KTH Electrical Engineering

EE – Automatic Control
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CONTROL THEORY AND PRACTICE
ADVANCED COURSE

Lå 12/13

COMPUTER EXERCISES
LABORATORY EXERCISES

EE – Reglerteknik

Control Theory and Practice
Advanced Course

Computer exercise:
CLASSICAL LOOP-SHAPING

Magnus Jansson och Elling W. Jacobsen, January 1998

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1 Introduction

Loop-shaping is a classical procedure for control design. In the basic course it was denoted lead lag design. Loop-shaping was introduced during world war II and it was used to construct single variable circuits, such as amplifiers in feedback (Bode). This knowledge has later been transferred to other areas of automatic control, and it has been extended to multivariable systems, i.e. systems with multiple input and output signals.

The idea is to shape the *open-loop* gain with a controller in order to achieve intended properties of the *closed-loop* system under feedback. In the 70'ies and 80'ies advanced methods for loop-shaping based on optimization were developed. However, in this computer exercise we will focus on basic classical loop-shaping. Frequency domain descriptions are fundamental in control design!

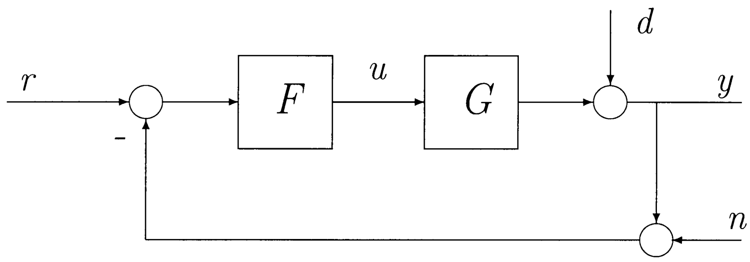
We will here only consider SISO systems (single input single output), but the ideas are also applicable to MIMO systems (multiple input multiple output).

Preparations: Chapters 7.1-7.4 in the course book (Ljung, Glad, "Control theory"). It is also recommended to repeat Chapter 5.5 in the basic course book (Glad, Ljung, "Reglerteknik-Grundläggande teori").

Presentation: All problems in this exercise should be solved and be presented in a written report. The date when it should be handed in is indicated in the course instructions. The report should contain all relevant figures.

2 Background

Consider the control system in Figure 1.



Figur 1: K–controller, G–system, r–reference signal, u–control signal, d–disturbance signal, y–output signal, n–measurement noise.

The loop gain is given by $L = GF$, the sensitivity function $S = (I + L)^{-1}$ and the complementary sensitivity function $T = (I+L)^{-1}L$. Remember that we have $S+T = I$. The control error depends on the input signals as

$$e = r - y = Sr - Sd + Tn.$$

Since we wish to have a small control error, we obtain the following conditions

$$e \approx 0 \Rightarrow \begin{cases} i) \;\; S \approx 0 \;\Rightarrow\; T \approx I \;\Rightarrow\; L \text{ large} \\ ii) \;\; T \approx 0 \;\Rightarrow\; S \approx I \;\Rightarrow\; L \text{ small} \end{cases}$$

We obviously have contradictive condtions! The case $i)$ corresponds to reference tracking and disturbance attenuation while case $ii)$ corresponds to noise attenuation (and sensitivity to model errors, robustness). For example, if we wish to track low frequency reference signals we have to design the loop gain to be large at low frequencies.

Apart from keeping the control error small, the control signal should not be too large or vary too much. Since

$$u = F(r - y - n)$$

this condition implies that the control gain must not be designed too large, F small $\Rightarrow \;\; L = GF$ small.

Stability is another important issue. The slope of the curve $|L(i\omega)|$ is coupled to the phase $\arg\{L(i\omega)\}$. For example, $L = a/s^n$ has slope $-n$ and phase $-n\pi/2$. In order to keep a reasonable stability margin, $|L|$ must not have too large slope around the cross-over frequency ω_c. Typically, $|L|$ is designed to have slope ≈ -1 at ω_c.

Also note that the phase margin is coupled to control performance. For example we have resonance peaks $M_S = \max_\omega |S|$ and $M_T = \max_\omega |T|$

$$M_T > \frac{1}{PM} \;; \quad M_S > \frac{1}{PM}$$

where the phase margin PM is given in radians. For example, if we demand the resonance peaks smaller than 2, the phase margin has to be larger than 30°.

Such contradictive constraints give rise to different strategies to shaping the loop L so that performance demands are met. They also provide limits of achievable control performance.

3 Introduction to Control System Toolbox

In this computer exercise we will use MATLAB to shape the loop, just as we did in the basic course. Most of the functions are in Control System Toolbox. Let us start by defining some useful function. Recall that you get access to the MATLAB help be typing `help function name`".

A transfer function

$$G(s) = \frac{s+2}{s^2 + 2s + 3}$$

is defined in MATLAB by typing

```
s=tf('s'); G=(s+2)/(s^2+2s+3);
```

The product of two transfer functions is obtained by

```
G12=G1*G2
```

For a system with 2 inputs and 2 outputs, the closed-loop transfer matrix is obtained with

```
S=feedback(eye(2),G*F) ;   T=feedback(G*F,eye(2))
```

For a SISO system this can be written

```
S=1/(1+G*F) ;   T=G*F/(1+G*F)
```

For numerical reasons it is **very important** to use the function `minreal`, for example `minreal(T)`. This creates an equivalent system where all cancelling pole/zero pair or non minimal state dynamics are eliminated.

The bode diagram for `G` is plotted by typing

```
bode(G)   or   bode(G,{wmin,wmax})
```

Amplitude and phase at a given frequency are obtained by

```
[m,p]=bode(G,w)
```

Phase margin, amplitude margin and corresponding frequencies are obtained by

```
[Gm,Pm,wp,wc]=margin(G*F)
```

To simulate a step response in the control signal, use the function

```
step(G)   or   step(G,tfinal)
```

In the same way, to simulate a step response in the reference signal, we type

```
step(T)
```

4 Exercises

4.1 Basics

Consider a system which can be modelled by the transfer function

$$G(s) = \frac{3(-s+1)}{(5s+1)(10s+1)}.$$

Exercise 4.1.1. Use the procedure introduced in the basic course to construct a lead lag controller which eliminates the static control error for a step response in the reference signal. The phase margin should be 30° at the cross-over frequency $\omega_c = 0.4$ rad/s.

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Exercise 4.1.2. Determine the bandwidth of the closed-loop system and the resonance peak M_T. Also, determine the rise time and the overshoot for step changes in the reference when the controller designed in 4.1.1. is used.

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Exercise 4.1.3. Modify the controller in 4.1.1. such that the phase margin increases to 50° while the cross-over frequency is unchanged. For the corresponding closed-loop system, determine the bandwidth and resonance peak. Also, determine the rise time and the overshoot of the step response.

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4.2 Disturbance attenuation

Now we will construct a controller which both tracks the reference and attenuates disturbances. The block diagram of the control system is given in Figure 2. We assume that the signals have been scaled such that $|d| < 1$, $|u| < 1$ and $|e| < 1$ where $e = r - y$.

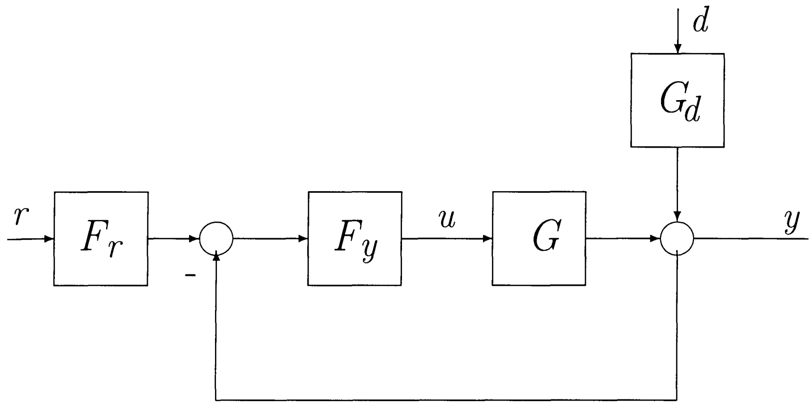
The exercise is about designing F_r and F_y in Figure 2 such that:

- The rise time for a step change in the reference signal less than 0.2 s and the overshoot is less than 10%.
- For a step in the disturbance, we have $|y(t)| \leq 1 \ \forall t$ and $|y(t)| \leq 0.1$ for $t > 0.5$ s.
- Since the signals are scaled the control signal obeys $|u(t)| \leq 1 \ \forall t$.

The transfer functions have been estimated to

$$G(s) = \frac{20}{(s+1)((\frac{s}{20})^2 + \frac{s}{20} + 1)}$$

$$G_d(s) = \frac{10}{s+1}$$



Figur 2: F_r–prefilter, F_y–feedback controller, G–system, G_d–disturbance dynamics, r–reference signal, u–control signal, d–disturbance signal, y–measurement signal.

Exercise 4.2.1. For which frequencies is control action needed? Control is needed at least at frequencies where $|G_d(j\omega)| > 1$ in order for disturbances to be attenuated. Therefore the cross-over frequency must be large enough. First, try to design F_y such that $L(s) \approx \omega_c/s$ and plot the closed-loop transfer function from d to y and the corresponding step response. (A simple way to find $L = \omega_c/s$ is to let $F_y = G^{-1}\omega_c/s$. However, this controller is not proper. A procedure to fix this is to "add" a number of poles in the controller such that it becomes proper. How should these poles be chosen?)

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A loop gain of slope -1 at all frequencies gives in our case poor disturbance attenuation. To understand the reason for this, note that the output is given by

$$y = SG_d d = (1 + L)^{-1} G_d d.$$

Provided the signals have been scaled we want $|(1 + L)^{-1}G_d| < 1$ for all ω. For frequencies where $|G_d| > 1$ this approximately implies $|L| > |G_d|$ or $|F_y| > |G^{-1}G_d|$. Most often we also want integral action and as a starting point we can choose

$$F_y = \frac{s + \omega_I}{s} G^{-1} G_d, \tag{1}$$

where ω_I determines the frequency range of efficient integral action. We see that if $G_d \approx 1$, the controller should contain the inverse of the system. On the other hand if $G_d \neq 1$ the controller should be designed in some other fashion. Especially, we observe that if the disturbance is on the input side to the system we have $G_d = G$ and then F_y should be chosen as a PI controller according to (1).

Note that the controller (1) cannot be used if it is not proper, causal and stable. To ensure these properties, approximations of (1) may be necessary.

Exercise 4.2.2. Let us now reconstruct F_y according to the instructions above. We will start with the disturbance attenuation. In a second step, adjustments can be made on

F_r to obtain the desired reference tracking properties. Start by choosing F_y according to (1). Try different approximations of the product $G^{-1}(s)G_d(s)$ and choose ω_I large enough so that step disturbances are attenuated according to the specifications.

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Exercise 4.2.3. To fulfil the reference tracking specifications, we can combine lead lag control and prefiltering of the reference signal. First, try to add lead action to F_y to reduce the overshoot. Then it can be necessary to add prefilter action to fulfil all specifications. Note that F_r should be as simple as possible (why?). Also, remember to check the size of the control signal ($u = F_y F_r S r - F_y G_d S d$)! Typically a low pass filter is chosen, for example

$$F_r = \frac{1}{1 + \tau s}.$$

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Exercise 4.2.4. Finally, check that all specifications are fulfilled. Plot the sensitivity and complementary sensitivity functions.

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ROYAL INSTITUTE
OF TECHNOLOGY

Control Theory and Practice
Advanced Course

Computer exercise: H_∞ control design

Marcus Berner and Peng Zhou, 2008

Automatic Control — KTH, Stockholm, Sweden

Goals & requirements

It is important to read this document and answer the preparation tasks, 1.1.1, 1.2.1, and 1.3.1 before the exercise. The preparation tasks should be included in the report.

The main goal of this lab is to get a feeling for how H_∞ control design can be used to obtain desired specifications on sensitivity and robustness. To perform this lab basic knowledge in control theory, corresponding to the basic control course, is required. It can be a good idea to look back on the following things:

- Sensitivity and Complementary sensitivity function
- Robustness and model errors
- How poles and zeros affect the dynamics of the system and how it is reflected in the Bode diagram.

1 Introduction

In H_∞ control design the sensitivity and complementary sensitivity function are shaped to meet certain desired specifications. To be able to form these specifications this section will begin with a brief summary of some important basic theory followed by theory more specific for the design method.

