 49.5 and 49.9 Hz) [MW/Hz]	Dimensioning fault [MW]
Aland	1 000	1 200
Beland	1 500	600
Celand	500	900

Table 5 Data för transmissionsförbindelserna.

Connection	Type	Transmission when the system frequency is 49.9 Hz [MW]	Maximal capacity [MW]
Aland ↔ Beland	Direct current (HVDC)	400 MW from Beland to Aland	600
Aland ↔ Celand	Alternating current	1 000 MW from Aland to Celand	2 000
Beland ↔ Celand	Alternating current	2 000 MW from Beland to Celand	2 500

Problem 8 (20 p)



Obygdens kraft AB owns five smaller hydro power plants, located as shown in the figure above. The company has not invested in a modern control room, but the power plants are still operated manually. This means that in order to change the discharge or spill in a power plant, it is necessary to send out a technician to the power plant in question. The power plant Sjön is always manned, whereas the other four power plants are unmanned most of the time. Data for the power plants are given in table 6.

The generated electricity is sold to the local power exchange ElKräng. Bids for all hours on Friday have to be submitted to ElKräng no later than 12 noon on Thursday. The company assumes that they can sell unlimited amounts to the prices stated in table 7. Normally, no technicians are sent out to the unmanned power plants during evenings and nights; therefore, in order to plan the operation during Friday, the company must also consider the expected prices at ElKräng during Saturday before noon, which are shown in table 8. (If it is decided to generate electrictiy in Friday evening, there will also be generation during Saturday morning, and then it is necessary to take the Saturday prices into account.) .

Table 6 Data for the hydro power plants of Obygdens kraft AB.

Power plant	Start contents of reservoir [HE]	Maximal contents of reservoir [HE]	Marginal production equivalents [MWh/HE]		Maximal discharge [HE]		Local inflow [HE]
			Segment 1	Segment 2	Segment 1	Segment 2	
Sele	700	800	0.165	0.140	40	10	42
Forsen	70	100	0.085	0.070	50	25	2
Fjärd	2 100	3 000	0.110	0.100	80	30	64
Fallet	100	600	0.090	0.080	80	35	5
Sjön	4 000	8 000	0.184	0.176	160	60	20

Table 7 Expected prices at ElKräng during Friday.

Hour	0-1	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10	10-11	11-12
Price at ElKräng [SEK/MWh]	310	310	305	305	315	330	340	355	370	360	355	345
Hour	12-13	13-14	14-15	15-16	16-17	17-18	18-19	19-20	20-21	21-22	22-23	23-24
Price at ElKräng [SEK/MWh]	345	340	335	335	340	345	340	335	330	325	315	305

Table 8 xpected prices at ElKräng during Saturday.

Hour	0-1	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10	10-11	11-12
Price at ElKräng [SEK/MWh]	305	295	285	290	300	315	330	340	340	340	340	340

Table 9 Schedule for the technician.

Sele	7:45–8:15	14:45–15:15
Forsen	6:45–7:15	13:45–14:15
Fjärd	8:45–9:15	15:45–16:15
Fallet	9:45–10:15	16:45–17:15

Table 10 Notation for the planning problem of Obygdens kraft AB.

Symbol	Explanation	Value
$M_{i,0}$	Start contents of reservoir *i*	See table 6
$\overline{M}_i$	Maximal contents of reservoir *i*	See table 6
$\mu_{i,j}$	Marginal production equivalent in power plant *i*, segment *j*	See table 6
$\overline{Q}_{i,j}$	Maximal discharge in power plant *i*, segment *j*	See table 6
V_i	Local inflow to power plant *i*	See table 6
λ_t	Expected price at ElKräng hour *t*	See tables 7 and 8
λ_f	Expected future electricity price	335

a) (14 p) Formulate the planning problem of Obygdens kraft AB as an LP problem under the condition that a technician from Sjön is going to the four umanned power plants twice a day according to the schedule in table 9. The plan for the last hour at Thursday evening is to discharge 40 HE in Sele, 50 HE in Forsen, as well as 80 HE each in Fjärd and Fallet. No discharge is planned in Sjön for this hour and there will be no spillage from any of the reservoirs. Stored water is assumed to be used for electricity generation at the best marginal production equivalent in each power plant and future electricity generation is valued 335 SEK/MWh. The water delay time between the power plants can be neglected.

