Navigable Small-World Networks

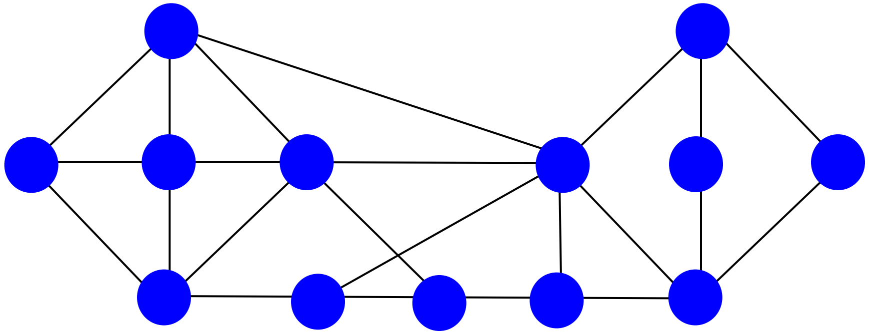
Based on slides by Šarūnas Girdzijauskas

Outline

- Small-Worlds vs. Random Graphs
- Navigation in Small-Worlds (Kleinberg's model)
- Small-World based Structured Overlays
- Non-uniform Structured Overlays

How is our society connected?

- Structure of our social networks
  - Most people's friends are located in their vicinity, be it colleagues, neighbours, or team mates in the local soccer club.
- Social networks were expected to be “grid-like“
  - Within a specific social dimension (e.g., profession, hobby, geographical distribution)



Implies the diameter of the social network is roughly O(√N).

Milgram's (Small-World) experiment

- Finding short chains of acquaintances linking pairs of people in USA who didn't know each other;
  - Source person in Nebraska and Kansas;
  - Target person in Massachusetts.
  - The letter could be only be given to persons one knows on a first name basis (*acquaintances*).

Milgram’s (Small-World) experiment

- Average length of the chains that were completed lied between 5 and 6 steps;
- Coined as “Six degrees of separation” principle.
- This was far less than assumed under the 'grid-like' assumption !
  - Similar results have been found in many other social networks

- **BIG QUESTIONS:**
  - Why are there short chains of acquaintances linking together arbitrary pairs of strangers?
  - That is, why is the diameter of the graph low?

Random Graphs

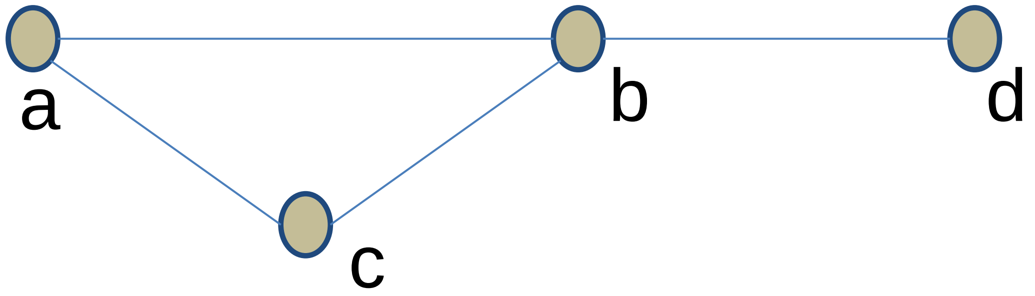
- **Previously believed to be a Random graph**
  - A **random graph** is a graph that is generated by some random process.
  - When pairs or vertices are joined uniformly at random -> then any two vertices are connected by a short chain with high probability.

- **However..**
  - If A and B have a common friend C it is likely that they themselves will be friends! (clustering)
  - **Random networks tend not to be clustered**

Graphs

- A **graph** G formally consists of a set of vertices *V* and a set of edges *E* between them. That is, G=(V,E).
- An **edge** e_{ab} connects vertex *a* with vertex *b*.
  - Edges can be directed or undirected.
- The neighbourhood *N* of a vertex *a* is defined as the set of its immediately connected vertices.
- The degree of a vertex is defined as the number of vertexes in its neighbourhood.
- Distance between two vertices is the number of edges in the shortest path connecting the vertices.
- The diameter of a graph is maximum distance for any vertex.

Example Unidirected Graph



$$V = (a, b, c, d)$$

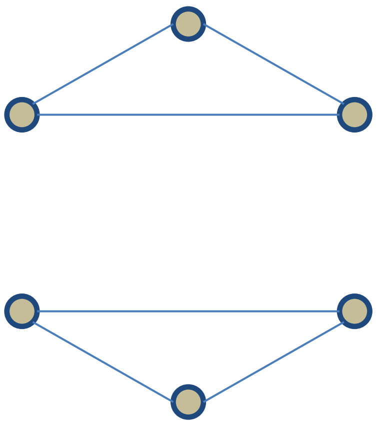
$$E = (e_{ab}, e_{ac}, e_{cb}, e_{bd})$$

$$N(a) = (b, c)$$

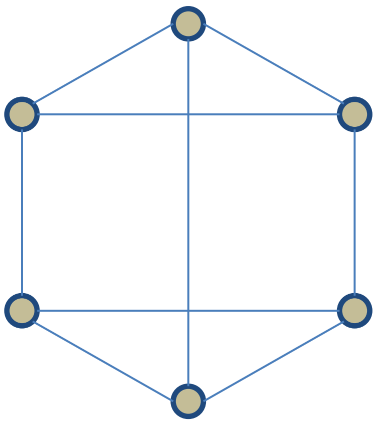
$$\deg(a) = 2$$

Regular Graph

- A regular graph is a Graph where each vertex has the same number of neighbors
  - In other words, nodes have the same degree
- Regular graphs can also be random
  - Random regular graph



2-regular graph



3-regular graph

Informally Clustering in Graphs

- High clustering => a given vertex's neighbours have lots of connections to each other

- Low clustering => a given vertex's neighbours have few connections to each other

Clustering coefficient

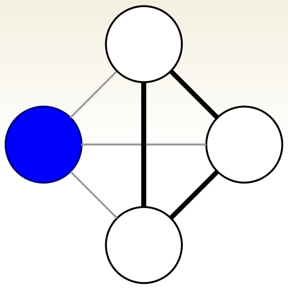
- The **local *clustering coefficient*** $C(v)$ of vertex v is a measure of how close v 's neighbours are to being a clique (a fully connected graph):

$$C(v) = \frac{e(v)}{deg(v)\,(deg(v)-1)\,/2}$$

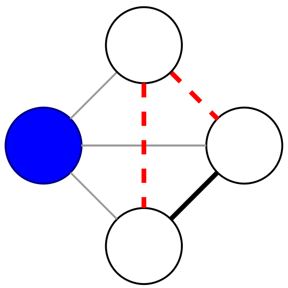
where $e(v)$ denotes the number of edges between the vertices in the v's neighbourhood.

- **Network average *clustering coefficient*** $\tilde{C}$ is given by the fraction of:

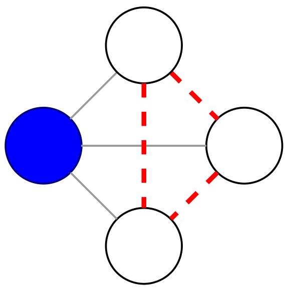
$$\tilde{C} = \frac{1}{N}\sum_{i=1}^{N} C(i)$$



c = 1



c = 1/3

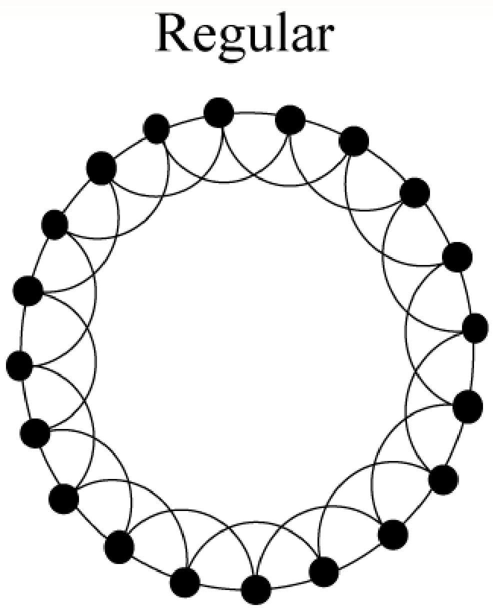


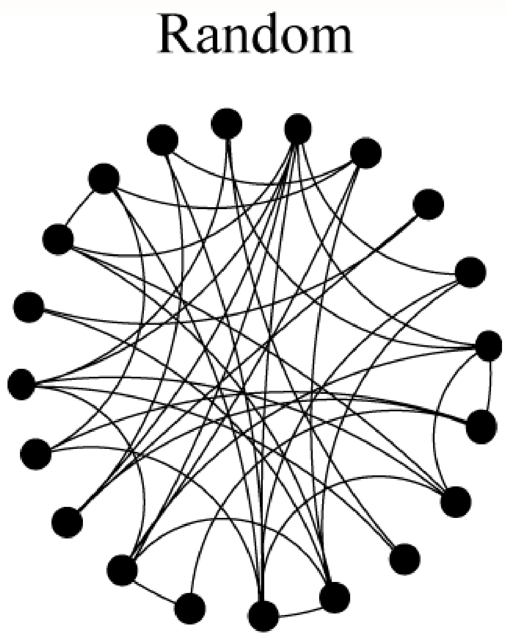
c = 0

Clustering

- Clustering measures the fraction of neighbours of a node that are connected among themselves
- Regular Graphs have a high clustering coefficient
  - but also a high diameter
- Random Graphs have a low clustering coefficient
  - but a low diameter
- Both models do match some properties expected from real networks - such as Milgram's!