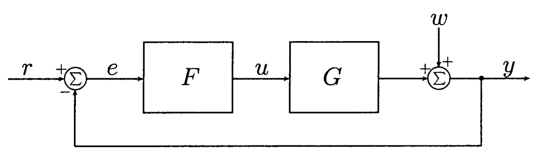

Figure 1: Block diagram of the feedback system used in this lab

The system used in the entire lab has the structure depicted in Figure 1. The signals indicated in the figure are:

r: Reference value

e: Control error

u: Control signal

w: Disturbance on the output

y: Output

1.1 Sensitivity function and reduction of disturbances

The sensitivity function, denoted S, is the transfer function from the disturbance to the output, see Equation (1). Note that the equation only describes the relation between the disturbance and the output. The reference is therefore assumed to be zero.

$$y = Sw, \quad S = (1 + GF)^{-1} \tag{1}$$

By making the amplification of the sensitivity function small, the effects of disturbances on the output can be reduced. As this lab will show it is not possible to make it arbitrary small for all frequencies. This can easily be realized by looking at the amplification of the controller required to make the sensitivity very small.

1.1.1 Preparation task 1

Use the block diagram in Figure 1 to show that the transfer function from the disturbance (w) to the output (y) satisfies Equation (1).

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1.2 Complementary sensitivity and robustness

The complementary sensitivity function T given by

$$T = (1 + GF)^{-1}GF \tag{2}$$

can be used to prove robustness to model errors. For that a new system with model uncertainty Δ_G is introduced. The new system is depicted in Figure 2.

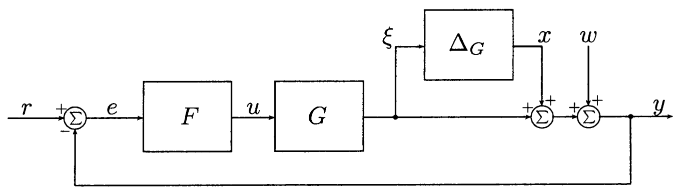

Figure 2: Block diagram of the feedback system with model error

The system in Figure 2 can be rewritten as the system depicted in Figure 3.

With the small gain theorem (lågförstärkningssatsen), see course book [1] or [2], closed loop stability can be guaranteed if Δ_G and T both are stable and condition (3) is satisfied.

$$|T(i\omega)| < \frac{1}{|\Delta_G(i\omega)|}, \quad \forall\omega \tag{3}$$

One important remark about the above result is that the small gain theorem is conservative. The condition on T is therefore sufficient but not necessary for stability.

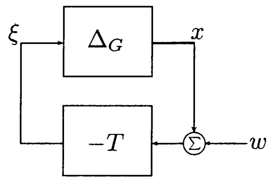

Figure 3: Block diagram of the feedback system with model error on the form used in the small gain theorem

1.2.1 Preparation task 2

Show that the systems in Figure 2 and Figure 3 are equivalent if the reference signal is zero.

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1.3 H_∞ control design

In H_∞ control design the three functions S, T and G_{wu} are shaped to meet desired performance. The first two have already been discussed. Here G_{wu} denotes the transfer function from the disturbance to the control signal, see Equation (4) (where it is assumed $r = 0$).

$$u = G_u w, \quad G_{wu} = -(1 + FG)^{-1}F = -FS \tag{4}$$

It is desirable to make the magnitude of S, T and G_{wu} small. That is unfortunately not possible because they are related to each other. To deal with that, the weights (transfer functions) W_S, W_T and W_U are introduced. They decide how much emphasis to put on minimizing each closed loop transfer function.

After choosing the weights the following problem is solved:

> Find F such that
>
> $$\left\| \begin{matrix} W_S S \\ W_T T \\ W_U F S \end{matrix} \right\|_\infty \leq \gamma$$
>
> Try to make γ as small as possible.

The above constraint can be rewritten as

$$\begin{cases} |S(i\omega)| & \leq \gamma |W_S^{-1}(i\omega)|, \quad \forall\omega \\ |T(i\omega)| & \leq \gamma |W_T^{-1}(i\omega)|, \quad \forall\omega \\ |F(i\omega)S(i\omega)| & \leq \gamma |W_U^{-1}(i\omega)|, \quad \forall\omega \end{cases} \tag{5}$$

To compute the controller that gives the smallest value of γ is far from trivial, especially for higher order systems and weights. It is therefore not done by hand in this lab. The computations are instead done numerically in the design tool.

1.3.1 Preparation task 3

Use the block diagram in Figure 1 to show that the transfer function from the disturbance (w) to the control signal (u) satisfies Equation (4).

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2 Software

This lab is run in MATLAB. The files needed can be found on the course homepage. A graphical design tool will be used to design the weights and compute the resulting controller. There is also a Simulink model used for simulations. How to use the design tool and do the simulations will be described in this section.

2.1 H_∞ graphical design tool

Before the design tool can be opened a transfer function for the system must be defined. The model has to be strictly stable and proper, which means that is has at least as many poles as zeros and all poles are in the left half plane. An example sequence of how the tool is started can be seen below.

```
s=tf('s')}; G=1e4*(s+2)/(s+3)/(s+100)^2; Hinf(G);
```

The tool should now open and look like in Figure 4.

There are three Bode plots, (1*a*), (1*b*), (1*c*). They show S, T, G_{wu} and the **inverse** of their respective weights.

To the right of each Bode there are lists of poles and zeros and some buttons, (4) in Figure 4. Notice that the poles corresponds to zeros in the inverse of the weight that is plotted and the opposite for the zeros.

The weights can be changed in two ways. One is to add and remove poles or zeros with the corresponding buttons. The other is to open a graphical editor for the poles and zero. It is done by clicking the "Edit Pole-Zero diagram" button. A new window, seen in Figure 5, will open. In the new window different tools can be chosen from the toolbar, (1) in Figure 5.

The tools from left to right are:
Add real pole: Add real pole by clicking in the diagram.
Add complex pole pair: Add pole pair by clicking in the diagram.
Add real zero: See Add real pole.
Add complex zero pair: See Add complex pole pair.
Remove: Removes pole or zero by clicking on it.
Move: Move pole or zero by clicking on it and holding the mouse button down while moving the cursor.

The weights are on the form (6).

$$W(s) = k\frac{(s-z(1))(s-z(2))...(s-z(m))}{(s-p(1))(s-p(2))...(s-p(n))} \tag{6}$$

To edit the constant k, just type the new value in the gain input field (3) in Figure 4 and press enter.

The weights on T and G_{wu} can be disabled. Just press the disable button (5) in Figure 4. It can be enabled again by pressing the enable button that replaces the disable button.

The magnitude scale is automatically fitted to the weight but the frequency scale has to be set by the user. There are two ways of doing it. One is to enter a maximum and minimum value in the fields (2a) and (2b) in Figure 4 and press enter. The other option is to use the "auto-set frequency" option in the plots menu (6) in Figure 4.

To compute the controller from the weights simply use the "compute controller" option in the controller menu, (6) in Figure 4. The controller will then be computed and the plots updated. Some information about the controller can be displayed in MATLAB command window.
The controller can then finally be exported to workspace with the "export controller" option in the controller menu (6) in Figure 4. It will be saved as F in workspace. If there already exists an varable with that name it will be over saved.

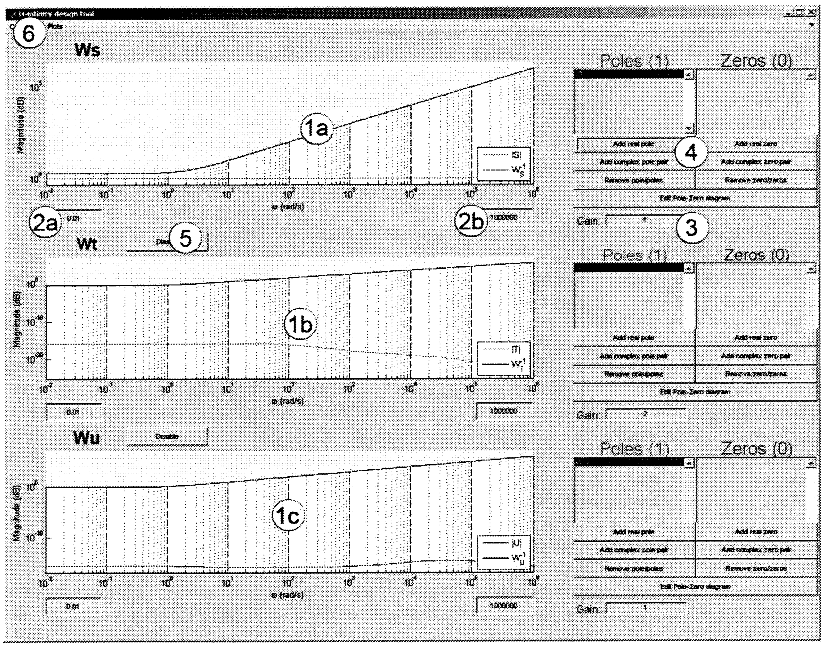

Figure 4: The H_∞ graphical design tool

2.2 Simulations

The Simulink model named *servo1.mdl* is used to simulate the system in Figure 1. A step is used as reference and disturbances can be added as band-limited white noise and a sinusoid.

To run the simulation the short macro, found on the homepage, can be used. In the beginning of the file *macro.m* there are some parameters that can be changed to customize the simulation. The parameters are described in the m-file.

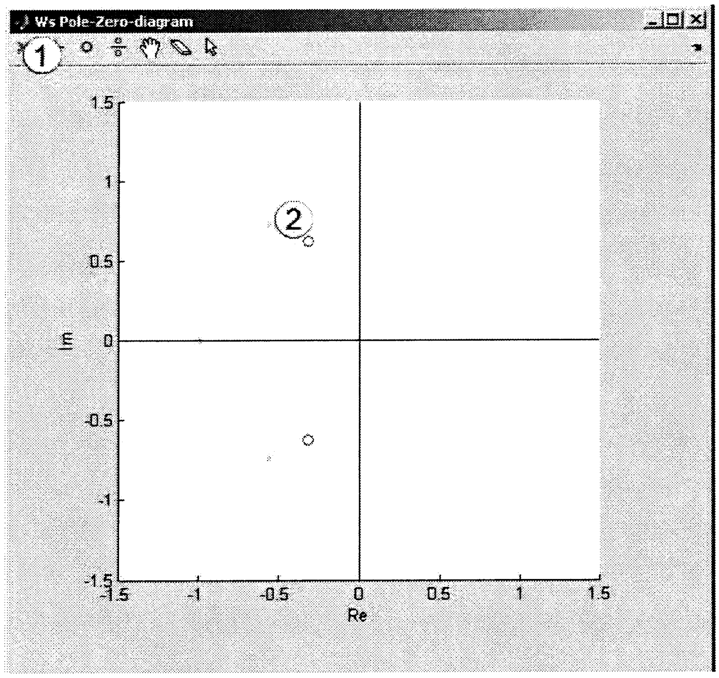

Figure 5: The Pole zero editor

To run the macro, the controller and system must first be saved as transfer functions named ***Fsim*** and ***Gsim*** in workspace. The macro runs the simulation and plots the results in a new figure.

Below is an example of a sequence that simulates the system. The system and controller are assumed to be defined as transfer functions in workspace with the names F and G.

```
Fsim=F; Gsim=G;
%Edit parameters in macro.m
macro
```

Now the simulation should start and plot the results when ready. If the simulation takes very long time it can be stopped by pressing *Ctrl-C* in the MATLAB command window.

3 Exercises

The system to control in this lab is an electrical device powered by the $50Hz$ (100π rad/s) net. A proposed model of the system is given by transfer function (6).

$$G(s) = \frac{10^4(s+2)}{(s+3)(s+100)^2} \tag{7}$$

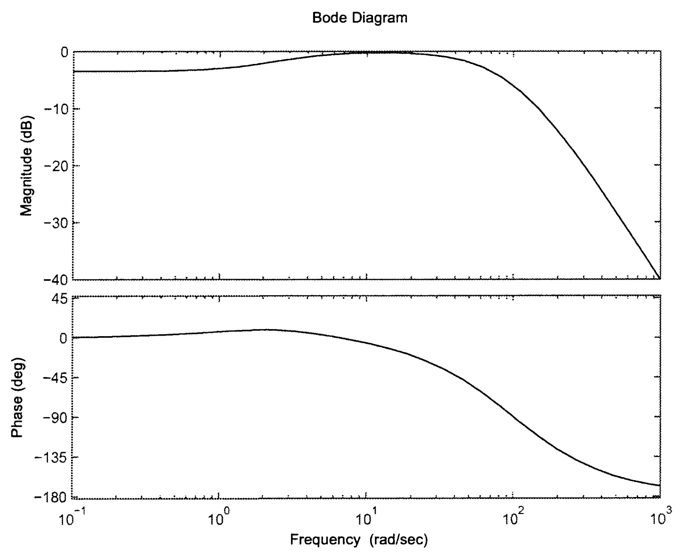

Figure 6: Bode diagram of the system in Equation (7)

The closed loop system is considered to be fast enough, when it comes to following the reference, without a controller, but not the suppression of disturbances. A controller will therefore be designed focusing on the disturbances.

