EL2520
Control Theory and Practice

Lecture 6:
Decentralized control and decoupling

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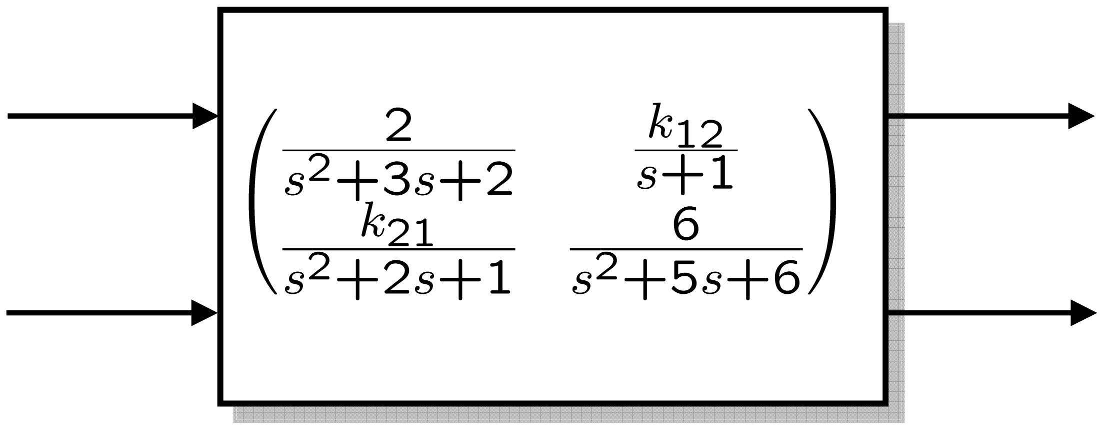
Course structure

Three parts

1. SISO control revisited
2. Multivariable control
    a. Multivariable linear systems
    b. H_∞ optimal control
    c. Linear quadratic control
    d. Design example and relation via H_2
    e. Decentralized control and decoupling
    f. Glover-McFarlane loop shaping
3. Systems with hard constraints

Motivating example

How could we control the following two-input/two output system?



$$\begin{pmatrix} \frac{2}{s^2+3s+2} & \frac{k_{12}}{s+1} \\ \frac{k_{21}}{s^2+2s+1} & \frac{6}{s^2+5s+6} \end{pmatrix}$$

If k_{12} and k_{21} are small, it is convenient to use decentralized control

– design controllers for loops in isolation

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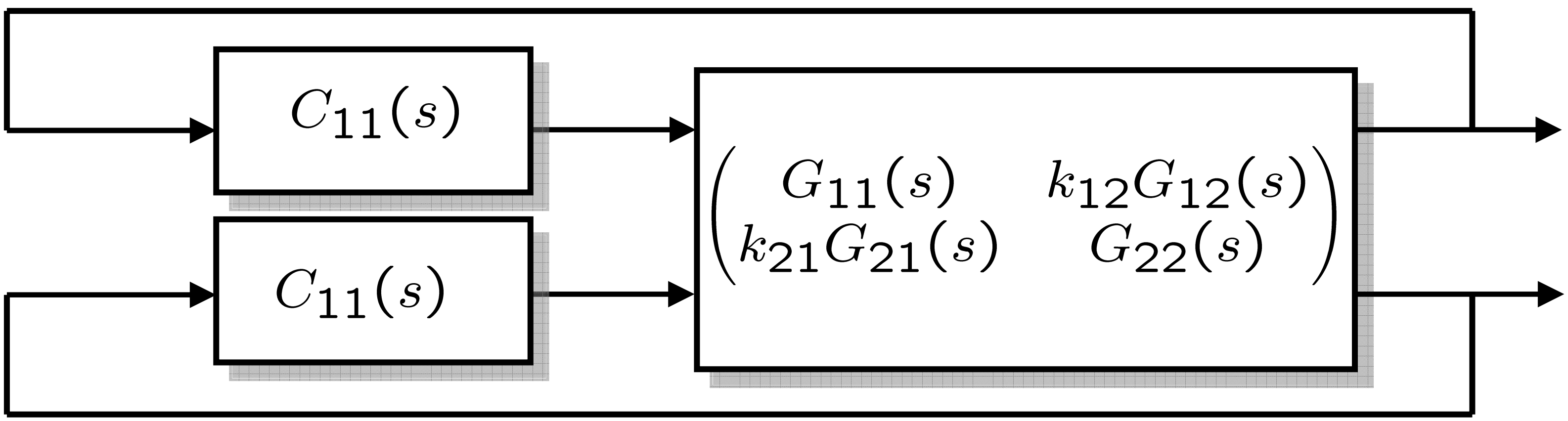
Motivating example cont'd