Random vs. regular graphs



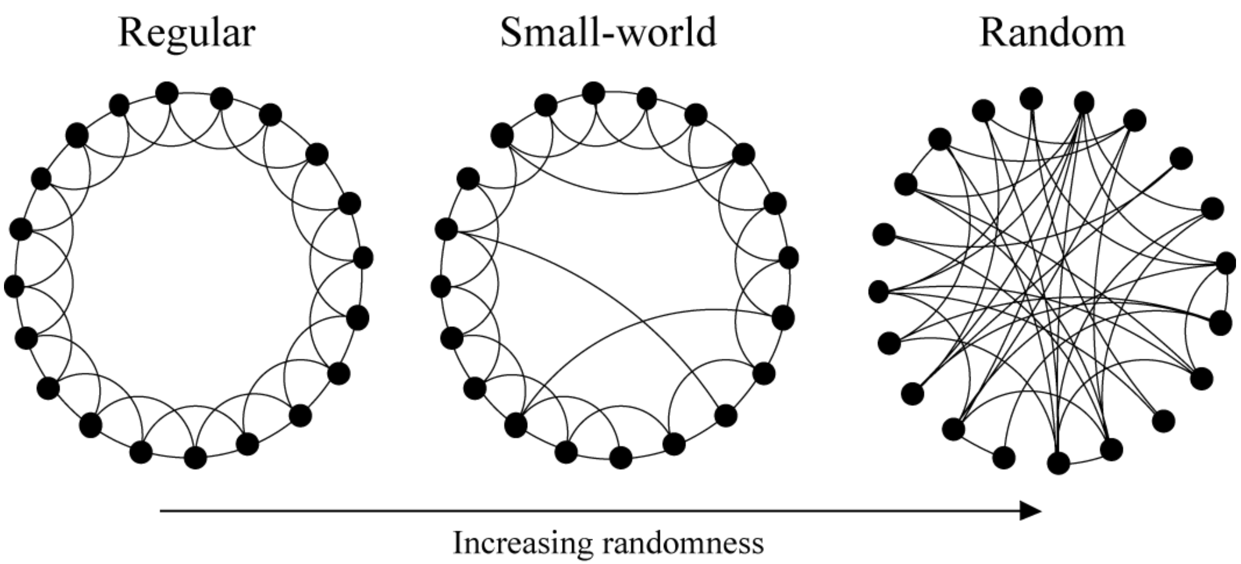


- Long paths
  - $L \sim N/(2k)$
- Highly clustered
  - $C \sim 3/4$

- Short path length
  - $L \sim \log_k N$
- Almost no clustering
  - $C \sim k/N$

Small-World networks

- Random rewiring procedure of regular graph (by Watts and Strogatz)
- With probability *p* rewire each link in a regular graph:
  - Exhibit properties of both: random and regular graphs:
    - High clustering coefficient;
    - Low diameter.

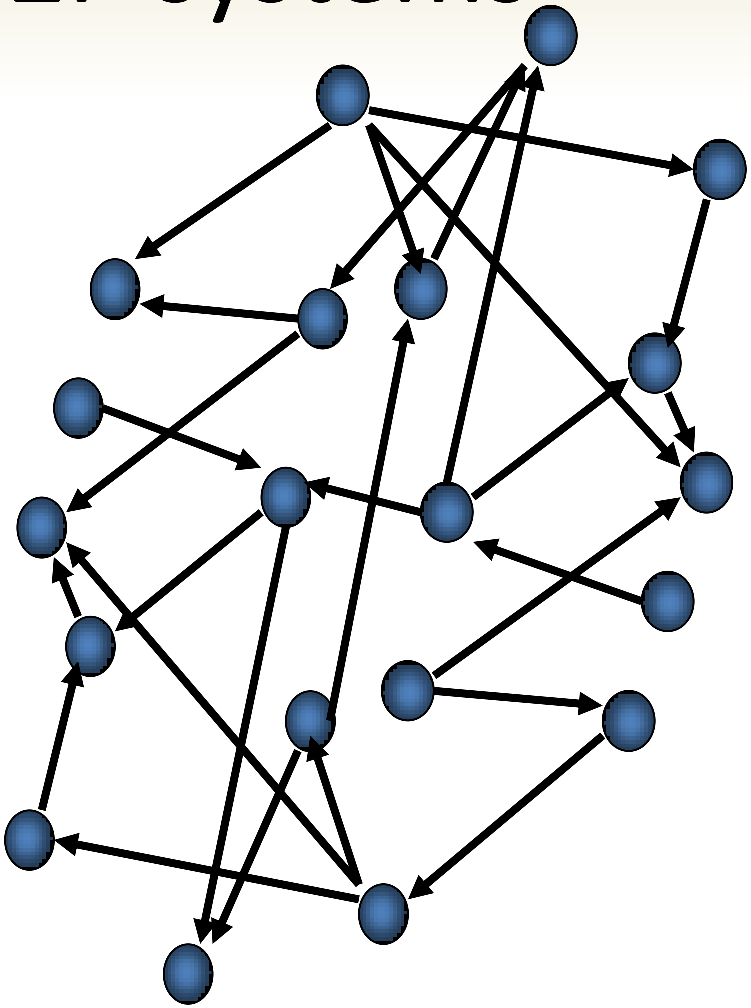


Small-World: remaining questions

- This is still not enough to explain Milgram's experiment:
  - If there exists a shortest path between any two nodes - where is the global knowledge that we can use to find this shortest path?
  - Why should arbitrary pairs of strangers be able to find short chains of acquaintances that link them together?
  - Why do decentralized "search algorithms" work?

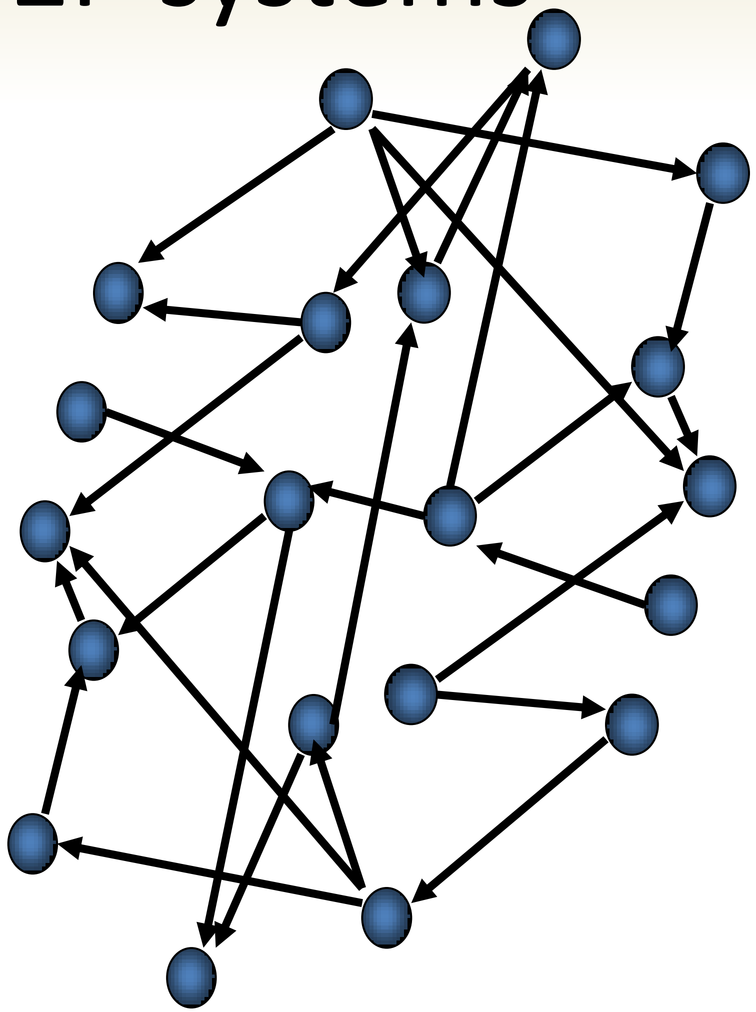
Implications for P2P systems

- Each P2P system can be interpreted as a directed graph where peers correspond to the nodes and their routing table entries as directed edges (links) to the other nodes