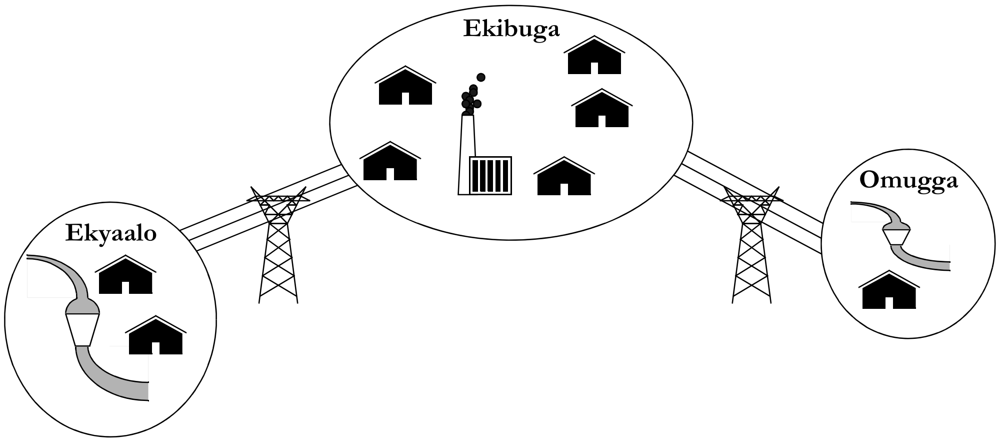
Use the notation in table 10 for the parameters (it is however permitted to add further symbols if you consider it necessary).

NOTICE! The following is required to get full score for this problem:

- The symbols for the optimisation variables must be clearly defined.
- The optimisation problem should be formulated so that it is easy to determine what the objective function is, which constraints there are and which limits there are.
- The possible values for all indices should be clearly stated for each equation.

b) (6 p) Assume that there is choice between sending the technician on the trip Forsen-Sele-Fjärd-Fallet (i.e., the trip described in table 9) or that the technician travels the same trip but the other way around, and that it should be possible to decide the direction for each trip. How must the planning problem from part a be reformulated in order to consider this? Do not forget to define all new variables and parameters that you introduce!

Problem 9 (20 p)



Ekibuga District is not connected to the national grid in Eggwanga, but there is a regional grid that comprises the population centres Ekibuga and Ekyaalo, as well as some smaller villages. The regional grid is supplied by a diesel generator set in Ekibuga, as well as a hydro power plant in Ekyaalo and a hydro power plant in Omugga (see the figure above). The diesel generator set in Ekibuga has a capacity of 500 kW, 90% availability and a variable operation cost of 10 ¤/kWh. The hydro power plants are run-of-the river units and have the installed capacities 400 kW (Ekyaalo) and 150 kW (Omugga) respectively. The risk of outages in the hydro power plants is negligible and the natural flow in the river passing by the power plant is always sufficient to generate the installed capacity.

Ekyaaro and Omugga are connected to Ekibuga via 33 kV lines. The losses on these lines can be computed according to

$$L = \beta_L \cdot P^2,$$

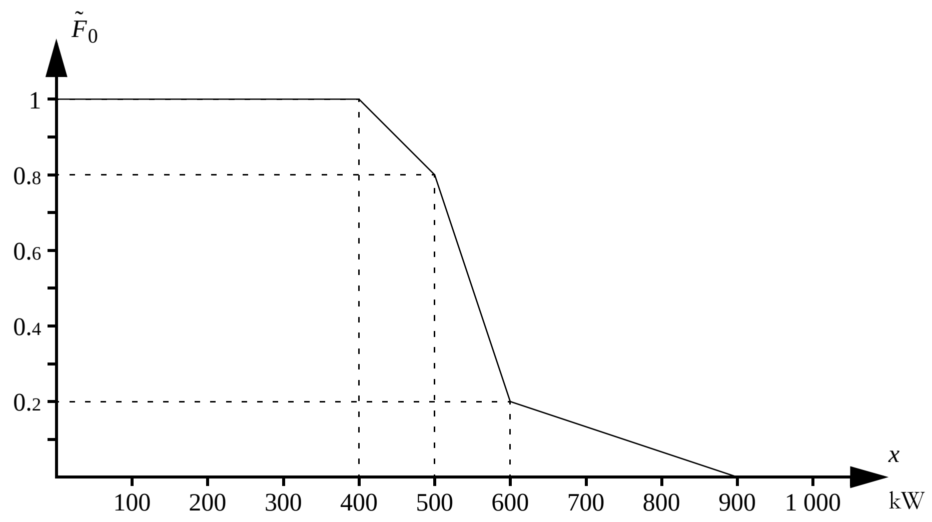
where

L = losses on the line [kW],

$$\beta_L = \text{loss coefficient [kW}^{-1}] = \begin{cases} 0.0001 & \text{for the line between Ekyaalo and Ekibuga,} \\ 0.0004 & \text{for the line between Omugga and Ekibuga,} \end{cases}$$

P = injected power on the line [kW].