3.1 Suppression of disturbances

i. The aim is to damp the $50Hz$ disturbances. Propose a suitable weight, W_S, by drawing it in the empty Bode diagram in Figure 7. Also draw the expected resulting sensitivity function. Keep in mind that the sensitivity function can't be small for all frequencies.

ii. Now try to design the weight in the software. In this part only the sensitivity is considered. The weights on T and G_{wu} can therefore be disabled. When the sensitivity function is satisfactory, export the controller to workspace and run the simulation with a $50Hz$ sinusoidal disturbance

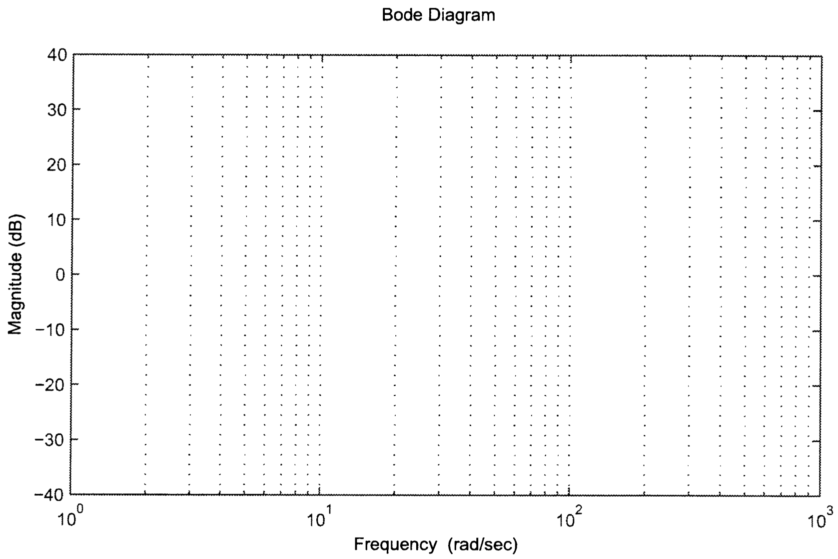

Figure 7: Draw W_S and S here

like described in section 2.2. Use the parameters in Table 1. Fill out Table 2 with the results from the simulation.
Hint: Placing poles in $s = -\epsilon \pm i\sqrt{\omega^2 - \epsilon^2})$, *where* ϵ *is small, gives a peak at* ω *rad/s*

Parameter	**Value**
u_max	*inf*
sin_dist_freq	100π
sin_dist_amp	1
white_noise_amp	0
step_size	0
sim_time	10

Table 1: Parameters to use in macro.m for simulation

Signal	**Amplitude**
disturbance (w)	
output (y)	
control signal (u)	

Table 2: Results from simulation in Exercise 3.1 ii.

How much is the disturbance damped on the output? ....
(*The rate between the disturbance amplitude and the output oscillations*)

Approximately what amplification is required for a P-controller to get the same rate and what are the advantages/disadvantages of such a controller?

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Hint: If $|FG| >> 1$, $|S| \approx |FG|^{-1}$.

3.2 Robustness

Unfortunately the model (7) is not accurate. It was obtained by sending sinusoids, with different frequency, into the system and measure the output amplitude. By ignoring the phase-shift some important dynamics was not detected. The true system is given by Equation (8).

$$G_o(s) = G(s)(1 + \Delta_G(s)) = \frac{10^4(s+2)}{(s+3)(s+100)^2} \cdot \frac{(s-1)}{(s+1)} \tag{8}$$

i. What influences will this error have on the system behavior, and will they be a limiting factor on achievable control performance?

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ii. Simulate the system with the controller designed in the previous exercise and the system (8). Run the simulation with the same parameters as before.
(The simulation time might need to be increased to see the results)

Comment on the results from the simulations:

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iii. What is the condition on T required to guarantee stability for the new system, using the small gain theorem?

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Is the condition fulfilled? ....
(Look at the Bode diagram in the graphical design tool.)

iv. Use the software to design a new controller that suppresses $50Hz$ disturbances but is stable with G_o. Keep the W_S used earlier but enable the weight on T. Then try to find a weight on T that makes the closed loop system stable and still has good suppression of the disturbance. Export the controller to workspace and run the simulation.

Remark: The controller should be designed for G in Equation (7) but simulated with G_o in Equation (8).

Compare the results to Table 2.

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3.3 The use of control signal

Enable the weight on G_{wu} and try to reduce the control signal. Try to reduce the amplitude to half of the one used in Exercise 3.2 iv. How is the amplitude of the output affected?

....
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....

References

[1] Torkel Glad & Lennart Ljung: *Control theory - Multivariable and Nonlinear Methods*, Taylor & Francis (2000)

[2] Torkel Glad & Lennart Ljung: *Reglerteori - Flervariabla och olinjära metoder*, Studentlitteratur (2007)

EE – Reglerteknik

Control Theory and Practice
Advanced Course

Computer exercise: Multivariable systems

Anders Hansson, Per Bodin och Elling Jacobsen, December 1998

1 Introduction

In this computer exercise we will investigate properties of multivariable systems. The application is a model of a four-tank-process. In particular we will consider pairings between different input and output signals and non-minimum phase dynamics. The pairings are analyzed using RGA and we will investigate their influence in decentralized PI control.

Preparations: Chapters 3.3, 3.5, 6.5, 7.3-7.5, 7.7 and 8.3 in Glad, T. and Ljung, L.: Control theory—multivariable and nonlinear methods.

Presentation: All problems in this exercise should be solved and be presented in a written report. The date when it should be handed in is indicated in the course instructions. The report should contain all relevant figures.

2 Theoretical overview

This section contains a short overview of basic theory for multivariable systems. The content is based on the course book.

2.1 Poles and zeros

As for SISO systms we can define poles and zeros for a linear MIMO system with transfer matrix $G(s)$.

The poles of $G(s)$ are defined as the eigenvalues of the system matrix A in a minimal state space realization of the system and they are calculated as the roots of the *pole polynomial*, $\det(sI - A)$. The pole polynomial can also be obtained by calculating the least common denominator of all subdeterminants of $G(s)$.

It is more difficult to extend the definition of zeros from the SISO case to the MIMO case. The most common definition of a zero of the system is a value of s where the transfer matrix $G(s)$ looses rank. For the special case of square transfer matrices, the zeros are given by the roots of $\det G(s) = 0$.

2.2 Singular values, directions and H_∞ norms

The singular values σ_i of a matrix A are defined as $\sigma_i = \sqrt{\lambda_i}$, where λ_i are the eigenvalues of the matrix A^*A. The largest singular value of A is denoted $\overline{\sigma}(A)$, and the smallest as $\underline{\sigma}(A)$. If $y = Ax$, it follows from the singular value definition that

$$\underline{\sigma}(A) \leq \frac{|y|}{|x|} \leq \overline{\sigma}(A)$$

where the relation between the norm of y and the norm of x, $|y|/|x|$, can be interpreted as the gain of the matrix A. If x is parallel with the eigenvector corresponding to the largest eigenvalue of A^TA then we have $|y| = \overline{\sigma}(A)|x|$, and if x is parallel to the eigenvector corresponding to the smallest then we obtain $|y| = \underline{\sigma}(A)|x|$. This way we can define directions corresponding to the largest and smallest singular value for A respectively.

For a linear stable multivariable system with transfer matrix $G(s)$ we have

$$Y(i\omega) = G(i\omega)U(i\omega)$$

where $Y(s)$ and $U(s)$ are the Laplace transforms of the system's output and input signals. According to the definition of singular values we therefore have

$$\underline{\sigma}(G(i\omega)) \leq \frac{|Y(i\omega)|}{|U(i\omega)|} \leq \overline{\sigma}(G(i\omega))$$

Introducing

$$|G(i\omega)| := \overline{\sigma}(G(i\omega))$$

it holds

$$|Y(i\omega)| \leq |G(i\omega)||U(i\omega)|.$$

This notation is analogous to the SISO case where the norm is interpreted as the absolute value of $G(i\omega)$, and the inequality above turns to an equality.

To understand the inequalities in the MIMO case we can choose the input parallel to the direction corresponding to the largest or smallest singular values of $G(i\omega)$. The directions decide the "mix" of the input signal components that results in the largest and smallest gain of the system respectively.

The largest gain of the multivariable system $G(i\omega)$ is dentoted $\|G\|_\infty$, which is given by

$$\|G\|_\infty = \max_\omega |G(i\omega)|.$$

$\|G\|_\infty$ is called the H_∞ norm of G. For output signal $y(t)$ and input signal $u(t)$ it holds

$$\|y\| \leq \|G\|_\infty \|u\|.$$

Therefore the H_∞ norm can be interpreted as the time domain gain of the system. It holds

$$\sup_u \frac{\|y\|}{\|u\|} = \|G\|_\infty,$$

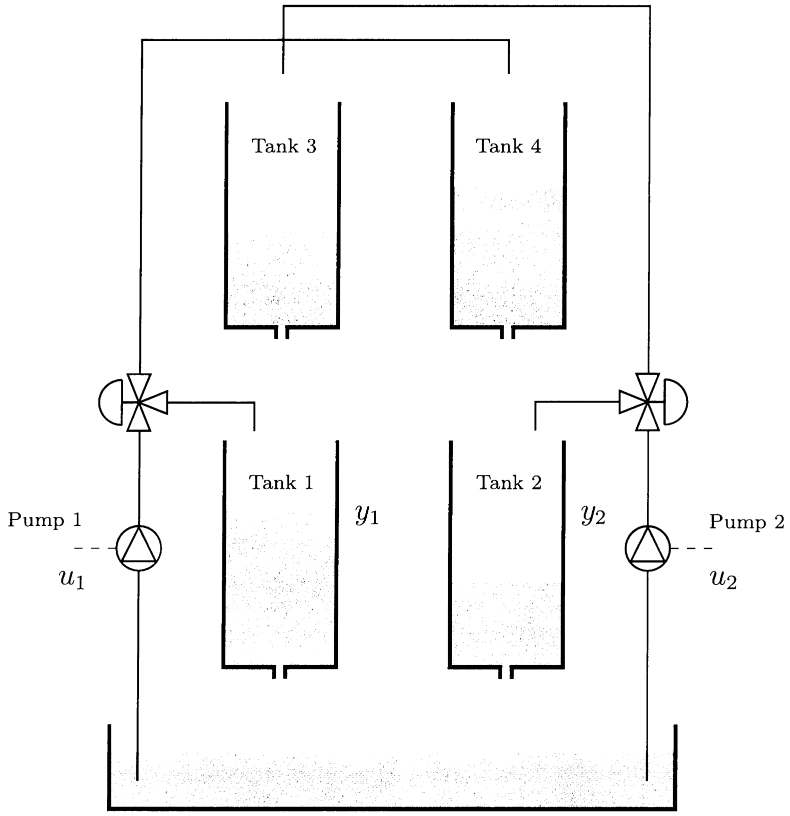
which can be seen as the definition of the H_∞ norm for G.

2.3 Decentralized control

A fundamental problem in multivariable control is the pairings between the inputs and outputs. This means that if one input changes there is generally a change in all outputs. A measure of the strength of the pairings in a multivariable system, $G(s)$, is given by the *Relative Gain Array*, RGA of the transfer matrix G. It is defined by

$$\text{RGA}(G(i\omega)) := G(i\omega). * \left[G^{-1}(i\omega)\right]^T,$$

where ".*" denotes element wise multiplication. We can use RGA to determine which input and output that are suitable to pair in a decentralized controller. Two rules of thumb:



Figur 1: The four-tank process

1. Find a pairing such that diagonal elements of $\mathrm{RGA}(G(i\omega_c))$ are as close to 1 as possible, where ω_c is the intended cross-over frequency.

2. Avoid pairings which correspond to negative elements in $\mathrm{RGA}(G(0))$.

3 Exercises

In this computer exercise linear models of a four-tank process will be investigated. The system is shown schematically in Figure 1. The input signals are the voltages of the pumps, u_1 and u_2. The output signals that we want to control are the levels in the lower tanks, y_1 and y_2. Connected to each pump there is a valve that divides the water to the upper and lower tanks. A linear multivariable model with 2 inputs and 2 outputs is given by $Y(s) = G(s)U(s)$ where

$$Y(s) = \begin{bmatrix} Y_1(s) \\ Y_2(s) \end{bmatrix}, \quad U(s) = \begin{bmatrix} U_1(s) \\ U_2(s) \end{bmatrix} \quad \text{and} \quad G(s) = \begin{bmatrix} g_{11}(s) & g_{12}(s) \\ g_{21}(s) & g_{22}(s) \end{bmatrix}$$

Depending on the settings of the valves we obtain different $G(s)$. In this computer exercise two different valve settings wil be investigated: in the first, most of the water will go directly to the lower tanks and $G(s)$ is minimum phase; in the second, most of the water go to the upper tanks and $G(s)$ will be non-minimum phase.