Implications for P2P systems

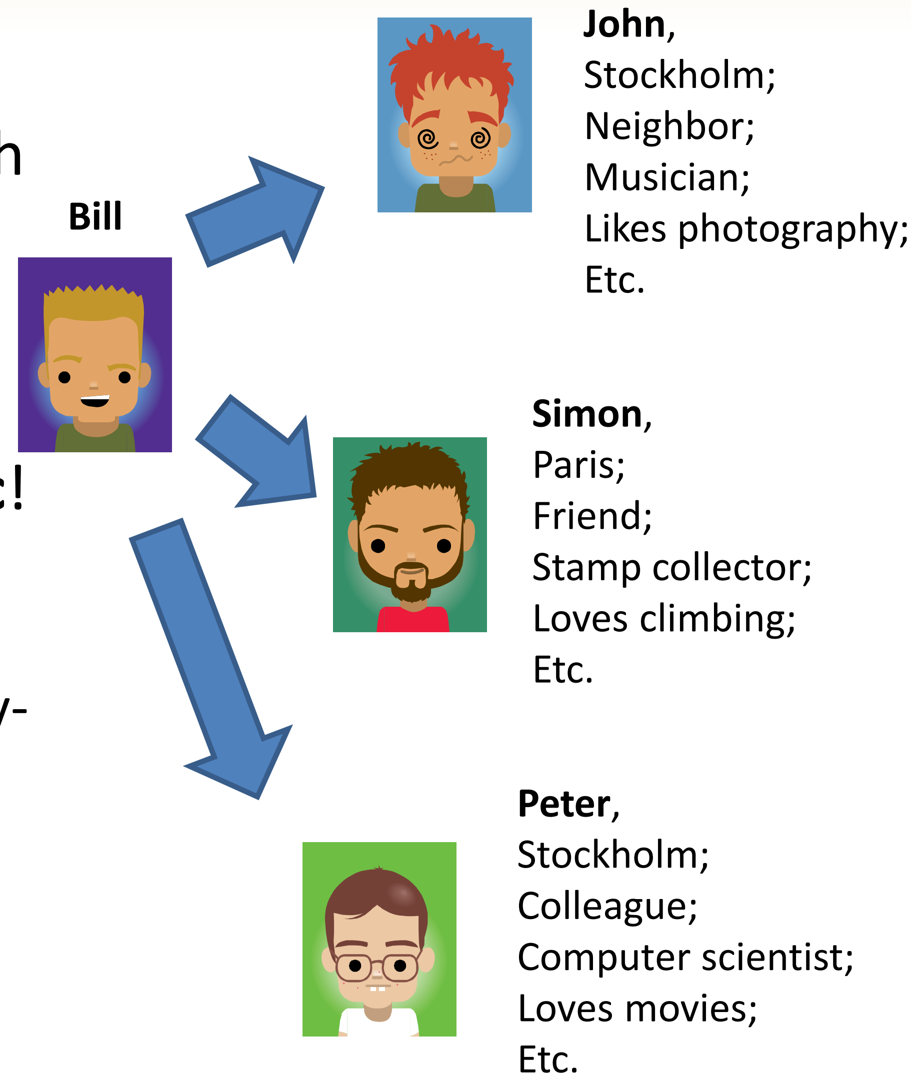
- Task for P2P:
  - Invent a decentralized search algorithm that would route message from any node A to any other node B with relatively few hops compared with the size of the graph



- Is it possible?
  - Milgram's experiment suggests YES!

Why did Milgram's experiment work?

- A social network is not a simple graph, but a graph with certain "labels"
  - "labels" representing various dimensions of our life
- We internalize a "**labeling space**" with a **distance metric**!
- We can greedily minimize the distance!
  - Decentralized search: a greedy-routing algorithm
  - We need to build the right graph where a decentralized search algorithm might perform the best



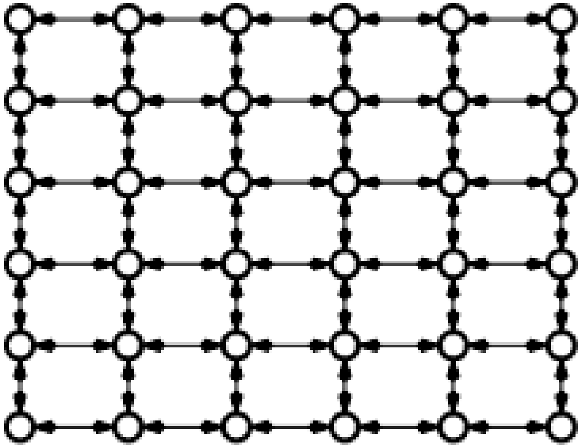
Kleinberg’s model of Small-Worlds

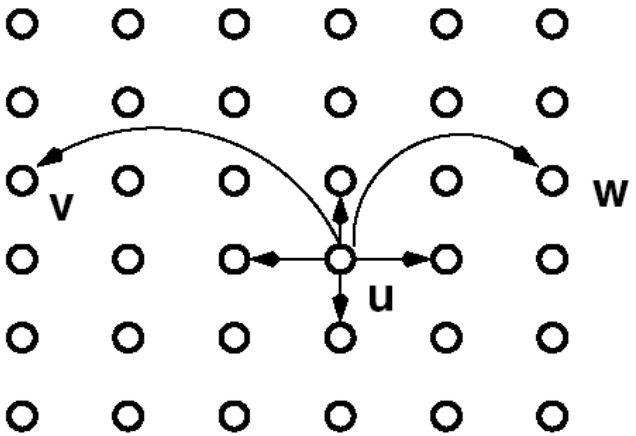
- **Research of Jon Kleinberg:**
  - Claim that there is no decentralized algorithm capable performing effective search in the class of SW networks according to Watts and Strogatz model;
  - J. Kleinberg presented the infinite family of Small World networks that generalizes Watts and Strogatz model and shows that decentralized search algorithms can find short paths with high probability;
  - It was proven that there exists only one unique model within that family for which decentralized search algorithms are effective.

Navigable Small-World networks

- Kleinberg’s Small-World’s model
  - 2-dimensional lattice
  - Lattice (Manhattan) **distance**
  - Two type of edges:
    - Lattice edges (short range)
    - Long range
      - Probability for a node *u* to have a node *v* as a long range contact is proportional to

$$P(u \to v) \sim \frac{1}{d(u,v)^r}$$





Influence of "r"

- Each peer *u* has an edge to the peer *v* with probability $\frac{1}{d(u,v)^r}$ where d(u,v) is the manhattan distance between *u* and *v.*

- **Tuning "r"**
  - When **r<dim** (dimension of the euclidean space) we tend to choose more far away neighbours (search algorithm quickly approaches the target area, but slows down till it finally reaches the target).
  - When **r>dim** we tend to choose closer neighbours (search algorithm reaches the target area slowly if it is far away, but finds the target quickly in it's neighbourhood).
  - When **r=0** – long range contacts are chosen uniformly. Random graph theory proves there exists short paths between every pair of vertices, **but there is no search algorithm capable of finding these paths.**
  - When **r=dim**, the algorithm exhibits optimal performance.