When $k_{12}=k_{21}=0$, then

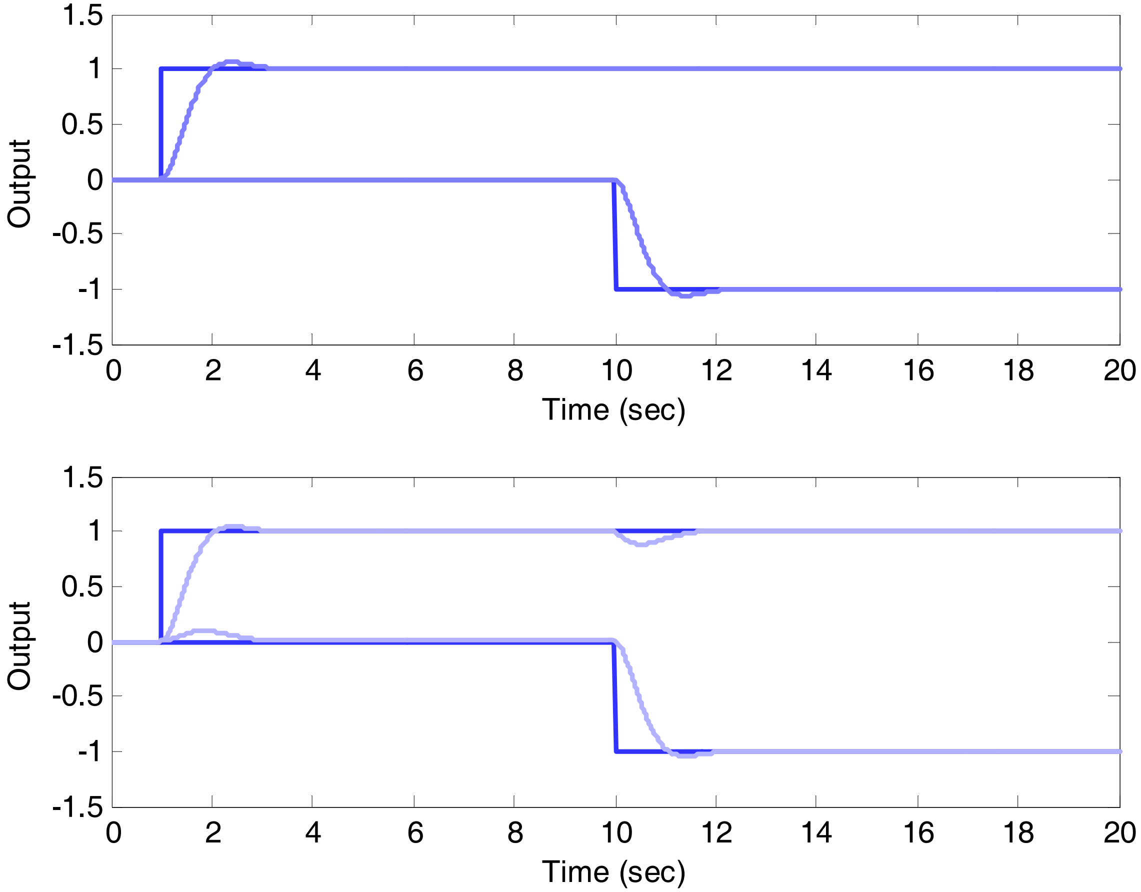
$$C_{11}(s) = \frac{4.5(s^2+3s+2)}{s(s+4)}, \qquad C_{22}(s) = \frac{1.5(s^2+5s+6)}{s(s+4)}$$