The figure on the next page shows the duration curve of the total load in the regional power system. The main part of the total load is in Ekibuga; the local consumption in Ekyaalo is in average 15% of the total load and the local consumption in Omugga is in average a 5% aof the total laod.



a) (6 p) Use probabilistic production cost simulation to compute the expected operation cost and the risk of power deficit in Ekibuga District.

b) (6 p) Suggest a strata tree which can be used for a Monte Carlo simulation of the power system in Ekibuga District. Moreover, calculate the stratum weights and state which values of the result variables you expect in each stratum.

c) (8 p) Some scenarios for a Monte Carlo simulation of Ekibuga District are given in table 11. Use these scenarios to estimate the expected operation cost and the risk of power deficit.

NOTICE! To get full score in this problem, you must use as many variance reduction techniques as possible!

Table 11 Some scenarios as well as selected results for a Monte Carlo simulation of the power system in Ekibuga District.

Scenario,	Load [kW]			Available capacity in the diesel generator set [kW]	Generation in the diesel generator set {kW]	Total transmission losses [kW]
	Ekibuga	Ekyaaro	Omugga			
1	446	56	30	0	0	17.6
2	447	59	36	0	0	16.8
3	436	89	23	500	14.1	16.1
4	485	72	29	500	52.6	16.6
5	413	100	35	500	12.3	14.3
6	411	102	20	500	0	15.6
7	647	119	33	500	262.4	13.4
8	450	98	39	500	51.0	14.0
9	405	96	37	0	0	14.3
10	434	69	33	0	0	16.4

KTH Electrical Engineering

Answer sheet for part I

Name: ....

Personal number: ....

Problem 1

a) Alternative .... is correct.

b) Alternative .... is correct.

Problem 2

a) .... TWh

b) .... TWh

c) .... MWh

Problem 3

a) Time ....

b) Time ....

c) Time ....

Problem 4

a) Parameters: ....

Optimisation variables: ....

b) ....

c) ....

d) .... m^3

Problem 5

a) ....

b) ....

c) ....

d) .... ¤/h

e) .... kW

f) *ETOC* .... ¤/h

LOLP .... %

Suggested solution for exam in EG2050 System Planning, 14 March 2013.

Problem 1

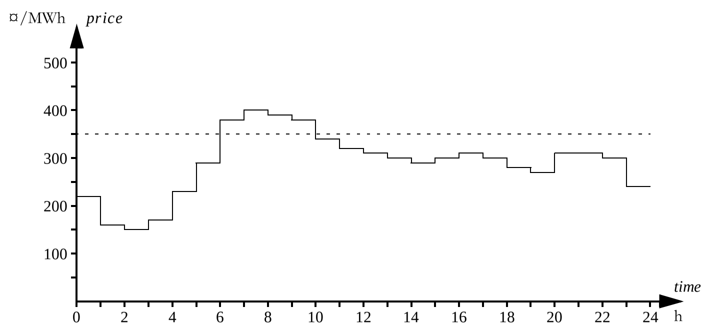
a) 2, **b)** 5.

Problem 2

a) Since the electricity price is higher during the second half of the year, the hydrso power producers want to store as much water as possible. Thus, the hydro reservoirs will hold 40 TWh after a half-year.

b) In total there is 20 (start content) + 50 (inflow) – 40 (final content) = 30 TWh available for hydro power generation during the first half-year. The entire capacity will be utilised, as the variable costs are negligible.

c) The power plant will generate its installed capacity during those hours when the electricity price is higher than 350 ¤/MWh. Drawing a line at the level 350 ¤/MWh shows that the power plant will generate 100 MW during 4 hours, which results in a total generation of 400 MWh.



Problem 3

a) The frequency is decreasing as the system has lost generation. The normal operation reserve cannot manage an outage of 500 MW, because we would then get a frequency decrease by $\Delta f = \Delta G/R = 500/2\,000 = 0.25$ Hz, which is larger than the frequency interval for which the normal operation reserve is available and consequently the disturbance reserve must be part of the response. The only occasion there the frequency rapidly drops below 49.9 Hz is time IV.

b) If the generation would increase by 250 MW then the frequency in the system is less than 49.9 Hz, we would get a frequency increase $\Delta f = \Delta G/R = 250/1\,500 = 0.167$ Hz. The only occasion then the frequency is rapidly increasing from a level below 49.9 Hz is time V, but then the frequency is increasing by almost 0.25 Hz; thus, this cannot be the time we are looking for.

If the generation is increased by 250 MW during normal operation, the frequency will increase by $\Delta f = \Delta G/R = 250/2\,000 = 0.125$ Hz. This corresponds well to time I.

c) Then the export on the HVDC line is reduced, there will be a continuous increase of the frequency for five minutes. This might however be difficult to detect in the frequency curve, as there are simultaneous small frequency deviations caused by variations in load and wind power generation. It is easier to search for a time period where the frequency several times is decreased by $\Delta f = \Delta G/R = 100/2\,000 = 0.05$ Hz as the electricity generation is reduced to compensate the export decrease—these changes are large enough to be clearly visible in the frequency curve. The only time that fits into this pattern is time VIII.