Performance with r=dim

- When q = 1 (there is one long range link)
  - The expected search cost is bounded **by O($\log^2$ N)**
- When q = k (there are a constant number of long range links)
  - The expected search cost is bounded by

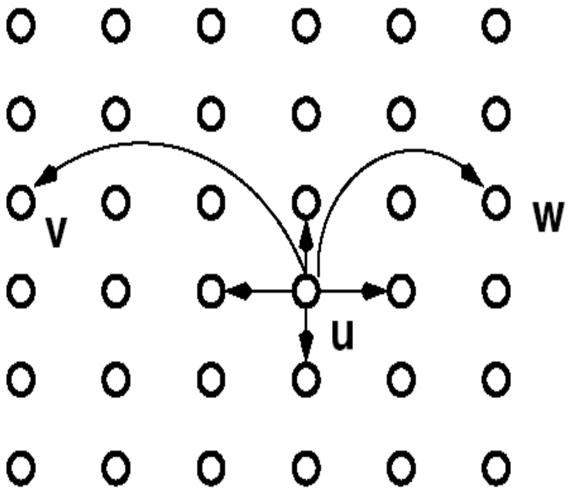
    O($\log^2$ N)/k
- When q = log N
  - The expected search cost is bounded **by O(log N)**

How does it work in practice?

$$P(u \to v) \sim \frac{1}{d(u,v)^r}$$

- Normalization constant has to be calculated:

$$\sum_{\forall i \neq u} \frac{1}{d(u,i)^r}, i \in N$$



$$P(u \to v) = \frac{1}{d(u,v)^r} \cdot \frac{1}{\sum_{\forall i \neq u} \frac{1}{d(u,i)^r}}, i \in N$$

Example

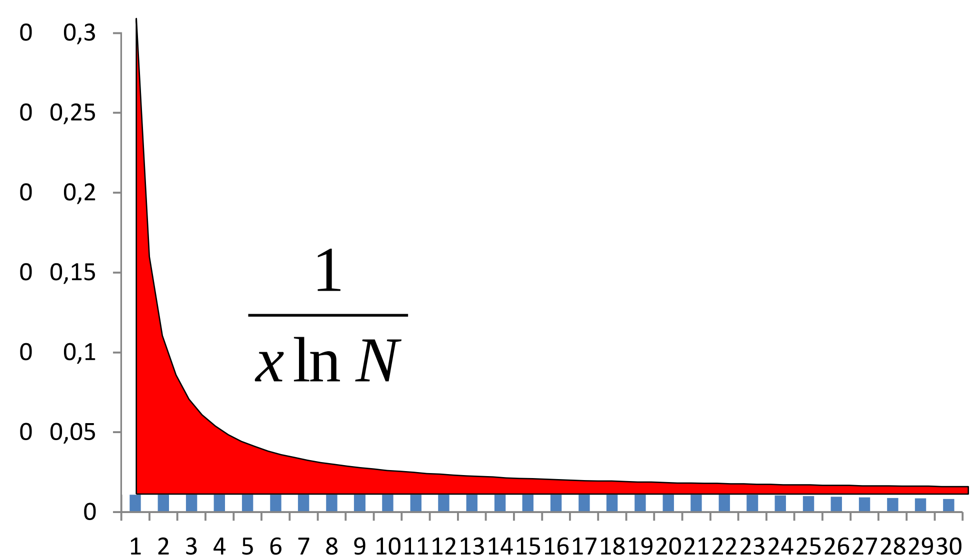
- Choose among 3 friends (1-dimension)
  - A (1 mile away)
  - B (2 miles away)
  - C (3 miles away)
- Normalization constant

$$\sum_{\forall i \neq u} \frac{1}{d(u,i)} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} = \frac{11}{6}$$

$$P(chooseA) = \frac{\frac{1}{1}}{\frac{11}{6}} = \frac{6}{11}$$

$$P(chooseB) = \frac{\frac{1}{2}}{\frac{11}{6}} = \frac{3}{11}$$

$$P(chooseC) = \frac{\frac{1}{3}}{\frac{11}{6}} = \frac{2}{11}$$



1-dimensional continuous case

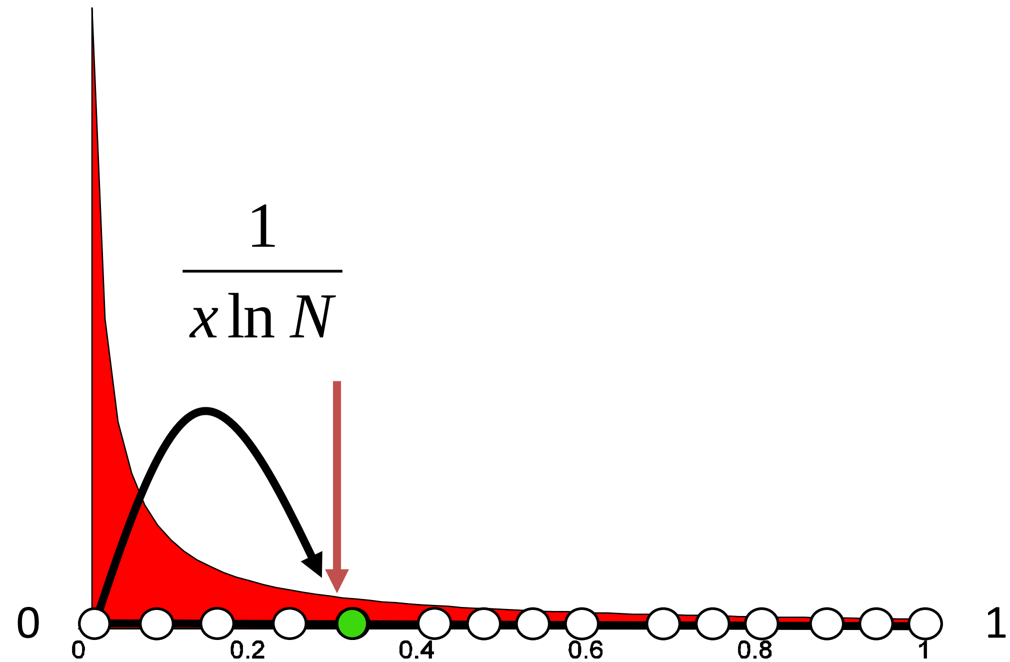
- Peers uniformly distributed on a unit interval (or a ring structure)
- Long range links chosen with the probability P~1/d

- Search cost

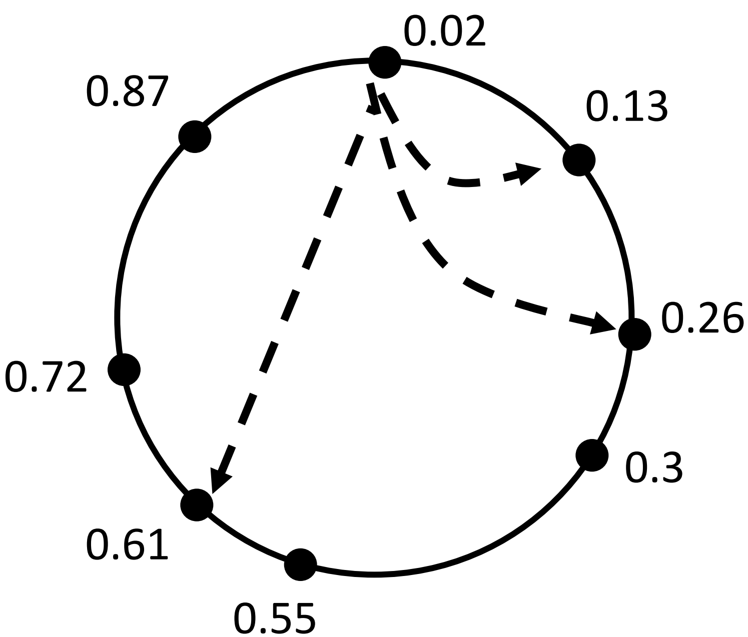
O($\log^2 N/k$) with k long-range links

O(logN) with O(logN) long-range links

- Systems:
  - Symphony (Manku et al, USITS 2003)
  - Accordion (Li et al, NSDI 2005)



Small-World based P2P Overlay



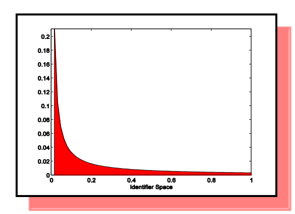
Systems:

Symphony (Manku et al, USITS 2003)

Accordion (Li et al, NSDI 2005)

- **Peers** mapped onto positions on the ring
  - Uniform hash function (e.g., SHA-1) for peerId
  - Establish successor and predecessor ring links
- Small-World **connectivity establishment**