3.1 Poles, zeros and RGA

A linear state space model for the four-tank process is generated by the Matlab functions `minphase` and `nonminphase`. To put the minimum phase model in the variable `sysmp` we write

```
sysmp = minphase
```

The follwing Matlab functions can be used in this exercise. The poles of the system `sys` are obtained with

```
pole(sys)
```

and its zeros with

```
tzero(sys)
```

The system should be given as a (minimal) state space description when these functions are used. (Otherwise numerical problems can appear in Matlab.)

The singular values for a system are calculated with `sigma`. To extract system matrices and transfer functions the functions `ssdata` and `tfdata` are used, respectively. The singular value decomposition are calculated using `svd`. To calculate step responses for linear systems the function `step` can be used. The Bode diagram of a system is plotted using `bode`. Note that dB scale is used. Use the Matlab Help to learn more about the functions.

The problems below should be solved both for the minimum phase and non-minimum phase case. It is suitable to start with the former.

Exercise 3.1.1. Calculate the transfer matrix $G(s)$. Investigate each element of the matrix (Hint: `G(1,1)` extracts element (1,1)). Calculate the poles and zeros of the elements.

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Exercise 3.1.2. Calculate the poles and zeros of the multivariable system. Do these imply any constraint on the achievable control performance?

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Exercise 3.1.3. Investigate the largest and smallest singular values for the system at different frequencies. Calculate the H_∞ norm of the system.

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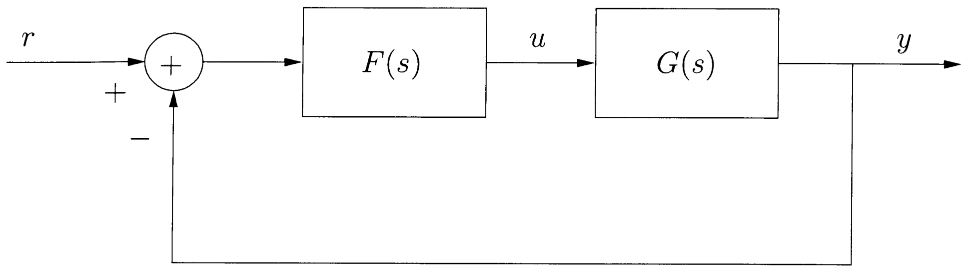
Exercise 3.1.4. Investigate the RGA of the system at frequency 0. What conclusions can we draw about the possibility of using decentralized control?



Figur 2: Control.

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Exercise 3.1.5. Plot the step response for one input at the time. Investigate the outputs: is the system coupled? Is this in line with the properties of RGA?

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Now solve the above problems above for the non-minimum phase case.

Exercise 3.1.6. Describe the most important differences between the two cases and discuss how it affects the control performance.

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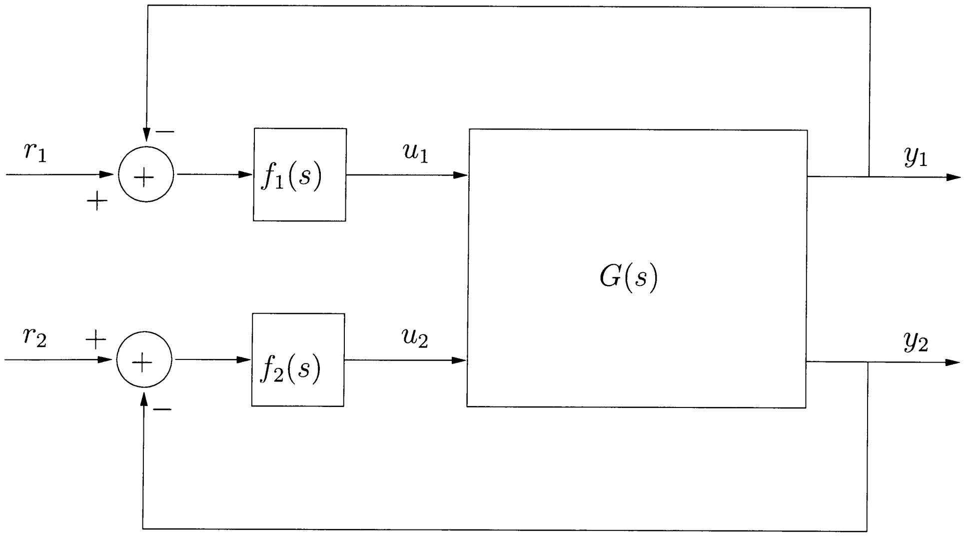
3.2 Decentralized control

We will now investigate control of the four-tank process as illustrated in Figure 2, where both the process $G(s)$ and the controller $F(s)$ are 2×2 matrices of transfer functions. The simplest way to control a system is to use decentralized control. This means that one input is paired with one output, which is controlled with a one-dimensional controller. An example is depicted in Figure 3, where output y_1 is controlled with the input u_1 through the controller $f_1(s)$. Similarly, the output y_2 is controlled with the input u_2 through $f_2(s)$. Here y_1 is paired with u_1 and y_2 is paired with u_2. This corresponds to Figure 2 with

$$F(s) = \begin{bmatrix} f_1(s) & 0 \\ 0 & f_2(s) \end{bmatrix}$$

The other way around, if y_1 is paired with u_2 and y_2 is paired with u_1, then $F(s)$ is given by

$$F(s) = \begin{bmatrix} 0 & f_1(s) \\ f_2(s) & 0 \end{bmatrix}$$



Figur 3: Decentralized control.

In the first case, the controllers $f_1(s)$ and $f_2(s)$ are designed using single-varible control design with the transfer functions between u_1 and y_1 and between u_2 and y_2. A procedure for one-dimensional control design was investigated in the computer exercise on loop-shaping. Here we will design PI controllers

$$f_j(s) = K_j\left(1 + \frac{1}{sT_{i_j}}\right), \quad j = 1,2$$

such that the intended phase margin φ_m and cross-over frequency ω_c are obtained. The loop gain is given by $L = GF$. Therefore we wish to shape l_{11} and l_{22} in such a way that given specifications are fulfilled.

Let us now investigate the case where y_1 is to be controlled with u_1. Denote the intended phase margin and cross-over frequency by φ_m and ω_c respectively. From l_{11} we see that K_1 and T_{i_1} are given by the following equations:

$$|g_{11}(i\omega_c)f_1(i\omega_c)| = 1 \quad (1)$$

$$\arg g_{11}(i\omega_c) + \arg f_1(i\omega_c) - \varphi_m = -\pi \quad (2)$$

Note that Equation (2) is equivalent to

$$\arg g_{11}(i\omega_c) + \arctan(\omega_c T_{i_1}) - \pi/2 - \varphi_m = -\pi$$

where $\arg g_{11}(i\omega_c)$ can be obtained from the Bode diagram of $g_{11}(s)$. This gives T_{i_1}. Then we can draw the Bode diagram for the loop gain

$$l_{11}(s) = g_{11}(s)\left(1 + \frac{1}{sT_{i_1}}\right)$$

Equation (1) now gives

$$K_1 = \frac{1}{|l_{11}(i\omega_c)|}$$

where $|l_{11}(i\omega_c)|$ is obtained from the Bode diagram of $l_{11}(s)$. Control design for other input/output pairings is performed similarly. Apart from Matlab functions already mentioned, the following ones can help you: `tf, zoom, append, inv` and `feedback`.

The problems below should be solved both for the minimum phase and non-minimum phase case. It is suitable to start with the former.

Exercise 3.2.1. Design a decentralized controller by pairing inputs and outputs according to the RGA analysis. The intended phase margin is $\varphi_m = \pi/3$ and the crossover frequency ω_c is 0.1 rad/s for the minimum phase case and 0.02 rad/s for the non-minimum phase case. (To make sure that the problem is correctly solved, investigate the Bode diagram of L.)

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Exercise 3.2.2. Calculate the singular values of the sensitivity function

$$S(s) = (I + G(s)F(s))^{-1}$$

and the complementary sensitivity function

$$T(s) = (I + G(s)F(s))^{-1}G(s)F(s)$$

Is the design good with respect to sensitivity and robustness?

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Exercise 3.2.3. Simulate the closed-loop system in Simulink by typing `closedloop`. A Simulink window will appear where the block diagram is shown. Make sure that the variables `F` and `G` in the Matlab work-space contain the controller and the process respectively. Go to the Simulation meny and click Start. On the screen the unit step responses from the references to the outputs $y_1(t)$ (at $t = 100$) and $y_2(t)$ (at $t = 500$) are plotted together with the inputs. The time instant of the steps can be modified by clicking on the step blocks and changing the Step time. The total simulation time can be modified by changing the Stop time in the menu Simulation/Parameters. Simulation data is saved in the variable `simout` in the Matlab workspace. Is the control good? Are the outputs coupled?

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Now solve the above problems for the non-minimum phase case.

Exercise 3.2.4. Describe the most important differences between the two cases.

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EE – Reglerteknik

Control Theory and Practice
Advanced Course

Computer exercise: Decoupling and Glover-McFarlane robust loop-shaping

Anders Hansson, Alf Isaksson och Magnus Jansson, January 1999

1 Introduction

The purpose of this computer exercise is to investigate different procedures for multivarible control design. The process is the same as in the computer exercise on multivariable systems, *i.e.* the four-tank process. First, we will investigate static and dynamic decoupling. The control performance will be evaluated. Then, the design will be robustified using a method proposed by Glover and McFarlane.

Preparation: Chapters 8.3 and 10.5 in Glad, T. and Ljung, L.: Control theory—multivariable and nonlinear methods.

Presentation: All problems in this exercise should be solved and be presented in a written report. The date when it should be handed in is indicated in the course instructions. The report should contain all relevant figures.

2 Theoretical overview

This section provides the theory that you will need to solve the problems. It is based on the course book.

2.1 Decentralized control and decoupling

A fundamental problem in multivariable control is that the input and output signals are coupled. This means that if one input is changed then, in general, all outputs are affected. A measure of the strength of the coupling in a multivariable system ($G(s)$) is given by the *Relative Gain Array*, RGA of the transfer matrix G, defined as:

$$\text{RGA}(G(i\omega)) := G(i\omega). * \left[G^{-1}(i\omega)\right]^T,$$

where ".*" denotes element wise multiplication. In decentralized control the RGA can help us to determine which inputs and outputs that are suitable to pair. Two rules of thumb:

1. Find a pairing such that the diagonal elements in $\text{RGA}(G(i\omega_c))$ are as close to 1 as possible, where ω_c is the intended cross-over frequency.

2. Avoid pairings that correspond to negative elements in $\text{RGA}(G(0))$.

If it is not possible to find a suitable pairing of inputs and outputs, one can try to make a better system using *decoupling*. Consider the following example: one output is a function of the difference of two inputs, while another output depends on the sum of these two inputs. In this case, it is suitable to introduce two new inputs which denote the sum and the difference respectively of the two original inputs. This is the main idea in decoupling. Generally, decoupling is performed in the following way. Introduce the new variables $\tilde{y} = W_2 y$ and $\tilde{u} = W_1^{-1} u$, so that the transfer function from $\tilde{u}$ to $\tilde{y}$ becomes

$$\tilde{G}(s) := W_2(s)G(s)W_1(s),$$

where we try to design $\tilde{G}$ as diagonal as possible. Typically, we let $W_2 = I$. The idea is to find a $\tilde{G}$ which is more suitable for decentralized control than the original system G. In general, a completely diagonal $\tilde{G}$ is not realizable. However, we can try to design $\tilde{G}$ to be as decoupled as possible in a certain frequency range with the dynamical transfer matrices $W_1(s)$ and $W_2(s)$. Alternatively, we can decouple the system for one frequency, *e.g.* $\omega = 0$ or $\omega = \omega_c$, with constant matrices W_1 and W_2.

2.2 Glover-McFarlane robust loop-shaping

The decentralized control can be robustified using the method proposed by Glover och McFarlane. It is described in Chapter 10.5 in the course book. A summary of the design procedure, step by step, is given below.

1. Start by pairing the input and output signals in such a way that the system becomes as diagonal as possible. A useful mathematical tool is RGA (relative gain array).

2. Design an initial controller using pre-compensation W_1 and post-compensation W_2. To start with, we can typically choose $W_2 = I$ and $W_1 = W_{dc}F_{diag}$ where W_{dc} decouples the system at a suitable frequency (*i.e.* $\omega = 0$ or the intended ω_c) and $F_{diag}(s)$ is a diagonal lead/lag controller designed using classical loop-shaping (*c.f.* computer exercises 1 and 2).

3. Robust stabilization. Design the Glover-McFarlane controller F_r for the system $G_s = W_2GW_1$. If $\gamma > 4$, redesign W_1.

4. The final controller is $F(s) = W_1F_rW_2$.

3 Exercises

In this computer exercise the four-tank process will be investigated. Please recall the Matlab functions introduced in the exercise on multivariable systems.