Problem 4

a) Parameters: β_{Gg}, C_g^+, $\overline{G}_g$, $\underline{G}_g$, λ_t, $u_{g,0}$ and W_t. Optimisation variables: $G_{g,t}$, $s_{g,t}^+$, and $u_{g,t}$.

b) The income of the wind power generation cannot be controlled by Stads energi AB and does therefore not need to be included in the objective function, which means that we get

$$\text{maximise} \sum_{t=1}^{24}\sum_{g=1}^{4}\left((\lambda_t - \beta_{Gg})G_{g,t} - C_g^+ s_{g,t}^+\right).$$

c) $u_{g,t} - u_{g,t-1} \le s_{g,t}^+$.

d) As the flow into the reservoir is 150 HE and the flow out from the reservoir is 160 HE, it must hold 1 990 HE = 7 164 000 m^3 water at the end of the hour.

Problem 5

a) $\tilde{F}_1(x) = 0.1\tilde{F}_0(x) + 0.3\tilde{F}_0(x-100) + 0.35\tilde{F}_0(x-200) + 0.25\tilde{F}_0(x-300)$
$\Rightarrow \tilde{F}_1(x) = 0.1 \cdot 0.6 + 0.3 \cdot 1 + 0.35 \cdot 1 + 0.25 \cdot 1 = 0.96$ for the interval $400 \le x < 500$.

b) $\tilde{F}_1(x) = 0.1\tilde{F}_0(x) + 0.3\tilde{F}_0(x-100) + 0.35\tilde{F}_0(x-200) + 0.25\tilde{F}_0(x-300)$
$\Rightarrow \tilde{F}_1(x) = 0.1 \cdot 0.6 + 0.3 \cdot 0.6 + 0.35 \cdot 1 + 0.25 \cdot 1 = 0.84$ for the interval $500 \le x < 600$.

c) $\tilde{F}_1(x) = 0.1\tilde{F}_0(x) + 0.3\tilde{F}_0(x-100) + 0.35\tilde{F}_0(x-200) + 0.25\tilde{F}_0(x-300)$
$\Rightarrow \tilde{F}_1(x) = 0.1 \cdot 0.1 + 0.3 \cdot 0.6 + 0.35 \cdot 0.6 + 0.25 \cdot 1 = 0.65$ for the interval $600 \le x < 700$.

d) The total electricity generation of the four diesel generator sets is calculated by

$$EG_{2345} = EENS_1 - EENS_5 = \int_{300}^{\infty}\tilde{F}_1(\xi)d\xi - \int_{1\,100}^{\infty}\tilde{F}_5(\xi)d\xi = 415 - 9 = 406 \text{ kWh/h};$$

hence, we get $ETOC = 0.5 \cdot 406 = 203$ ¤/h.

e) If the given random number is denoted U then the load is calculated by $D = \tilde{F}_0^{-1}(U)$ = {use the figure} = 660 kW.

f) The following estimates are obtained of the expectation value in each stratum:

$m_{TOC1} = 1\,200/5 = 240$ $\quad m_{LOLO1} = 0$
$m_{TOC2} = 1\,560/5 = 312$ $\quad m_{LOLO2} = 1/5 = 0.2$
$m_{TOC3} = 1\,500/5 = 300$ $\quad m_{LOLO3} = 5/5 = 1$

Thus, we get

$$m_{TOC} = \sum_{h=1}^{3} \omega_h m_{TOCh} = 0.9 \cdot 240 + 0.025 \cdot 312 + 0.075 \cdot 300 = 246.3 \text{ ¤/h},$$

$$m_{LOLO} = \sum_{h=1}^{3} \omega_h m_{LOLOh} = 0 + 0.025 \cdot 0.2 + 0.075 \cdot 1 = 8\%.$$

Problem 6

The claim by Grönfrid that the nuclear power plant in Strålinge is not profitable is based on a simple analysis of income, variable costs and fixed costs. The method is reasonable; hence, the question is whether they have used reasonable inputs in the computations. Let us therefore have a closer look on the studied data:

- If a 1 000 MW power plant would be running at installed capacity for a year, it would generate 8.76 TWh. Grönfrid has assumed an annual generation of 8 TWh (i.e., a utilisation factor slightly above 90%), which is reasonable for a well-managed nuclear power plant.
- The price of nuclear fuel has been quite stable around 30 000 ¤/kg for many years. Counting backwards based on this price and the heat content 880 MWh/kg, we find that Grönfrid has assumed the efficiency

  $$\frac{30\,000 \text{ ¤/kg}}{880 \text{ MWh/kg} \cdot 100 \text{ ¤/MWh}} \approx 34\%,$$

  which seems fully reasonable.
- The investment cost 30 000 million ¤ is from AB Industriel and Kärnkraftbyggarna AB, and should be trustworthy. (Notice that the power plant is built for a fixed price, which means that the owner AB Industriel does not risk cost increases if the construction is delayed.) The depreciation time 15 years seems plausible—a nuclear power plant may indeed have a longer technical life, but it is questionable if any player is prepared to make such an extremely long-term investment; moreover, there may be additional large investments in maintenance during the later part of the technical life of the power plant. It is more difficult to judge if the required rate of return 5% is reasonable (it depends on the economical situation in Land, for which we have no further details). Just in case, we can apply the annuity formula on some alternative values of required rate of return and depreciation time:
  - r = 1%, n = 30 year $\Rightarrow AC \approx$ 1 162 M¤/year
  - r = 1%, n = 15 year $\Rightarrow AC \approx$ 2 164 M¤/year
  - r = 5%, n = 30 year $\Rightarrow AC \approx$ 1 952 M¤/year
  - r = 5%, n = 15 year $\Rightarrow AC \approx$ 2 890 M¤/year
  - r = 10%, n = 30 year $\Rightarrow AC \approx$ 3 182 M¤/year
  - r = 10%, n = 15 year $\Rightarrow AC \approx$ 3 944 M¤/year

  We can see that more favourable assumption concerning required rate of return or depreciation time can reduce the annual cost to about 20 000 million ¤. If we assume a low required rate of interest as well as a long depreciation time, we might reduce the costs as far as to 10 000 million ¤, but that seems overly optimistic.

  It should be noted that although the investment costs are a major part of the fixed costs, the power plant will also have fixed costs for staff, maintenance, grid connection, etc. For simplicity, we choose to neglect these costs in this analysis.
- It is true that the electricity price has been about 300 ¤/MWh during the last years, but as we can see in the statistics, there has also been considerably higher price levels in the electricity market of Land. We do not have any information about the background of the development of electricity prices in Land; hence, it is impossible to state anything about likely future electricity prices. We can only conclude that if we accept the other figures provided by Grönfrid, the electricity price must be higher than 3 000 M¤/8 TWh (fixed costs divided per generated MWh) + 100 (variable costs) = 475 ¤/MWh to make Strålinge become profitable. If we use a fixed cos of 20 000 million ¤ per year then an electricity price above 350 ¤/MWh is required.

There is no reason to question that AB Industriel is expecting the investment in Strålinge to yield a reasonable profit. The difference between their analysis and the one by Grönfrid is that the company may have used a lower required rate of return or a longer depreciation time. Whether the power plant is going to be profitable or not is difficult to predict given the information in the problem text, but is something that will have to be determined by the future development in Land.

Problem 7

We start by noticing that Aland provides a third of the gain in the disturbance reserve, Beland provides half of the gain, and Celand a sixth. An outage ΔG will therefore result in the generation increasing by $\Delta G/3$, $\Delta G/2$ and $\Delta G/6$ in the three countries respectively.

An outage of 1 200 MW in Aland will thus be compensated by 400 MW in Aland, 600 MW in Beland and 200 MW in Celand. As it is only power flows on AC lines that will change, we gbet the flow 2 600 MW between Beland and Celand, which means that the line will be overloaded; hence, the system does not fulfil the requirements. If we would increase the transmission from Beland to Aland to 500 MW (which is possible as the capacity is 600 MW) we would instead get the flow 1 900 MW from Beland to Celand and would then have a sufficient reserve on this line to manage a dimensioning fault in Aland. The change on the HVDC line would also increase the flow from Aland to Celand up to 1 100 MW, which is not a problem. A dimensioning fault in Aland would require the import to Aland to increase by 800 MW, which means that the flow on the line between Aland and Celand would be reduced to 300 MW (neither that is a problem).

Let us now verify that the dimensioning faults in Beland and Celand can be managed provided that the flow at 49.9 Hz is 1 900 MW from Beland to Celand and 1 100 MW from Aland to Celand. An outage of 600 MW in Beland is compensated by 200 MW in Aland, 300 MW in Beland and 100 MW in Celand. The flow from Aland to Celand must then increase to 1 300 MW and the flow from Celand to Beland must decrease to 1 600 MW. None of these changes is causing any problem with overloading. An outage of 900 MW in Celand is compensated by 300 MW in Aland, 450 MW in Beland and 150 MW in Celand. The flow from Aland to Celand must then increase to 1 400 MW and the flow from Celand to Beland must increase by 2 350 MW. None of these changes is causing any problem with overloading. Thus, the suggested change of the flow on the HVDC line assures that the system fulfils the requirements.

Problem 8

a) The problem we want to solve is

maximise	*value of sold electricity + value of stored water*,
subject to	*hydrological balance of the reservoirs*,
	limitations in changes of discharge,
	limitations in changes of spillage,

limitations in reservoirs, discharge and spillage.