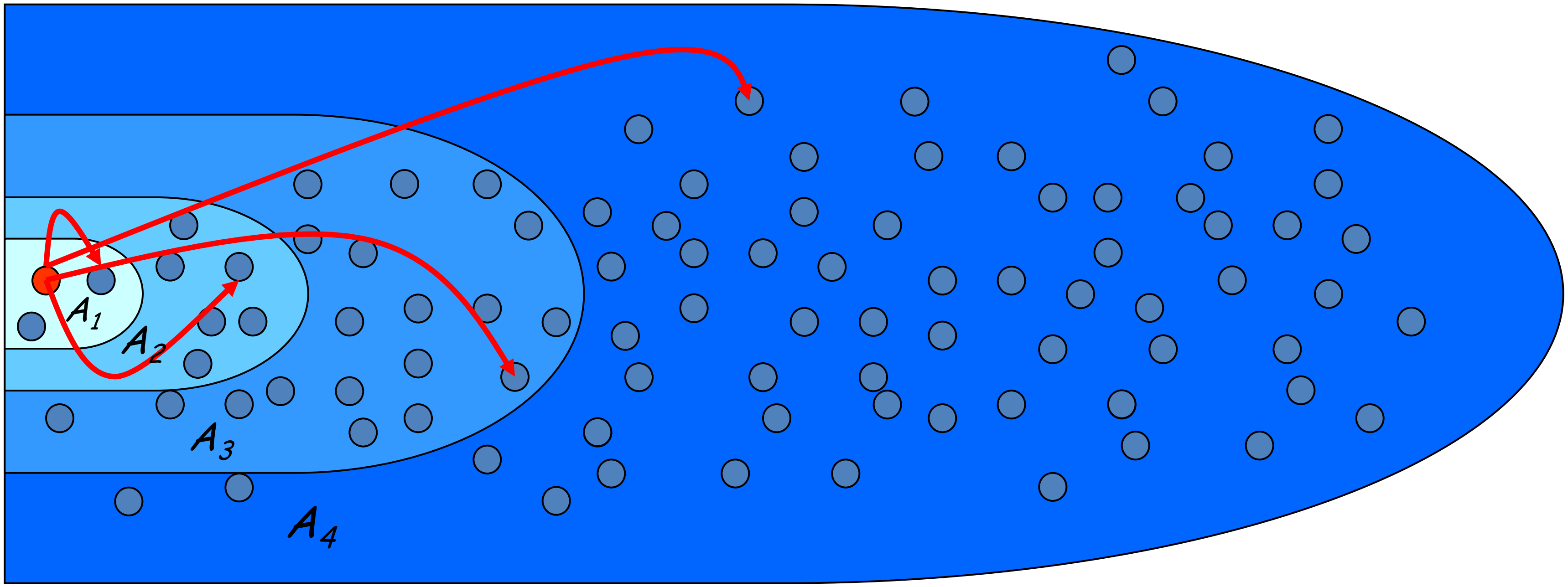
$$\frac{1}{x \ln N}$$



- No restrictions on peer-degree
- Implicit load balancing

Approximation of Kleinberg's model

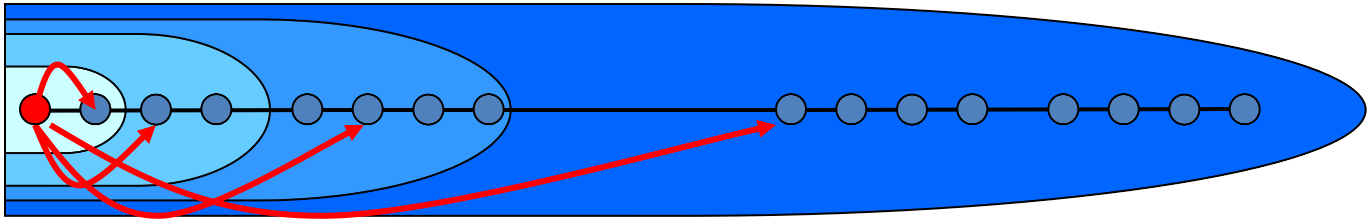
- Given node *u* if we can partition the remaining peers into sets $A_1, A_2, A_3, \ldots, A_{logN}$, where A_i, consists of all nodes whose distance from *u* is between 2^{-i} and 2^{-i+1}.
  - *Then given r=dim each long range contact of u is nearly equally likely to belong to any of the sets* A_i
  - *When q=logN – on average each node will have a link in each set of* A_i



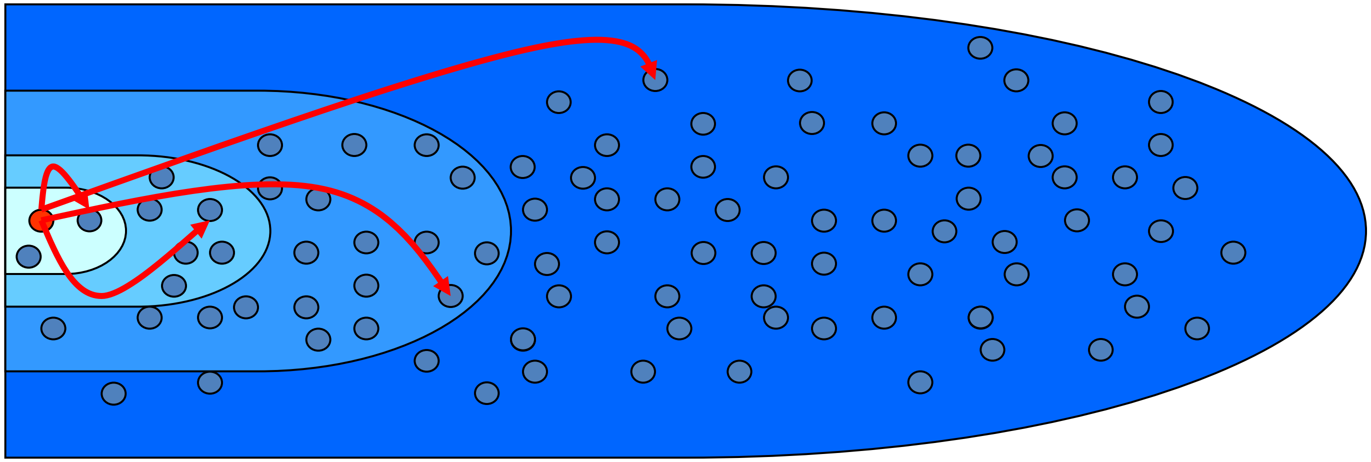
Traditional DHTs and Kleinberg model

- Most of the structured P2P systems are similar to Kleinberg's model and are called logarithmic-like approaches. E.g.
  - Chord (randomized version) q=logN, r=1
  - Gnutella q=5, r=0

- Randomized Chord's model

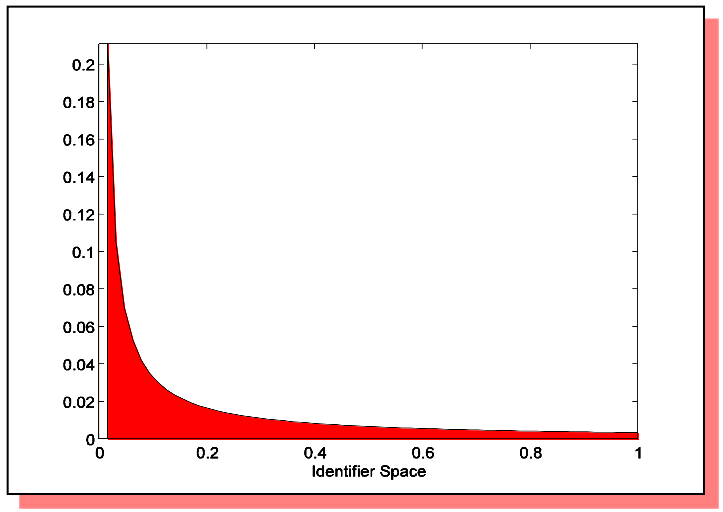


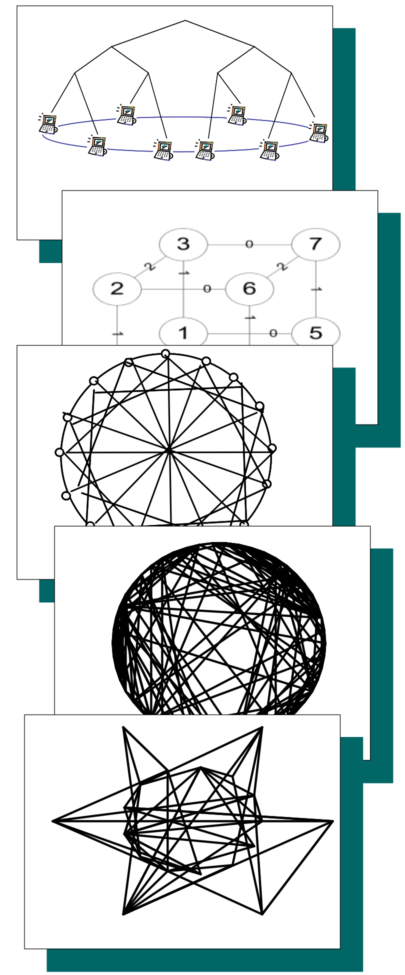
- Kleinberg's model



Similarity to other P2P construction techniques

- Many existing P2P are just the "special cases" of Kleinberg's navigable Small-World
  - Ring based
  - Tree based
  - Hybercube
  - Torus
  - Etc.,





- Kleinberg's Small-World
  - Randomized construction
  - No restrictions on peer-degree
  - "choice-of-two" possibility
  - Effective nonGreedy routing

What to take away from the small-world tour?