NB: numerical problems in Matlab can occur if you work with systems as transfer functions (`tf` objects in Matlab). It is therefore important that you instead work with state space representations (`ss`). When performing multiplication and division of systems, it is highly recommended to use the function `minreal`, which creates an equivalent system where all cancelling pole/zero pairs or non minimal state dynamics are eliminated. Numerical properties can depend on the Matlab version that you use.

3.1 Static decoupling

Static decoupling is obtained by choosing $W_2(s) = I$ and $W_1(s) = G^{-1}(0)$. This implies that $\tilde{G}(s) = G(s)G^{-1}(0)$ is decoupled at $s = 0$.

The problems below should be solved both for the minimum phase and non-minimum phase case. It is suitable to start with the former.

Exercise 3.1.1. Calculate the static decoupling for the system and plot the Bode diagrams of $\tilde{G}(s)$ for verification.

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Exercise 3.1.2. Design a diagonal controller $\tilde{F}(s)$ for $\tilde{G}(s)$. Design the controllers $\tilde{f}_i(s)$ as PI controllers. The intended phase margin is $\varphi_m = \pi/3$. The intended cross-over frequency ω_c is 0.1 rad/s for the minimum phase case and 0.02 rad/s for the non-minimum phase case.

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The controller is now given by

$$F(s) = G^{-1}(0)\tilde{F}(s)$$

Exercise 3.1.3. Calculate the singular values of the sensitivity function

$$S(s) = (I + G(s)F(s))^{-1}$$

and the complementary sensitivity function

$$T(s) = (I + G(s)F(s))^{-1}G(s)F(s)$$

Is the design good with respect to sensitivity and robustness?

....

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Exercise 3.1.4. Simulate the closed-loop system in Simulink by typing `closedloop`. A Simulink window will appear where the block diagram is shown. Make sure that the variables `F` and `G` in the Matlab work-space contain the controller and the process respectively. Go to the Simulation meny and click Start. On the screen the unit step responses from the references to the outputs $y_1(t)$ (at $t = 100$) and $y_2(t)$ (at $t = 500$) are plotted together with the inputs. The time instant of the steps can be modified by clicking on the step blocks and changing the Step time. The total simulation time can be modified by changing the Stop time in the menu Simulation/Parameters. Simulation data is saved in the variable `simout` in the Matlab workspace. Is the control good? Are the outputs coupled?

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Now perform the exercises above for the non-minimum phase case.

Exercise 3.1.5. Describe the most important differences between the two cases.

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3.2 Dynamical decoupling

Dynamical decoupling can be obtained for example by choosing $W_2(s) = I$ and $W_1(s)$ in such a way that $\tilde{G}(s) = G(s)W_1(s)$ is a diagonal matrix. The conditions for $\tilde{G}(s)$ to be diagonal are the following:

$$g_{11}(s)w_{12}(s) + g_{12}(s)w_{22}(s) = 0$$
$$g_{21}(s)w_{11}(s) + g_{22}(s)w_{21}(s) = 0$$

where $w_{ij}(s)$ denote the elements of the matrix $W_1(s)$. Since there are four unknowns and and two equations we have additional degrees of freedom. A suitable procedure is to let the diagonal elements of $W(s)$ be equal to one if the RGA of $G(s)$ indicates the pairings (u_1, y_1) and (u_2, y_2). Accordingly, for other pairings it is suitable to set the other two elements equal to one. For the case $w_{11}(s) = w_{22}(s) = 1$, we have

$$w_{12}(s) = -g_{12}(s)/g_{11}(s)$$
$$w_{21}(s) = -g_{21}(s)/g_{22}(s)$$

Divisions with `ss` object is not possible in Matlab. After the divisions have been performed, we can return to state space descriptions using the function `ss`. (Notice that analytically, $\tilde{G}(s)$ is diagonal. However, numerically we can have off diagonal elements of size 10^{-16}, which can cause problems if we work with `tf` objects in Matlab).

If, by some reason, the static gain of $\tilde{G}(s)$ happens to be negative, this can be modified by changing signs of $W_1(s)$. If $W_1(s)$ becomes non-proper (for example if it contains derivations), we can still realize the dynamical decoupling for frequencies up to approximately 10 times the intended cross-over frequency ω_c using the following modification

$$W_1(s) \leftrightarrow W_1(s)\frac{10\omega_c}{s + 10\omega_c}$$

The problems below should be solved both for the minimum phase and non-minimum phase case. It is suitable to start with the former.

Exercise 3.2.1. Calculate a dynamical decoupling $W(s)$ for the system and plot the Bode diagrams of $\tilde{G}(s)$ for verification.

....

....

Exercise 3.2.2. Design a controller $\tilde{F}(s)$ for $\tilde{G}(s)$. Design the controllers $\tilde{f}_i(s)$ as PI controllers so that we have phase margin $\varphi_m = \pi/3$. The intended cross-over frequency ω_c is 0.1 rad/s for the minimum phase case and 0.02 rad/s for the non-minimum phase case.

....

....

The controller is now given by

$$F(s) = W_1(s)\tilde{F}(s)$$

Exercise 3.2.3. Calculate the singular values of the sensitivity function

$$S(s) = (I + G(s)F(s))^{-1}$$

and the complementary sensitivity function

$$T(s) = (I + G(s)F(s))^{-1}G(s)F(s)$$

Is the design good with respect to sensitivity and robustness?

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Exercise 3.2.4. Simulate the closed-loop system in Simulink. Is the control good? Are the outputs coupled?

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Now solve the above problems for the non-minimum phase case.

Exercise 3.2.5. Describe the most important differences between the two cases.

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Exercise 3.2.6. Which type of decoupling is the best for the minimum phase system and the non-minimum phase system respectively? What are the advantages and disadvantages of the static and dynamical decoupled designs?

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3.3 Glover-McFarlane robust loop-shaping

In the above exercises we combined static and dynamical decoupling with decentralized PI control. In this exercise we will continue with the design that showed to be best for each of the two cases. Alternatively, we could start all over as described above in 2.2, but we will not do that. The reason is that the Glover-McFarlane method works well for reasonably well-tuned nominal controllers.

Therefore assume that we have a nominal loop gain

$$L_0(s) = G(s)W_1(s)\tilde{F}(s)$$

obtained in the exercises above. The Glover-McFarlane method adds a link $F_r(s)$ to this loop gain so that the final controller becomes

$$F(s) = W_1(s)\tilde{F}(s)F_r(s)$$

In Matlab this link is calculated with the function

```
[Fr,gam] = rloop(L0,alpha)
```

A suitable choice for `alpha` is 1.1.

The problems below should be solved both for the minimum phase and non-minimum phase case. It is suitable to start with the former.

Exercise 3.3.1. Calculate L_0 corresponding to the best previous design procedure and plot the Bode diagrams to verify that L_0 has the intended cross-over frequency ω_c and that it is reasonably decoupled at ω_c. For the minimum phase case, ω_c is 0.1 rad/s and for the non-minimum phase case it is 0.02 rad/s.

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Exercise 3.3.2. Calculate the Glover-McFarlane controller for L_0. Are you satisfied with the γ that you have obtained?

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The controller is now given by

$$F(s) = W_1(s)\tilde{F}(s)F_r(s)$$

Exercise 3.3.3. Calculate the singular values of the sensitivity function

$$S(s) = (I + G(s)F(s))^{-1}$$

and the complementary sensitivity function

$$T(s) = (I + G(s)F(s))^{-1}G(s)F(s)$$

Describe the differences between the robust design and the nominal design in terms of the sensitivity functions. Is the robust design better with respect to sensitivity and robustness?

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Exercise 3.3.4. Simulate the closed-loop system in Simulink. Compare with the result that you obtained simulating the nominal design. What are the differences and similarities?

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Now solve the above problems for the non-minimum phase case.

Exercise 3.3.5. Describe the most important differences between the two cases.

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VETENSKAP OCH KONST
KTH

S3 – Automatic Control

2E1252 Control Theory and Practice

Computer Exercise: Model Predictive Control

Elling W. Jacobsen, February 2005

Introduction

The purpose of this computer exercise is get a hands-on experience with Model Predictive Control – MPC, and to get a feeling for how the various MPC parameters affect performance and stability.

Preparation: Chapter 16 in "Glad, T. och Ljung, L.: Reglerteori-flervariabla och olinjära metoder" and copies of slides from Lecture 13. Read through this booklet before going to the computer lab!

1 Brief Introduction to MPC

The basic idea in MPC is to use the control input u to optimize the future response of the system, given information about the current state. A finite horizon is usually considered for the future and the objective function is typically quadratic in the states and in the control inputs. If the state of the system can not be measured, one needs an observer to estimate the state. A key feature is that constraints on states, inputs and outputs can be considered in the optimization, i.e., using constrained optimization methods.

In MPC the input u is optimized over the whole horizon considered, but only the optimal value obtained for the current time is actually implemented. Then the the system is allowed to evolve one sample, new measurements are collected and the optimization is repeated. With a fixed length of the horizon, the horizon extends one sample further at each new sample. Because of this, MPC is often termed *moving horizon* control.

All computations in MPC are based on a discrete time representation of the system dynamics. On state space form the model is

$$x_{k+1} = Ax_k + Bu_k \tag{1}$$

$$y_k = Cx_k \tag{2}$$

where 1 time step corresponds to the sample time T. There is never a direct term D from input to output in MPC models. This follows from the fact that, at a given sample k, the output is already given and can hence not be affected by the present input u_k which is to be calculated by the optimizer.

A general MPC formulation is

$$\min_u f(x,u)$$

with the objective function $f(x,u) =$

$$\sum_{i=0}^{N_P-1} [(x_i - x_{ref,i})^T Q_x (x_i - x_{ref,i}) + (u_i - u_{ref,i})^T Q_u (u_i - u_{ref,i})] + (x_{N_P} - x_{ref,N_P})^T S (x_{N_P} - x_{ref,N_P}) \tag{3}$$

and subject to the equality and inequality constraints

$$x_0 = \texttt{given} \tag{4}$$

$$u_{min} \le u_i \le u_{max}, \quad i = 0, N_P - 1 \tag{5}$$

$$y_{min} \le Hx_i \le y_{max}, \quad i = 1, N_P \tag{6}$$

in which $i = 0$ corresponds to the present time (sample). Note that the same horizon is used for both inputs and outputs in this formulation. However, a penalty is imposed on the first state x_{N_P} following the optimization horizon. This plays a similar role to a longer horizon for the output than the input, as used in many MPC formulations. Also note that output errors $(y - y_{ref})$ are easily included by letting $Q_x = C^T Q_y C$ where C is the mapping from states to outputs in the state space model. The references for the states can then be obtained e.g., from $x_{ref} = pinv(C)y_{ref}$ where $pinv$ denotes the pseudoinverse.

The free variable considered in the optimization is the input vector u of length $nu \cdot N_P$ where n_u is the number of inputs and N_P is the number of samples in the considered horizon. The optimization problem is to be solved at every sample, and hence one needs efficient solvers for the problem.

The MPC problem as formulated above can be recast as a *Quadratic Program* (QP) for which highly robust and efficient solvers exist. The general QP problem is formulated as

$$\min_u u^T H u + h^T u \; ; \qquad \texttt{s.t.} \quad Lu \le b \tag{7}$$

in which the objective function is square (+linear term) and the constraint is a linear inequality. Here H (the Hessian) and L are constant matrices, h and b are constant vectors, and u is a vector variable.

To recast the MPC problem as a standard QP problem, we first note that the MPC objective function (3) and constraints (4)- (6) contain both the states x and the free variables u, while in QP only the free variables u are allowed. Thus, we need to "translate" the objectives and constraints for x into corresponding objectives and constraints for u. This is in principle trivial since the state-space model (1)-(2) provides x as a function of u (and x_0). The states x_i for $i = 1, N_P$ as a function of the current state x_0 and inputs u_i for $i = 0, N_P - 1$ are given by applying the state-space model (1)-(2) repeatedly for $i = 0, N_P$

$$x = \underbrace{\begin{pmatrix} A \\ A^2 \\ \vdots \\ A^{N_P-1} \\ A^{N_P} \end{pmatrix}}_{\hat{A}} x_0 + \underbrace{\begin{pmatrix} B & 0 & \dots & 0 & 0 \\ AB & B & \dots & \vdots & \vdots \\ \vdots & \vdots & \ddots & 0 & 0 \\ A^{N_P-2}B & A^{N_P-3}B & \dots & B & 0 \\ A^{N_P-1}B & A^{N_P-2}B & \dots & AB & B \end{pmatrix}}_{\hat{B}} u \tag{8}$$

Thus, we can insert this relationsship into the objective function and constraints to arrive at the QP form. In addition, the summing in the MPC objective function as well as the various constraints at each sample must be put on a compact matrix form by simply stacking all the relevant matrices and vectors on top of each other. See slides from Lecture 13 for details.