Indices for the power plants

Sele 1, Forsen 2, Fjärd 3, Fallet 4, Sjön 5.

Parameters

In addition to the parameters defined in table 10 of the problem text, we introduce the following:

$$Q_{i,j,0} = \text{discharge in power plant } i\text{, segment } j\text{, at the start of the planning period} = \begin{cases} 40 & i = 1, j = 1, \\ 50 & i = 2, j = 1, \\ 80 & i = 3, j = 1, \\ 80 & i = 4, j = 1, \\ 0 & \text{annars,} \end{cases}$$

$S_{i,0}$ = spillage from reservoir i at the start of the planning period = 0, $i = 1, \ldots, 5$.

Optimisation variables

$Q_{i,j,t}$ = discharge in power plant i, segment j, during hour t, $i = 1, \ldots, 5, j = 1, 2, t = 1, \ldots, 36$,

$S_{i,t}$ = spillage from reservoir i during hour t, $i = 1, 2, t = 1, \ldots, 24$,

$M_{i,t}$ = contents of reservoir i at the end of hour t, $i = 1, \ldots, 5, t = 1, \ldots, 36$.

Objective function

$$\text{maximise} \quad \sum_{t=1}^{36}\sum_{i=1}^{5}\sum_{j=1}^{2} \mu_{i,j} Q_{i,j,t} + \lambda_f((\mu_{1,1} + \mu_{2,1} + \mu_{5,1})M_{1,36} + (\mu_{2,1} + \mu_{5,1})M_{2,36}) + (\mu_{3,1} + \mu_{4,1} + \mu_{5,1})M_{3,36} + (\mu_{4,1} + \mu_{5,1})M_{4,36} + \mu_{5,1}M_{5,36}$$

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Hydrological balance for Sele:

$$M_{1,t} = M_{1,t-1} - Q_{1,1,t} - Q_{1,2,t} - S_{1,t} + V_{1,t}, \qquad t = 1, \ldots, 36.$$

Hydrological balance for Forsen:

$$M_{2,t} = M_{2,t-1} - Q_{2,1,t} - Q_{2,2,t} - S_{2,t} + Q_{1,1,t} + Q_{1,2,t} + S_{1,t} + V_{2,t}, \qquad t = 1, \ldots, 36.$$

Hydrological balance for Fjärd:

$$M_{3,t} = M_{3,t-1} - Q_{3,1,t} - Q_{3,2,t} - S_{3,t} + V_{3,t}, \qquad t = 1, \ldots, 36.$$

Hydrological balance for Fallet:

$$M_{4,t} = M_{4,t-1} - Q_{4,1,t} - Q_{4,2,t} - S_{4,t} + Q_{3,1,t} + Q_{3,2,t} + S_{3,t} + V_{4,t}, \qquad t = 1, \ldots, 36.$$

Hydrological balance for Sjön:

$$M_{5,t} = M_{5,t-1} - Q_{5,1,t} - Q_{5,2,t} - S_{5,t} + Q_{2,1,t} + Q_{2,2,t} + S_{2,t} + Q_{4,1,t} + Q_{4,2,t} + S_{4,t} + V_{5,t}, \qquad t = 1, \ldots, 36.$$

Limitations in changes of discharge at Sele:

$$Q_{1,1,t} + Q_{1,2,t} = Q_{1,1,t-1} + Q_{1,2,t-1}, \qquad t = 1, \ldots 8, 10, \ldots 15, 17, \ldots, 32, 34, 35, 36.$$

Limitations in changes of spillage at Sele:

$$S_{1,t} = S_{1,t-1}, \qquad t = 1, \ldots 8, 10, \ldots 15, 17, \ldots, 32, 34, 35, 36.$$

Limitations in changes of discharge at Forsen:

$$Q_{2,1,t} + Q_{2,2,t} = Q_{2,1,t-1} + Q_{2,2,t-1}, \qquad t = 1, \ldots 7, 9, \ldots 14, 16, \ldots, 31, 33, \ldots, 36.$$

Limitations in changes of spillage at Forsen:

$$S_{2,t} = S_{2,t-1}, \qquad t = 1, \ldots 7, 9, \ldots 14, 16, \ldots, 31, 33, \ldots, 36.$$

Limitations in changes of discharge at Fjärd:

$$Q_{3,1,t} + Q_{3,2,t} = Q_{3,1,t-1} + Q_{3,2,t-1}, \qquad t = 1, \ldots 9, 11, \ldots 16, 18, \ldots, 33, 35, 36.$$

Limitations in changes of spillage at Fjärd:

$$S_{3,t} = S_{3,t-1}, \qquad t = 1, \ldots 9, 11, \ldots 16, 18, \ldots, 33, 35, 36.$$