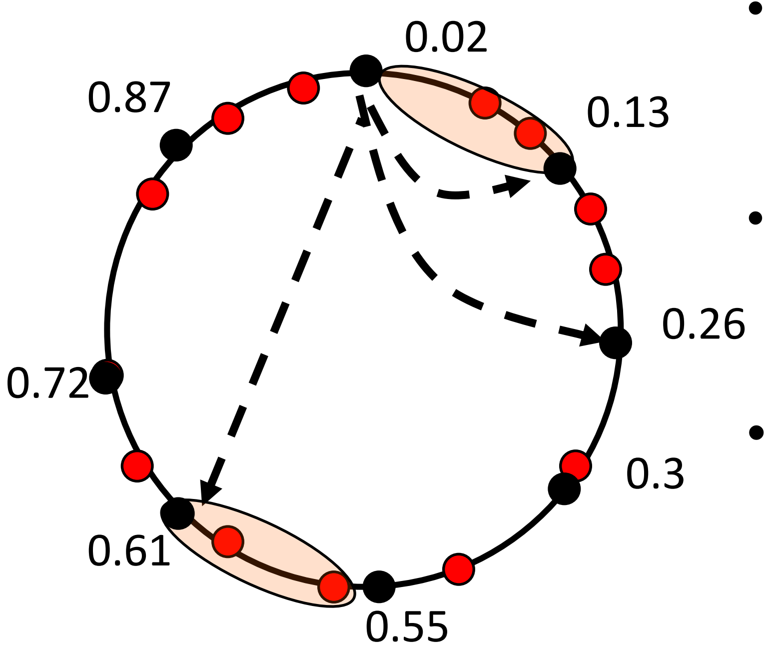
- How can we characterize P2P overlay networks such that we can study them using graph-theoretic approaches?
- What is the main difference between a random graph and a SW graph?
- What is the main difference between Watts/Strogatz and Kleinberg models?
- What is the relationship between structured overlay networks and small world graphs?
- What are possible variations of the small world graph model?
- How does it relate to our social networks?

Non-Uniform Structured Overlays

Structured overlay

- Build a routing table
  - Each peer has a well-defined neighbourhood and information about its immediate neighbours (in contrast to unstructured topologies)
- The search operation is performed efficiently (in contrast to unstructured)

Data on a Ring-Structured Overlay



- **Peers** mapped onto an **ID** the **ring identifier space**
  - Uniform hash function (e.g., SHA-1)
- **Resources** mapped on to an **ID** the **ring identifier space**
  - Uniform hash function
- **Peers** are **responsible** for a **range** of the **ring identifier space**
- **Connectivity establishment**
- E.g., Symphony [Manku et al. 2003]
- **Uniform** peer key (id) distribution
  - Implicit load balancing

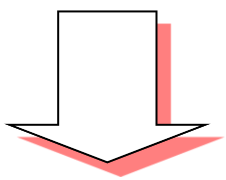
Problems with range queries

- Point queries
  - E.g. “ABBA Waterloo”
  - “ABBA Mamma Mia”
- Range queries:
  - What about all the files with prefix “ABBA..”?
  - Uniform hashing assigns all the files to random locations on the ring
  - Uniform distribution of IDs (keys)
  - **Inefficient lookup!**
- Order preserving (Lexicographic) hashing
  - If a > b then id(a)>id(b) (in uniform hashing id(a)=rand; id(b)=rand)
  - Non-uniform distribution (depends on the data)!

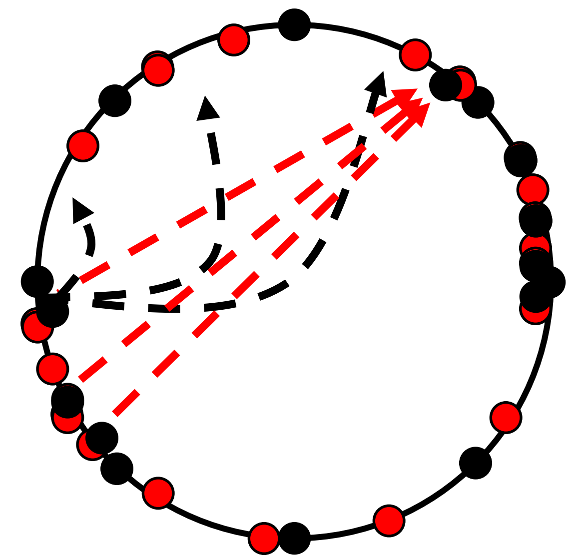
Problems with uniform key distributions

- Order preserving hash functions (e.g. Lexicographical ordering)
- Peer clustering in key space

Etc.

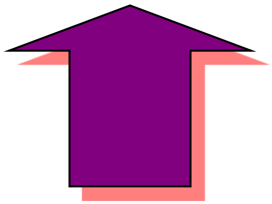


- For skewed key distributions
  - How do we make SW long range links?