The main tuning parameters to consider in the MPC formulation considered here are

N_P	– optimization horizon
Q_x, Q_y	– weight on the states, or outputs
Q_u	– weight on the control input
S, S_y	– weight on "terminal" state, or output (often used to ensure stability)
u_{min}, u_{max}	– constraints on input, often given by system
y_{min}, y_{max}	– constraints on output, usually imposed by design

In addition, one usually needs to design an observer for the states and then this will also contain tuning parameters.

Below we will consider the impact of some of the above parameters on the performance and stability of MPC for a simple SISO (Single-Input-Single-Output) example.

2 Tasks

Start by downloading the files `mpc_setup.m`, `mpc_sim.m` and `mpc_calc.m` from `http://www.s3.kth.se/control/kurser/2E1252/download.shtml`
Make sure to put these in your working directory, i.e., from where you start `Matlab`.

The MPC calculations in `mpc_calc.m` involve solving a QP problem and for this task it relies on the function `quadprog` from the `Matlab Optimization Toolbox`. If you do not have this toolbox available you should do the exercise in the XQ computer labs.

We shall consider controlling a system described by the model

$$Y(s) = \frac{1}{s(s+0.1)}\frac{10}{10s+1}U(s)$$

and using the sampling time $T = 0.2\ s$. For simplicity we will assume that all the states are available, e.g., from a separate observer.

Throughout the exercise we will assume that the following parameters apply

$$u_{min} = -1,\ u_{max} = 1,\ u_{ref} = 0,\ Q_y = 1$$

while we will vary the other parameters.

To run the simulations in the tasks below, you should follow the procedure

1. Modify the relevant parameters in `mpc_setup.m` and then run `mpc_setup`.
2. Perform a simulation by running `mpc_sim`. The number of samples considered in a simulation can be changed by changing the parameter `NN` in `mpc_sim.m` prior to running the simulation.

2.1 Prediction horizon

We shall here use the parameters

$Q_y = 1,\ Q_u = 1,\ S_y = 1,\ u_{min} = -1,\ u_{max} = 1,\ y_{min} = -10,\ y_{max} = 10,\ u_{ref} = 0,\ y_{ref} = 1$

Here S_y is the weight on the "terminal" output y_{NP} and corresponds to a weight $S = C^T S_y C$ on the terminal state x_{NP}.

Task 2.1. Edit the file `mpc_setup.m`: enter the model $G(s)$ and the samping time T, as given above. Also enter the parameters as given above. Run `mpc_setup` and then `mpc_sim` to verify that the programs work properly.

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Task 2.2. We shall start with a prediction horizon of 20 samples, i.e., $N_P = 20$. Change the corresponding parameter `Np` in `mpc_setup.m` and then run a simulation (run `mpc_setup` and then `mpc_sim` as described above). Describe the response; rise time, overshoot and active constraints (if any).

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Task 2.3. To reduce the computational time for the optimization one may try to reduce the prediction horizon N_P (the horizon is also often considered the most important tuning parameter in MPC). Try to reduce the prediction horizon to $N_P = 10$ and repeat the simulation. What happens? Consider also reducing it further to $N_P = 7$. Try to explain the observed behavior.

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Task 2.4. Try now with a very long horizon, $N_P = 200$. What happens with the computational time? Are any constraints active at any time? Considering the objective function in (3), what type of control do we in principle get with a long horizon and no active constraints?

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2.2 Weighting the "terminal" state

The weight S on the terminal state x_{NP}, i.e., the state at the end of the prediction horizon, is also an important tuning parameter. In particular, by increasing S one can in general improve stability. We here consider a weight S_y on the terminal output y_{NP} which can be translated into S as described above.

Task 2.5. Change the prediction horizon to $N_P = 7$ and increase the terminal weight to $S_y = 5$. Run a simulation. Comparing with the result for $N_P = 7$ and $S_y = 1$ above, what is the effect of increasing S_y? How large must S_y be chosen to obtain stability?

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Task 2.6. Use $N_P = 7$ and $S_y = 20$. Perform first a simulation with $y_{ref} = 1$ (as before), and then a simulation with $y_{ref} = 3$. What happens when you increase the size of the setpoint change? Why is stability dependent on the size of the reference?

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Task 2.7. Consider now increasing the prediction horizon to $N_P = 20$ and decreasing the terminal weight to $S_y = 1$. Perform setpoint changes corresponding to $y_{ref} = 1$ and $y_{ref} = 3$ as above. From a stability point of view, what would you recommend;

- a short prediction horizon with large penalty on the terminal state, or
- a long horizon with no particular penalty on the terminal state?

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2.3 Weighting the input

Just like in LQG, the relative weighting of the output and input in the objective function is an important tuning parameter. We here consider a fixed weight on the output $Q_y = 1$ and study the effect of changing the input weight Q_u.

Task 2.8. Use $N_P = 20$ and $S_y = 1$. Perform step responses for $y_{ref} = 1$ and with input weight $Q_u = 1,\ 0.1.,\ 0.01$ and $1d - 6$, respectively (4 simulations in total). How does Q_u affect the response? What does the control look like when the weight on the input approaches zero?

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2.4 Imposing Constraints on the Output

Task 2.9. Use $N_P = 20,\ S_y = 1,\ Q_u = 0.01$. Peform step responses for $y_{ref} = 1$ while imposing constraints $y_{max} = -y_{min} = 1.05,\ 1.0$ and 0.9 respectively. Is the controller able to satisfy all constraints for all time?

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2.5 Infeasible Constraints

A constrained optimization problem, as the one solved in MPC at every sample, may not have a solution that satisfies all constraints. In this case, the computations break down. In some algorithms, like `quadprog` used in `mpc_calc`, an approximate "best" solution is provided when the problem is infeasible.

The problem with infeasibility is more likely to occur with a short prediction horizon ("bad planning")[1].

Task 2.10. Use $N_P = 10$, $S_y = 1$, $-y_{min} = y_{max} = 1.5$. Perform simulations with $y_{ref} = 1.5$ and $Q_u = 0.1$ and $Q_u = 0.01$, respectively. What happens when the weight on the input is reduced?

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2.6 Fine Tuning the MPC

Task 2.11. Try to tune the MPC controller, i.e., select a set of parameters for the MPC problem, that gives the "best" response to setpoint changes y_{ref} in your opinion. Motivate your choice.

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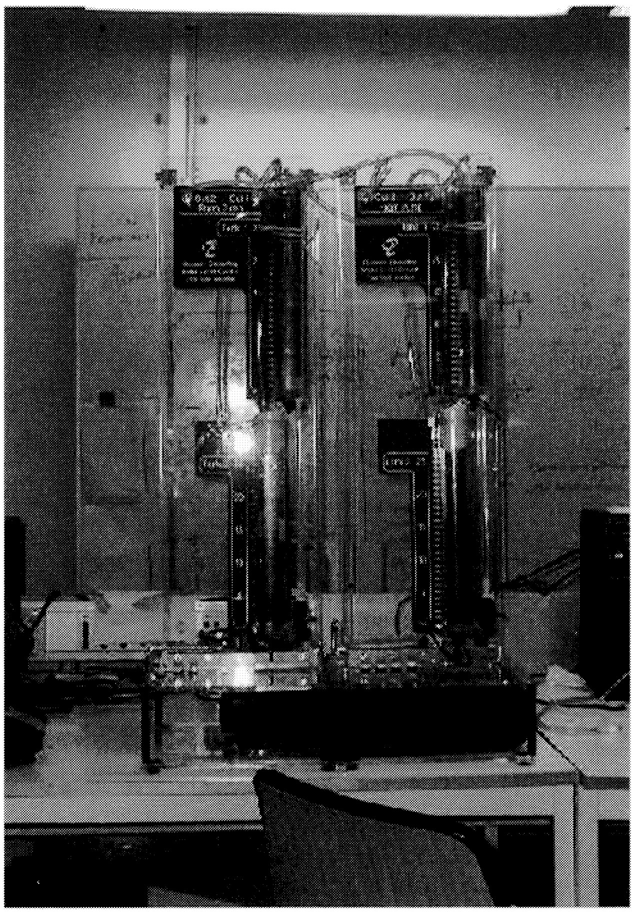
[1]In some MPC algorithms, constraints are imposed also beyond the optimization horizon to deal with this problem. In this case one must make assumptions on the input for $i > N_P$, i.e., $u = u_{ref}$ or $u_i = -Lx_i$ (state feedback).

EE – Reglerteknik



Control Theory and Practice
Advanced Course

Laboratory experiment: The four-tank process

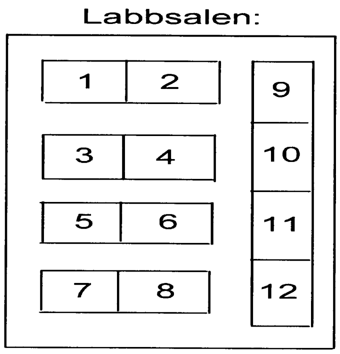
Anders Hansson, Ola Markusson och Magnus Åkerblad, December 1999
Revised by Jonas Wijk, December 2002

1 Introduction

In this laboratory experiment we will control the four-tank process, which is a multivariable system. In particular we will investigate interactions between inputs and outputs as well as properties of non-minimum phase dynamics.

The experiment is divided into two occasions. On the first occasion, the modelling and the manual control is performed. This will give you numerical values of some important parameters. Relevant parts of the computer exercises are then repeated, using the identified parameters, to obtain model based controllers. On the second occasion, the control design is investigated. In the Appendix you find information about the computer program used to control the process.

Signing up for the laboratory experiment: It is very important that you sign up for the same "Group letter" on both laboratory occasions. The Group letter determines which double-tank processes to be connected into a four-tank process. Group A uses process 1 and 2, group B uses process 3 and 4... and group F uses process 11 and 12. The figure to the right defines the double-tank processes in the laboratory hall.

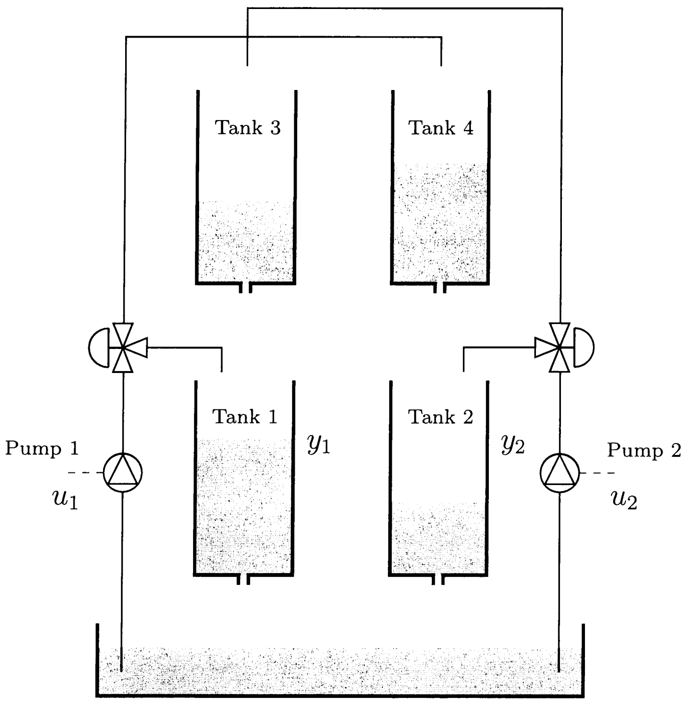


Preparations for occasion 1: Chapters 3.3, 3.5, 3.6, 6.5, 7.3-7.5, 7.7, 8.3, 10.5 in Glad, T. and Ljung, L.: Control theory—multivariable and nonlinear methods. Read the lab instructions and solve the problems that can be solved in advance. Also read the Appendix. (This way you will save **much** time.) Bring a watch or a cell phone with a stopwatch to the lab.

Preparations for occasion 2: Relevant parts of the computer exercises are repeated using the parameters identified during lab occasion 1. Design suitable decentralized controllers according to the instructions in the computer exercises. One controller for the minimum phase case and one for the non-minimum phase case. In addition to this, construct a robustified controller (for both cases) using the Glover-McFarlane method. Altogether, you will use four controllers. Bring a USB stick to the lab containing Matlab files and generate the controllers on the lab computer. On page 8 it is described how to name and save the controllers. **NB: you have to bring you own USB stick to the laboratory hall.**

Presentation: All problems in this laboratory experiment should be solved and presented in a written report. The date when the report should be handed in is indicated in the course instructions. The report should contain all relevant figures. Don't forget to save data for the plotting of figures.

In the exercises below a physical model of the four-tank process will be constructed. Then, we will investigate manual control and coupling between the tanks. Performance limitations due to non-minimum dynamics will be investigated. Finally, we will design model based controllers, more specifically decentralized PI control and robust control using the Glover-McFarlane method.