Limitations in changes of discharge at Fallet:

$$Q_{4,1,t} + Q_{4,2,t} = Q_{4,1,t-1} + Q_{4,2,t-1}, \qquad t = 1, \ldots 10, 12, \ldots 17, 19, \ldots, 34, 36.$$

Limitations in changes of spillage at i Fallet:

$$S_{4,t} = S_{4,t-1}, \qquad t = 1, \ldots 10, 12, \ldots 17, 19, \ldots, 34, 36.$$

Variable limits

$$0 \le Q_{i,j,t} \le \bar{Q}_{i,j}, \qquad i = 1, \ldots, 5, j = 1, 2, t = 1, \ldots, 36,$$

$$0 \le S_{i,t} \qquad i = 1, \ldots, 5, t = 1, \ldots, 36,$$

$$0 \le M_{i,t} \le \bar{M}_i, \qquad i = 1, \ldots, 5, t = 1, \ldots, 36.$$

b) The technician will make three trips during the planning period, and for each trip we should be able to choose in which direction the technician is passing the power plants. This should best be done by introducing a new integer variable:

d_r = direction of trip r (1 if the technician is travelling Forsen-Sele-Fjärd-Fallet and 0 if the technician is travelling the other way around), $r = 1, 2, 3$,

where $r = 1$ refers to Friday morning, $r = 2$ Friday afternoon and $k = 3$ Saturday morning.

We must now modify the constraints for limitations in changes of discharge during the hours the technician might have possibility to visit a power plant:

$$-1\,000 d_r \le Q_{1,1,t} + Q_{1,2,t} - Q_{1,1,t-1} - Q_{1,2,t-1} \le 1\,000 d_r, \qquad t = 9, r = 1, \quad t = 16, r = 2, \quad t = 33, r = 3,$$

$$-1\,000(1 - d_r) \le Q_{1,1,t} + Q_{1,2,t} - Q_{1,1,t-1} - Q_{1,2,t-1} \le 1\,000 d_r, \qquad t = 10, r = 1, \quad t = 17, r = 2, \quad t = 34, r = 3,$$

$$-1\,000 d_r \le Q_{2,1,t} + Q_{2,2,t} - Q_{2,1,t-1} - Q_{2,2,t-1} \le 1\,000 d_r, \qquad t = 8, r = 1, \quad t = 15, r = 2, \quad t = 32, r = 3,$$

$$-1\,000(1-d_r) \le Q_{2,1,t} + Q_{2,2,t} - Q_{2,1,t-1} - Q_{2,2,t-1} \le 1\,000 d_r, \quad t = 11, r = 1,\; t = 18, r = 2,\; t = 35, r = 3,$$

$$-1\,000 d_r \le Q_{3,1,t} + Q_{3,2,t} - Q_{3,1,t-1} - Q_{3,2,t-1} \le 1\,000 d_r, \quad t = 10, r = 1,\; t = 17, r = 2,\; t = 34, r = 3,$$

$$-1\,000(1-d_r) \le Q_{3,1,t} + Q_{3,2,t} - Q_{3,1,t-1} - Q_{3,2,t-1} \le 1\,000 d_r, \quad t = 9, r = 1,\; t = 16, r = 2,\; t = 33, r = 3,$$

$$-1\,000 d_r \le Q_{4,1,t} + Q_{4,2,t} - Q_{4,1,t-1} - Q_{4,2,t-1} \le 1\,000 d_r, \quad t = 11, r = 1,\; t = 18, r = 2,\; t = 35, r = 3,$$

$$-1\,000(1-d_r) \le Q_{4,1,t} + Q_{4,2,t} - Q_{4,1,t-1} - Q_{4,2,t-1} \le 1\,000 d_r, \quad t = 8, r = 1,\; t = 15, r = 2,\; t = 32, r = 3.$$

Notice that each row above corresponds to two constraints: one for increase of discharge and one for decrease of discharge. The value 1 000 is an arbitrarily chosen value larger than the maximal discharge in the largest power plant (the limits of the discharge are controlled anyway by the variable limits of the discharge).

Similarly, we need new limitations for the changes of spillage:

$$-1\,000 d_r \le S_{1,t} - S_{1,t-1} \le 1\,000 d_r, \quad t = 9, r = 1,\; t = 16, r = 2,\; t = 33, r = 3,$$

$$-1\,000(1-d_r) \le S_{1,t} - S_{1,t-1} \le 1\,000(1-d_r), \quad t = 10, r = 1,\; t = 17, r = 2,\; t = 34, r = 3,$$

etc.