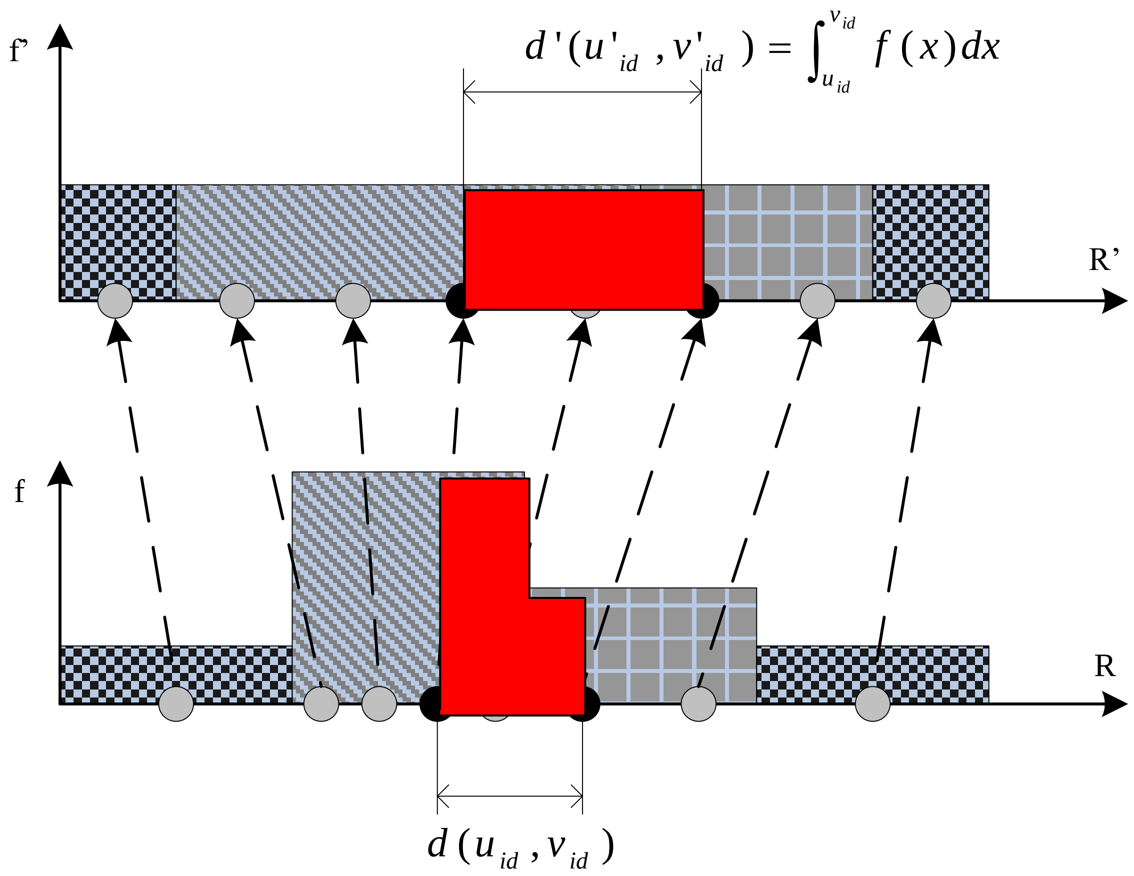


Extending Kleinberg's model

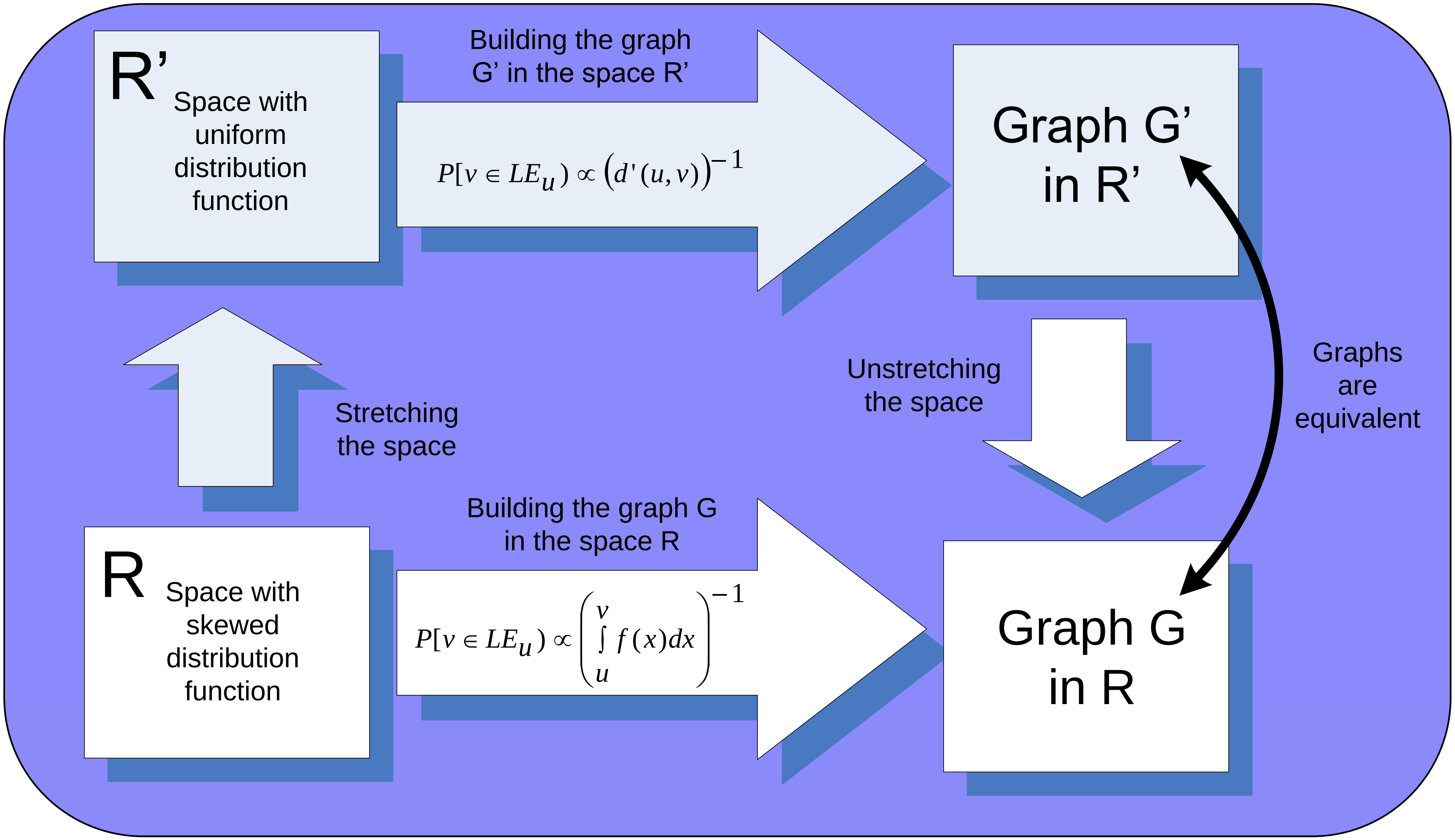
Stretched space



Original space



Extension of the "uniform model" to nonuniform case (1)



Small-World P2P in non-uniform spaces

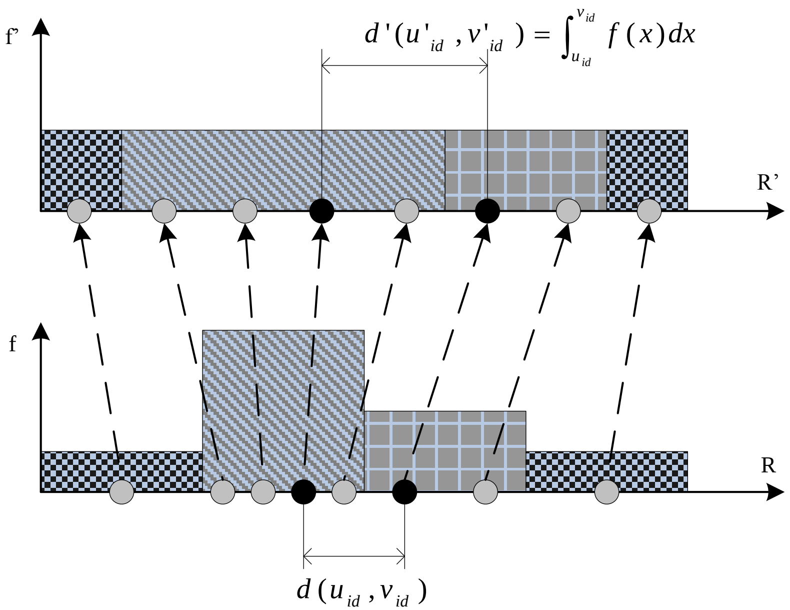
- *f(x)* – probability density function of the peer keys.
- Long range neighbours are chosen inversely proportional to the integral of *f(x)* between the two nodes

$$P[\mathrm{v} \leftrightarrow \mathrm{u}] \sim \frac{1}{\int_{u_{id}}^{v_{id}} f(x)dx}$$

Expected routing cost in such a network using greedy routing is O(log N) when the network degree is O(log N).

Acquiring Peer Key Distribution

- Problem:
  - Need to acquire the **global** key distribution function **locally.**

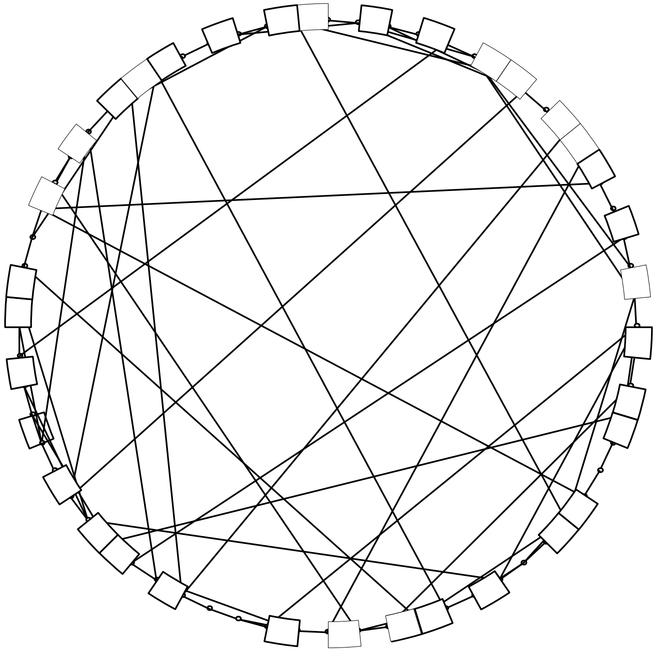


- Uniform sampling
  (Mercury [Bharambe et al.,2004])
  - $\log^2 N$ samples
  - Cannot cope with
  complex distributions! ☹

- Non-uniform (Scalable Sampling)
  - **OSCAR** (**O**verlays using **SCA**lable sampling of **R**ealistic distributions)
  - [DBISP2P06, ICDE07, TAAS10]