Figur 1: The four-tank process

2 Laboratory occasion 1

2.1 Modelling

Here the nonlinear differential equations which describe the four-tank process will be derived. The process is shown schematically in Figure 1. For each tank the following relation holds:

$$dV = (q_{in} - q_{out})dt$$

where dV is the change in water volume during the time dt. Divide this equation by dt and assume that $V = Ah$ where A is the cross section area of the tank and h its water level. The we obtain

$$A\frac{dh}{dt} = q_{in} - q_{out}$$

For the outflow of water, Bernoulli's law holds:

$$q_{out} = a\sqrt{2gh}$$

where a is the cross section area of the outlet hole and $g = 981\text{cm/s}^2$. The flow q generated by a pump is considered proportional to the applied pump voltage u:

$$q = ku$$

where k is the constant. This flow is then divided according to

$$q_L = \gamma ku, \qquad q_U = (1-\gamma)ku, \qquad \gamma \in [0,1]$$

where γ indicates the setting of the valve which is connected to the pump. q_L denotes the flow to the lower tank and q_U is the flow to the upper tank.

Exercise 2.1.1. Show that the following equations describe the water levels in the four tanks.

$$\begin{aligned}
\frac{dh_1}{dt} &= -\frac{a_1}{A_1}\sqrt{2gh_1} + \frac{a_3}{A_1}\sqrt{2gh_3} + \frac{\gamma_1 k_1}{A_1}u_1 \\
\frac{dh_2}{dt} &= -\frac{a_2}{A_2}\sqrt{2gh_2} + \frac{a_4}{A_2}\sqrt{2gh_4} + \frac{\gamma_2 k_2}{A_2}u_2 \\
\frac{dh_3}{dt} &= -\frac{a_3}{A_3}\sqrt{2gh_3} + \frac{(1-\gamma_2)k_2}{A_3}u_2 \\
\frac{dh_4}{dt} &= -\frac{a_4}{A_4}\sqrt{2gh_4} + \frac{(1-\gamma_1)k_1}{A_4}u_1
\end{aligned}$$

where index i in A_i, a_i and h_i refer to tank i and index j in k_j and γ_j refer to pump j and valve j.

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Assume that the levels in the lower tanks are measured by sensors for which the output voltages y_i are proportional to the water levels h_i:

$$y_i = k_c h_i$$

where k_c is a constant.

Exercise 2.1.2. Write down the equations which describe an equilibrium $u_1^0, u_2^0, h_1^0, h_2^0, h_3^0, h_4^0, y_1^0, y_2^0$ for the tanks.

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Let $\Delta u_i = u_i - u_i^0$, $\Delta h_i = h_i - h_i^0$ and $\Delta y_i = y_i - y_i^0$ denote the deviations from an equilibrium. Introduce

$$u = \begin{bmatrix} \Delta u_1 \\ \Delta u_2 \end{bmatrix}, \quad x = \begin{bmatrix} \Delta h_1 \\ \Delta h_2 \\ \Delta h_3 \\ \Delta h_4 \end{bmatrix}, \quad y = \begin{bmatrix} \Delta y_1 \\ \Delta y_2 \end{bmatrix}$$

Exercise 2.1.3. Show that the linearized system is given by

$$\begin{aligned}
\frac{dx}{dt} &= \begin{bmatrix} -\frac{1}{T_1} & 0 & \frac{A_3}{A_1 T_3} & 0 \\ 0 & -\frac{1}{T_2} & 0 & \frac{A_4}{A_2 T_4} \\ 0 & 0 & -\frac{1}{T_3} & 0 \\ 0 & 0 & 0 & -\frac{1}{T_4} \end{bmatrix} x + \begin{bmatrix} \frac{\gamma_1 k_1}{A_1} & 0 \\ 0 & \frac{\gamma_2 k_2}{A_2} \\ 0 & \frac{(1-\gamma_2)k_2}{A_3} \\ \frac{(1-\gamma_1)k_1}{A_4} & 0 \end{bmatrix} u \\
y &= \begin{bmatrix} k_c & 0 & 0 & 0 \\ 0 & k_c & 0 & 0 \end{bmatrix} x
\end{aligned}$$

where

$$T_i = \frac{A_i}{a_i}\sqrt{\frac{2h_i^0}{g}}$$

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Exercise 2.1.4. Show that the transfer matrix from u to y is given by

$$G(s) = \begin{bmatrix} g_{11}(s) & g_{12}(s) \\ g_{21}(s) & g_{22}(s) \end{bmatrix} = \begin{bmatrix} \frac{\gamma_1 k_1 c_1}{1+sT_1} & \frac{(1-\gamma_2)k_2 c_1}{(1+sT_3)(1+sT_1)} \\ \frac{(1-\gamma_1)k_1 c_2}{(1+sT_4)(1+sT_2)} & \frac{\gamma_2 k_2 c_2}{1+sT_2} \end{bmatrix}$$

where $c_i = T_i k_c / A_i$.

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Exercise 2.1.5. The zeros of $G(s)$ are given by the zeros of

$$\det G(s) = \frac{k_1 k_2 c_1 c_2}{\Pi_{i=1}^4 (1+sT_i)} \left[(1+sT_3)(1+sT_4) - \frac{(1-\gamma_1)(1-\gamma_2)}{\gamma_1 \gamma_2} \right]$$

Show that $G(s)$ is minimum phase if $1 < \gamma_1 + \gamma_2 \leq 2$ and that $G(s)$ is non-minimum phase if $0 < \gamma_1 + \gamma_2 \leq 1$.

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Exercise 2.1.6. Show that the RGA of $G(0)$ is given by

$$\begin{bmatrix} \lambda & 1-\lambda \\ 1-\lambda & \lambda \end{bmatrix}$$

where $\lambda = \gamma_1\gamma_2/(\gamma_1 + \gamma_2 - 1)$. In the minimum phase case we have $\gamma_1 = \gamma_2 = 0.625$ and in the non-minimum phase case we have $\gamma_1 = \gamma_2 = 1 - 0.625 = 0.375$. Calculate the RGA matrix for both these cases.

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All tanks have cross section area $A = 15.52$ cm^2. However, their effective outlet hole areas vary slightly, and therefore these have to be determined experimentally. We will use different outlet hole sizes in the upper tanks depending on if we are studying the minimum phase or non-minimum phase case. (The outlet holes in the two lower tanks should always have the same size). This means that we will have to determine six

(effective) outlet hole areas altogether.
The level sensors have the proportionality constant $k_c = 0.2\ \frac{V}{cm}$. For the minimum phase case, $\gamma_1 = \gamma_2 = 0.625$ and for the non-minimum phase case we have $\gamma_1 = \gamma_2 = 1 - 0.625 = 0.375$.

In order to determine the remaining parameters, (a_1 a_2 $a_{3,min}$ $a_{3,nonmin}$ $a_{4,min}$ $a_{4,nonmin}$ k_1 k_2), experiments will be performed.
It is important to prepare proposals for suitable experiments (in order to solve the problems stated below) before performing the laboratory exercise. Then, in order to perform the experiments, the four-tank process will be set up according to the following instructions:

1. Connect the components of the four-tank process according to the instructions in Appendix A.1.
2. Turn on the computer[1] and login as "student" with password "sommar".
3. Connect the minimum phase case according to the instructions in Appendix A.2.
4. Double-click the icon "Quadrupletank" at the Desktop.
5. The program starts by asking if any controllers are to be loaded. Answer no by typing "n" and pressing enter.
6. Start the program by pressing the green Start button.
7. Turn on the two UPM's by pushing the buttons at the back.
8. Choose 50 (% of maximum voltage) of the control signals in the boxes "Control sig. pump 1/2", and check that water is pumped into all tanks.

Exercise 2.1.7. Propose a suitable experiment[2] to determine k_1 and k_2 $[\frac{cm^3}{s \cdot V}]$, and perform it.

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[1]If the computer does not start this is probably because the electricity is turned off. The main switch is turned on by turning the key "TILL" on the electric board at the front of the hall to the left.

[2]Tip: there might be air in the tubes, even when the pumps are on. Let the water flow and squeeze the tubes carefully to eliminate the air. A good idea is to run the experiment with u_1=u_2=7.5 V (50% of maximum voltage).

Exercise 2.1.8. Propose a suitable experiment to determine the four effective outlet hole areas a_i for the minimum phase case, and perform it. (In order to save time, we will determine $a_{3,nonmin}$ and $a_{4,nonmin}$ later.)

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2.2 Manual control

Solve the problems below for both the minimum phase and the non-minimum phase case. It is suitable to start with minimum phase.

Exercise 2.2.1. Set the pumps on 50 (% of maximum voltage). Wait until stationarity and read the levels on all four tanks from the scale indicated in cm [3] on the four-tank process. Are the levels (fairly) in accordance with the calculations on the equations in Exercise 2.1.2?

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Exercise 2.2.2. Study the step responses (the two outputs) from one input at a time for the two cases (minimum and non-minimum phase). Does the system seem to be coupled? Is this in accordance with the indications of the RGA?

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Exercise 2.2.3. Choose suitable reference levels for the two lower tanks, for example 15 cm (60% of full tank). Try to manually set the pump voltages so that the values displayed on the computer screen [4] become equal to the reference values. How long is the transient time? Hint: patience is required for the non-minimum phase case. (If you have not succeeded after 10 minutes, skip it and move on.)

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[3] The level sensors are not calibrated exactly, and therefore the value you read from the scale does not correspond exactly to the value displayed on the screen.

[4] Because of sensitive technics of measurement, it might happen that the signals from the level sensors are subject to small jumps every now and then, so called "offset jumps".

1. Connect the components according to the non-minimum phase case described in Appendix A.2.

2. Go back to Exercise 2.1.8 and determine $a_{3,nonmin}$ and $a_{4,nonmin}$.

3. Now repeat the exercises above for the non-minimum phase case.

Exercise 2.2.4. For the above exercises, what are the most important differences between the minimum phase and the non-minimum phase case?

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The four-tank process will now be disconnected in the following way:

1. Turn off the computer and the UPM:s.

2. Disconnect the four-tank process according to the instructions in Appendix B.

3. Make sure that the laboratory spot is nice and clean.

4. If nobody else is working in the lab hall, turn off the main switch of the hall. It is done by pushing the red button "FRÅN" at the electric switch board located at the front of the hall to the left. Turn the light off if you are the last person to leave the hall.

5. Do the preparation tasks for the next laboratory occasion (they are described on the next page.)

3 Calculation of controllers

Before laboratory occasion 2, four controllers will be calculated.

Exercise 3.0.5. The values of the effective outlet hole areas, the k_i and γ_i (i = 1,2) that you obtained will now be used to calculate controllers for the next laboratory occasion. Therefore, change the values in the files `minphase.m` and `nonminphase.m`. After that, repeat relevant parts of the computer exercise to obtain four controllers. Two controllers for the minimum phase case and two for the non-minimum phase case. In each phase-case, the first controller should be the decentralized controller which you thought was the best when performing the computer exercise. Do not forget to motivate your choice. The second controller should be a robustified Glover-McFarlane controller.

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The controllers are saved as `.MAT` files using the function `save` (type `help save` for more information). The files must be named `reg1.MAT`, `reg2.MAT`, `reg3.MAT` and `reg4.MAT` and must contain state space representations of controllers. The state space matrices must be named A, By, C and Dy. **NB:** because different versions of MATLAB are not compatible, the controller F should be generated in MATLAB **on the computer in the laboratory hall**. This means that you bring your code on an USB stick and run it on the lab computer. Then, the controllers can be saved as `.MAT` files using the `save` function on the lab computer.

If the controller F is available on transfer function form, the following matlab code can be used:

```
F=ss(F,'min');
[A,By,C,Dy]=ssdata(F);
save regX.MAT A By C Dy
```

The transfer of the matlab code to the laboratory computer is made with a portable disc or a USB memory stick.

4 Laboratory occasion 2

4.1 Decentralized control

The exercises below require that you have repeated the design process in the computer exercises with the parameters obtained on the previous lab occasion. Make sure that small step responses and load disturbances do not cause saturation of the control signals. Start by setting up the four-tank process in the following way:

1. Use the same laboratory equipment as in laboratory occasion 1 to connect the four-tank process according to the instructions in Appendix A.1.
2. Turn on the computer and login as "student" with the password "sommar".
3. Connect the minimum phase case according to the instructions in Appendix A.2.
4. Insert the USB stick, generate and save the four controllers (according to the instructions in Section 3).
5. Double-click the icon "Quadrupletank" on the Desktop.
6. The program starts by asking if you want to load controllers. Answer yes by typing "y" and then press enter. Locate and select `reg1.MAT` (the remaining controllers will then load automatically).
7. Start the program by pushing the green Start button.
8. Turn on both UPM:s by pushing the button at the back.
9. Choose 50 (% of maximum voltage) of the control signals in the boxes"Control sig. pump 1/2", and check that water is being pumped into all tanks.

Solve the exercises below for both the minimum phase and non-minimum phase case. It is suitable to start with the minimum phase case.