Finally, we need to state the variable limits of the new optimisation variables:

$$d_r \in \{0, 1\}, \qquad r = 1, 2, 3.$$

Problem 9

a) As the hydro power plants are considered to have 100% availability, we get $\tilde{F}_2(x) = \tilde{F}_1(x) = \tilde{F}_0(x)$. The expected generation in the diesel generator set can then be computed according to

$$EG_3 = 1 \cdot 0.9 \int_{550}^{1\,050} \tilde{F}_1(x)dx = 0.9 \cdot ((0.5 + 0.2) \cdot 50/2 + 0.2 \cdot 300/2) = 38.75 \text{ kWh/h},$$

which gives the expected operation cost

$$ETOC = 10EG_3 = 10 \cdot 38.75 = 387.5 \text{ ¤/h}.$$

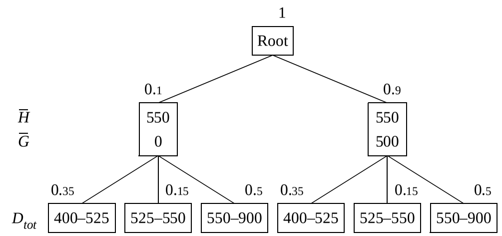
The risk of power deficit is computed by

$$LOLP = \tilde{F}_3(1\,050) = 0.9\tilde{F}_0(1\,050) + 0.1\tilde{F}_0(1\,050 - 500) = 0.9 \cdot 0 + 0.1 \cdot 0.5 = 5\%.$$

b) We start by estimating the maximal transmission losses in the system. The available generation capacity is considerably larger than the local load in both Ekyaaro and Omugga. The largest losses are thus obtained then we transfer as much as possible from each hydro power plant to Ekibuga. To simplify the calculations, we disregard the local load in Ekyaaro and Omugga (this means that we slightly overestimate the maximal losses, but that does not matter):

$$\bar{L} = 0.0001 \cdot 400^2 + 0.0004 \cdot 150^2 = 25 \text{ kW}.$$

The properties of the scenarios can be predicted by comparing the available generation capacity and the total load. Therefore, we create a strata tree with available generation capacity on the level below the root and the total load on the bottom level, as shown in the figure below:



h	1	2	3	4	5	6
ω_h	0.035	0.015	0.05	0.315	0.135	0.450
TOC	0	0	0	0	≥ 0	> 0
TOC_{PPC}	0	0	0	0	0	> 0
$LOLO$	0	0 or 1	1	0	0	0
$LOLO_{PPC}$	0	0	1	0	0	0

The node weights of the available generation capacity is determined by the unavailability and the availability of the diesel generator set. The node weights of the load intervals are obtained from the load duration curve. The stratum weights are the products of the node weights along each branch of the tree.

c) We cannot apply complementary random numbers, as the scenarios are already given to us. However, we can apply control variates as well as stratified sampling.

We start by identifying to which stratum each of the given scenarios belong. We also compute *TOC* and *LOLO* from the given partial results according to

$$TOC = 10G,$$

where G is the generation in the diesel generator set,

$$LOLO = \begin{cases} 0 \text{ if } D_{tot} + L_{tot} \le \bar{H} + \bar{G}, \\ 1 \text{ if } D_{tot} + L_{tot} > \bar{H} + \bar{G}, \end{cases}$$

where D_{tot} is the total load and L_{tot} are the total transmission losses.

Scenario	Stratum	Total load [kW]	TOC [¤/h]	TOC_{PPC} [¤/h]	$LOLO$	$LOLO_{PPC}$
1	2	532	0	0	0	0
2	2	542	0	0	1	0
3	5	548	141	0	0	0
4	6	586	526	360	0	0
5	5	548	123	0	0	0
6	5	533	0	0	0	0
7	6	799	2 624	2 490	0	0
8	6	587	510	370	0	0
9	2	538	0	0	1	0
10	2	536	0	0	1	0

Thus, we get the following estimates of the difference between the multi-area model and the PPC model:

Stratum, h	Estimate of m_{TOCDh}	Estimate of m_{LOLDh}
1	0 (analytical result)	0 (analytical result)
2	0 (analytical result)	$(0 + 1 + 1 + 1)/4 = 0.75$
3	0 (analytical result)	0 (analytical result)
4	0 (analytical result)	0 (analytical result)
5	$(141 + 123 + 0)/3 = 88$	0 (analytical result)
6	$(166 + 134 + 140)/3 \approx 146{,}67$	0 (analytical result)
Estimated difference between the multi-area model and the PPC mode, $m = \sum_{h=1}^{6} \omega_h m_{(X-Z)h}$	$0.135 \cdot 88 + 0.45 \cdot 146.67 \approx 77.9$	$0.015 \cdot 0{,}75 \approx 0{,}011$

The expectation values of the control variates were calculated in problem 9a, which yields the following final estimates:

$$ETOC = m_{TOCD} + \mu_{TOCPPC} = 77.9 + 387.5 = 465.4 \text{ ¤/h},$$

$$LOLP = m_{TOCD} + \mu_{TOCPPC} = 0.011 + 0.05 = 0.061 = 6.1\%.$$