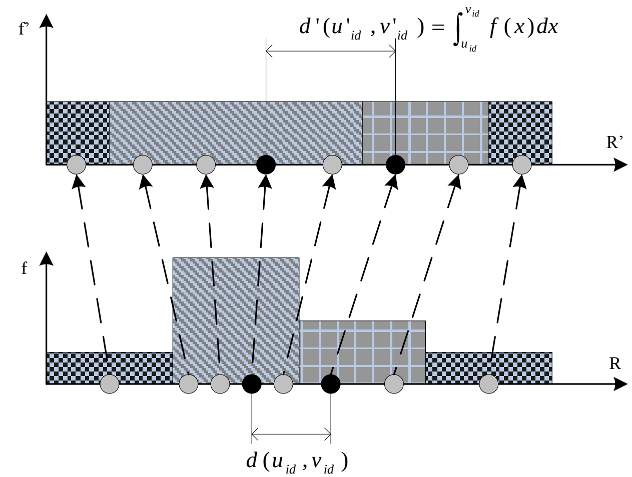
Sampling by random walks

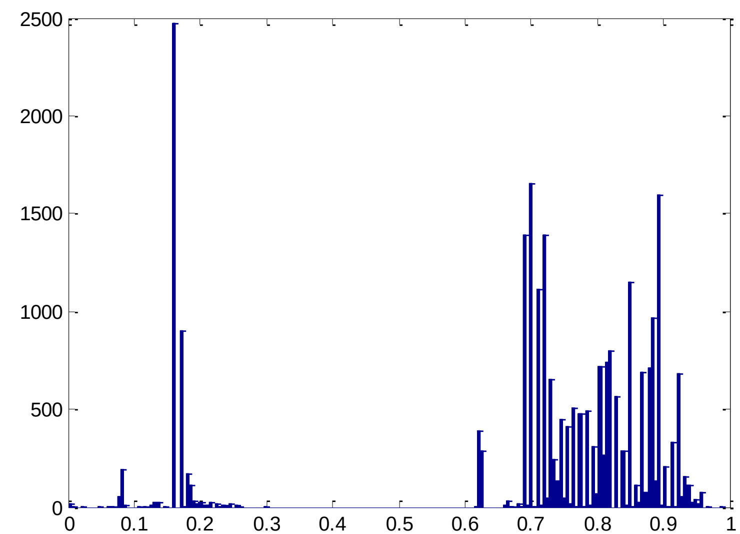
- Bharambe et al. 2004 ”**Mercury: supporting scalable multi-attribute range queries**”
- Sampling by random walks
  - A random walk with TTL at least logN ends up in a uniform random node (on expander graphs)



Sampling in Mercury

- Every node periodically issues k1 samples
  - The sampled nodes return its ID and their own collected samples (k2)
  - It is suggested k1=logN
- Over time the ID distribution histogram is built.
- Real world distributions are far more complex!





Oscar: Modifying Kleinberg's method

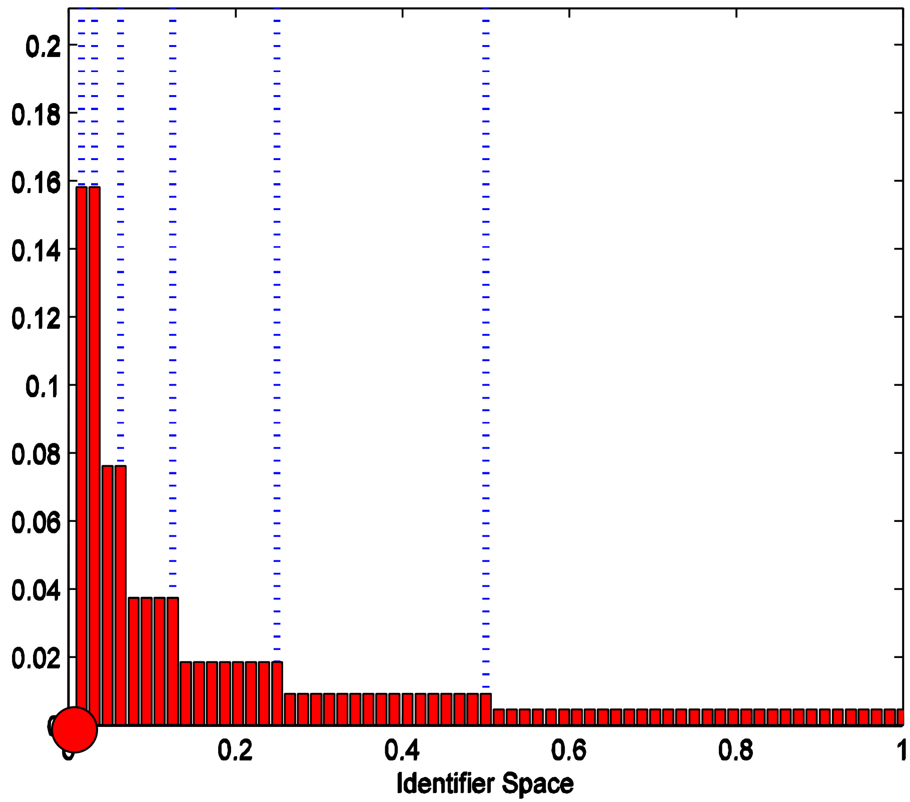
- Choosing long-range link:

  1) u.a.r. choose a partition

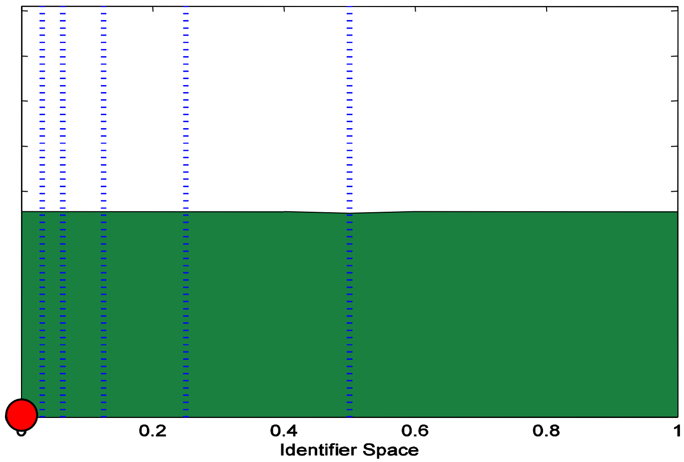
  2) u.a.r. choose a peer within that partition

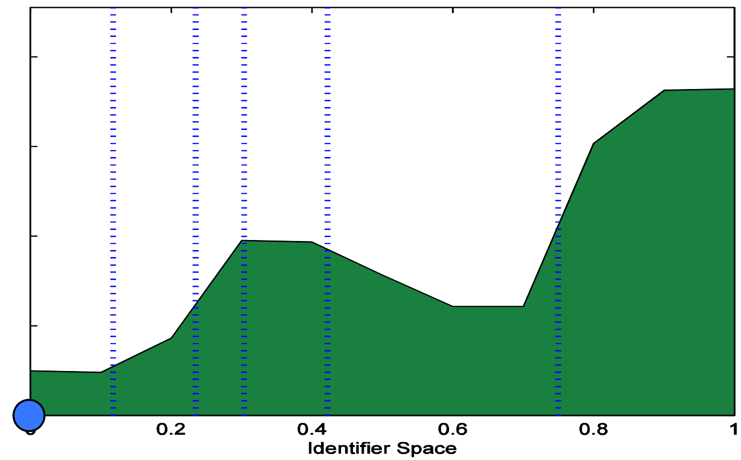
- It can be proven that **search cost** remains **O($\log^2$N)**

  O(logN) with O(logN) degree



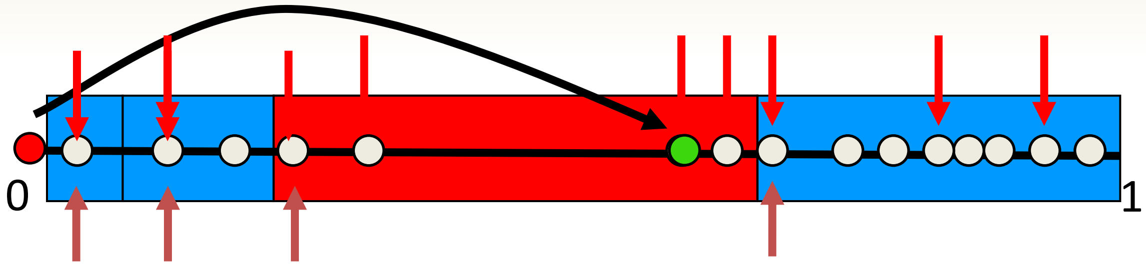
Oscar: Dealing with Skewed Spaces





- Find the boundaries between partitions!
- Uniform sampling by random walks.
- *k* samples for each boundary.

Oscar: an Example



- O(k*log N) samples is needed to construct a routing efficient network.

- Does not depend on the complexity of the distribution

- “The view” can be copied from a ring-neighbour

(by contacting median peers and requesting their ring-neighbour ids)

Recap of non-uniform structured overlays

- What is the relationship between the resource placement and the network structure in structured P2P overlays?
- What is a challenge for enabling range queries for structured overlays?
- What is the main difference between a regular Kleinberg's model Small-World model for non-uniform id spaces?
- How does random sampling work?
- What are the possible solutions to solve the non-uniform resource placement problem?

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