Exercise 4.1.1. Wait until stationarity. Choose the best decentralized PI controller. Choose Automatic in the popup menu "Operational Mode". Make sure that you work with small deviations from these levels, about 5 percentage points. Investigate the system's response from a step in one of the reference signals. What is the rise time and the overshoot? Also, investigate the system's response to different load disturbances: pour a cup of water in one of the lower tanks; open an extra outlet in one of the upper tanks. How long time does it take for the controller to eliminate the load disturbances?

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Now connect the non-minimum phase case according to the instructions in Appendix A.2. Then repeat the exercise above for the non-minimum phase case. (Problems can occur when opening an extra outlet in one of the upper tanks. In that case, specify what kind of problems that you get.)

Exercise 4.1.2. For the exercises above, what are the most important differences between the minimum phase and non-minimum phase case?

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4.2 Robust control

The exercises below require that you have repeated the design procedures in computer exercise 4 using the parameters obtained previously in this laboratory experiment.

Solve the problems below for both the minimum phase and the non-minimum phase case. It is suitable to start with the latter, since the laboratory equipment now is connected according to the non-minimum phase case.

Exercise 4.2.1. Wait until stationarity. Choose the Glover-McFarlane controller calculated according to the instructions in computer exercise 4. Choose Automatic in the pop up menu "Operational Mode". Make sure that you work with small devations from these levels. Investigate the system's response to a step in one of the reference signals. What is the rise time and the overshoot? Also, investigate responses from different load disturbances: pour a cup of water in one of the lower tanks; open an extra outlet in one of the upper tanks. How long time does it take for the controller to eliminate the load disturbances?

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Exercise 4.2.2. What are the most important differences in performance when comparing the different controllers?

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Connect the minimum phase case according to the instructions in Appendix A.2. Then repeat the exercises above for the minimum phase case.

Exercise 4.2.3. For the above exercises, what are the most important differences between the minimum phase and the non-minimum phase case?

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Before you start writing the laboratory report, you have to disconnect the laboratory process. Perform the following steps:

1. Turn off the computer and the UPM:s.
2. Unscrew the plugs which are located under the two upper tanks. For each process, place the plug at the bottom to the left with the other extra plugs.
3. Plug the middle hole in both upper tanks. It is <u>not</u> necessary to pull tight. (Remember that the processes are fragile.)
4. Disconnect the four-tank process according to the instructions in Appendix B.
5. Make sure that the laboratory spot is nice and clean.
6. If there is nobody else working in the laboratory, turn off the main switch of the hall. This is done by pushing the red button "FRÅN" at the electric switch board at the front of the hall to the left. Turn off the light if you are the last person to leave the hall.
7. Write the report!

A Manual for the four-tank process

This manual is divided into three parts. The first part describes how to connect the process. The second describes how to connect the minimum phase and non-minimum phase case. The third part deals with the graphic user interface.

A.1 How to connect the components of the four-tank process

The four-tank process consists of two double-tank processes connected to each other. The double-tanks are used for the laboratory experiments in the basic control course. Now we will describe how to connect the components of the four-tank process.

- Find the two double-tank processes that you are going to use. (Your group letter decides which two process that you are going to use, see "Signing up for the laboratory experiment" on page 1.) From now on, we will denote these two process "left" and "right", as seen from the front. Carefully remove the blue water bowls. Release the right process by disconnecting the two cords connected to it. Carefully put the right process as close to the left process as possible. (Put both processes at the same table, so that there is no differences in altitude between them). On top of the cupboard at the back of the hall, there is a bigger water bowl which you will place under both the two lower tanks. Fill it with water, almost to the top.
- We will only use the computer of the left process, and we will soon connect the measurement and control signals. However, we will use the UPM:s (Universal Power Module) of both processes. There is an I/O card which belongs to the computer, and you find it at the back of the computer. We will use the card's analogue Input and Output sockets. The left process should already be correctly connected, but to be sure we will verify it. Its control signal [5] should be connected to the card's "Analog Output kanal 0". Its level sensor for the lower tank[6] should be connected to the card's "Analog Input kanal 5". The remaining three level sensor cords should not be connected. Disconnect the cords which are connected to the I/O card of the right computer (not the broad grey flat band cables). Move the UPM of the right process closer to the left one, so that its cords can be connected to the I/O card of the left computer. Connect the control signal of the right process to the card's "Analog Output kanal 1" and the lower tank level sensor of the right process to "Analog input kanal 4". Finally, connect the two non-connected cords [7] from the right UPM to the right process. If you are unsure: check once again that you have connected the process correctly!

(At the end of both laboratory occasion 1 and 2, you will disconnect the four-tank laboratory. Reserve 20 minutes for that procedure.)

[5] The black cord which is connected to "From D/A" at the UPM.

[6] The white socket, (marked with 2), at the broad socket with 4 channels which is connected to "To A/D" at the UPM.

[7] The black cord connected to "To Load" at the UPM and the grey cord.

A.2 Minimum phase and non-minimum phase settings

Depending on which phase case you are working with, you should use different outlet holes of the upper tanks, and connect the cords differently. Below you find a description of this procedure.

For each double-tank process, there are three extra outlet plugs (besides the those that are already screwed under the tanks). One located far to the left which has no holes, one in the middle which has a small hole and finally one to the right which has a large hole.

In the minimum phase case, each pump pumps most of the water into "its own" lower tank, and only a smaller fraction of water into the upper tank at the other side. With this setting, we obtain a γ larger than 0.5. In the non-minimum phase case we have the opposite situation, and γ is therefore less than 0.5.

The minimum phase case is connected in the following way.

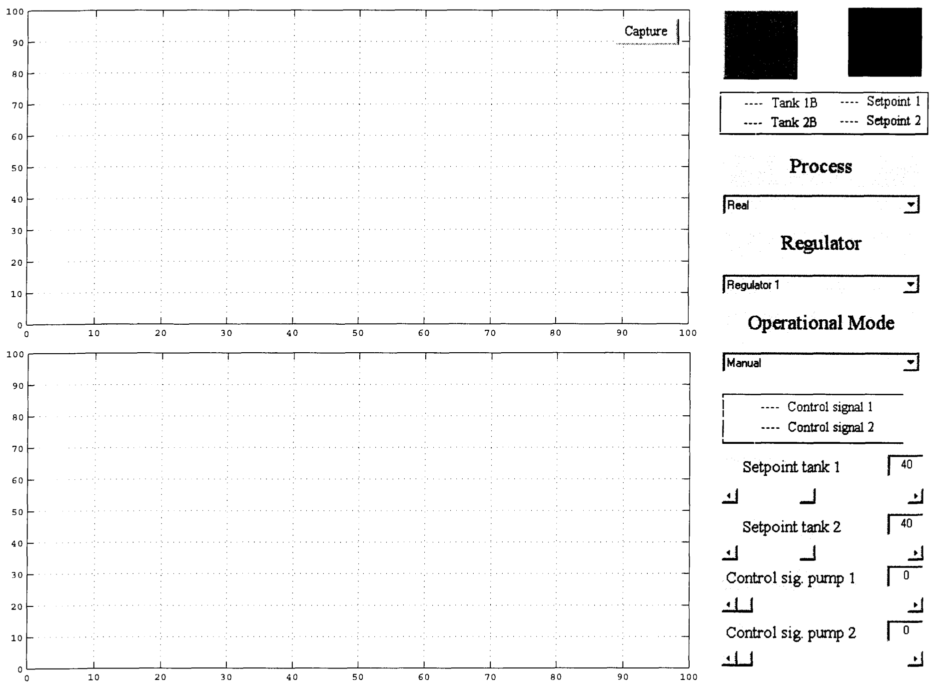
- Put something on top of the two lower tanks, for example a paper towels. (To make sure that nothing falls down into them [8].)
- Use the key attached to the process to unscrew the plugs under both the upper tanks. Place each screw among the set of extra screws on each process (to the left of the plug located at the far right side).
- Put the small outlet holes in both the upper tanks. It is **not** necessary to pull tight. (Remember that the processes are fragile.)
- The four tubes, which will be connected according to the instructions below, have to end approximately 27 cm above the bottom of the upper tanks[9]. It is a good idea to pull the tubes through the holes located at the top of the four-tank process, so that the tubes go partly at the back of the process.
- Pull the tube from "Out 1" at the left process to the extra plastic pipe next to tank 3, so that its water falls directly into tank 1. (The numbering of the tanks is given in Figure 1 on page 2.)
- Pull the tube from "Out 2" at the left process to tank 4.
- Pull the tube from "Out 1" at the right process to the extra plastic pipe next to tank 4, so that its water falls directly down in tank 2.
- Pull the tube from "Out 2" at the right process to tank 3.

[8] If anything falls into the tanks, one has to disconnect tubes and cords from that process. Then one has to tilt the process very carefully so that the item that fell into the tank falls out.

[9] If the tubes end at different altitudes, the constants k_1, k_2, γ_1 and γ_2 can be affected, because the driving force acting on the water is changed.

The non-minimum phase setting is obtained in the following way.

- Put something on top of the two lower tanks, for example a paper towels. (To make sure that nothing falls down into them).
- Unscrew the plugs located under the two upper tanks. For each process, place the plug among the set of extra screws (to the right of the plug located at the far left side).
- Put the medium size holes in the two upper tanks. It is **not** necessary to pull tight. (Remember that the processes are fragile.)
- The four tubes, which will be connected according to the instructions below, have to end approximately 27 cm above the bottom of the upper tanks. It is a good idea to pull the tubes through the holes located at the top of the four-tank process, so that the tubes go partly at the back of the process.
- Pull the tube from "Out 2" at the left process to the extra plastic pipe next to tank 3, so that its water flows directly into tank 1.
- Pull the tube from "Out 1" at the left process to tank 4.
- Pull the tube from "Out 2" at the right process to the extra plastic pipe next to tank 4, so that its water falls directly into tank 2.
- Pull the tube from "Out 1" at the right process to tank 3.



Figur 2: The graphic user interface.

A.3 The graphic user interface

- The interface is opened by double-clicking on the Quadrupletank matlab icon at the desktop.
- The program starts by asking if you want to load controllers. Choose yes or no depending on if you have designed controllers or not.
- Click on the green or red button at the top to the right to start or stop the process.
- Click on the popup menu `Regulator` to choose controller. (This popup menu is only displayed if you have loaded controllers.)
- The tank levels and the reference signals are plotted in the upper graph, and the control signals are plotted in the lower. The upper graph is scaled in percentage of full tank, so that 100 corresponds to 25 cm. The lower graph is scaled in percentage of maximal control signal, so that 100 corresponds to 15 V.
- In the popup menu `Operational Mode` you can switch between manual and automatic control.
- In order to change the manual control signal you could either pull the handle or type directly in the box.
- To change the reference value you can either pull the handle or type directly in the box.

- By right-clicking at an axis, a dialog box for zooming is opened. You can also use the capture button to study and zoom collected data.
- The `Time Offset` at the bottom to the left shows the time between 0 and the time displayed on the x axis. To obtain the true time at the x axis, you therefore add the offset value. (The time $t = 0$ is the time when you start the program using the green start button.)
- The Capture button is used to study collected data during operation. Click on the button to obtain a figure with reference signals and measured signals. (Use the zoom tool in the figure menu to zoom).
- Save data by clicking on the capture button. Then you obtain a box where you can save data. Data (time, measured signals, reference signals and control signals) is saved on the USB stick as "data.mat". If you already have a file with that name, the name becomes data1.mat etc up to data3.mat. (After that you have to use another USB stick). The data is loaded into Matlab with the function "load dataX.mat". Then you obtain a variable containing the saved information.

B How to disconnect the four-tank process

When disconnecting the four-tank process it is very important that the middle size holes are screwed in the upper tanks. Check this and therefore go through the items below:

1. Empty the large water bowl and put it on top of the cupboard at the back of the hall.

2. Disconnect the tubes between two double-tank processes.

3. Make sure that no tube is connected to "Out 2".

4. Connect a tube to "Out 1", put it through some hole so that it comes out at the back of the process, and then through the hole which is just above the upper tank. Then put the tube into the upper tank. Do this for both processes.

5. Remove the cables of the right UPM from the I/O card of the left computer. Also, disconnect the right process by disconnecting the two cords connected to it.

6. Put back each process (with its UPM) at its original spot and put a blue water bowl under each double-tank process.

7. Connect the level sensors to each tank[10] at the card's "Analog Input kanal 4" and "... 5" respectively. (The yellow at 4 and the white at 5.)

8. Connect each control signal [11] to "Analog Output kanal 0" at each I/O card.

9. Finally, connect the two non-connected cords[12] from the right UPM to the right process.

10. Go back to the instructions on page 7 (laboratory occasion 1) or page 11 (laboratory occasion 2).

[10]The yellow and white sockets, (marked with 1 and 2 respectively), at the broad cord with 4 channels connected to "To A/D" at the UPM

[11]The black cord connected to "From D/A" at the UPM.

[12]The black cord connected to "To Load" at the UPM and the grey cord.