gives

$$T_{11}(s) = T_{22}(s) = \frac{9}{s^2+4s+9}$$

Motivating example cont'd

Step responses for $k_{12}=k_{21}=0$ and $k_{12}=k_{21}=0.25$



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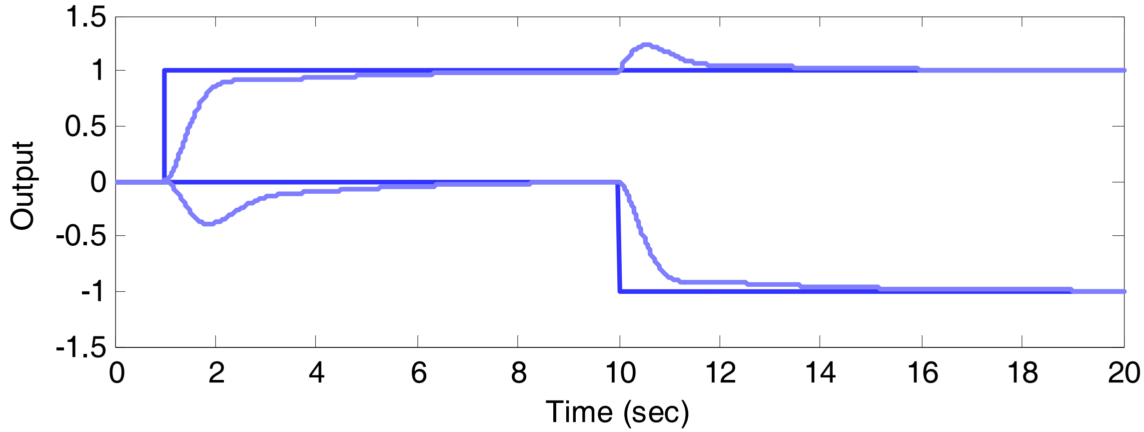
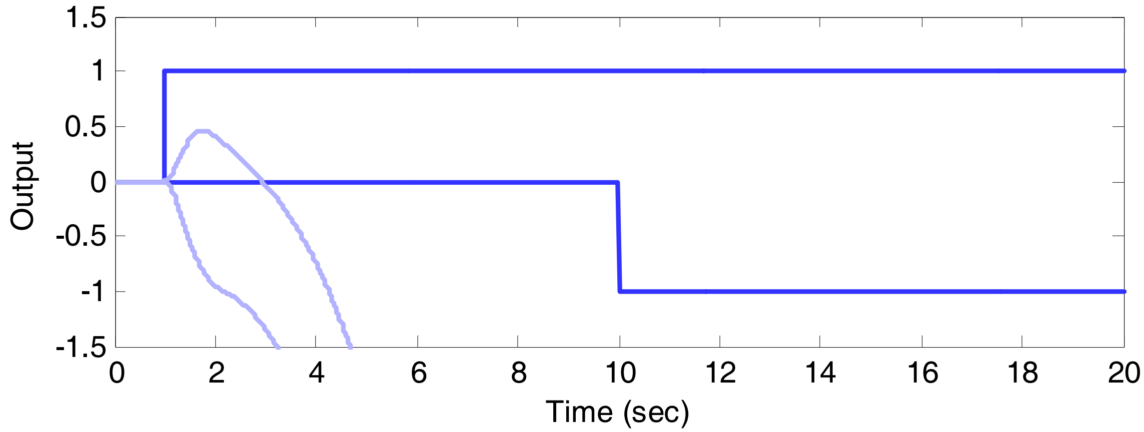
Motivating example

Step responses for $(k_{12},k_{21})=(-0.5, -1)$ and $(-1,-2)$ respectively.

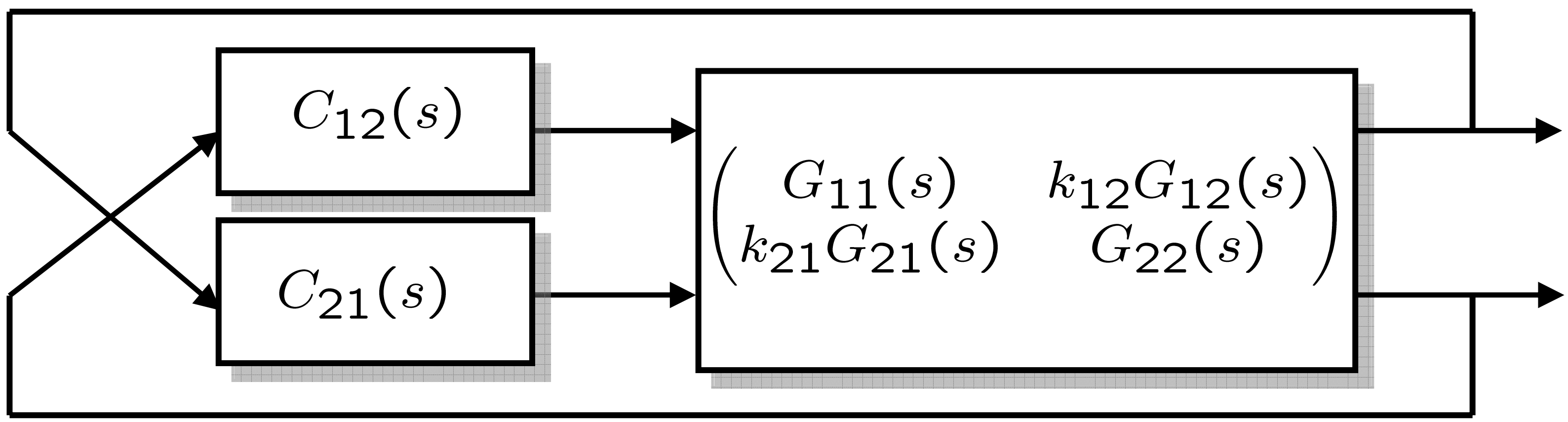




Motivating example cont'd

Controller can no longer maintain stability – what to do?

- Redesign controller (how?)
- Try other pairing of inputs and outputs



- Go full MIMO

How to pair inputs and outputs?

MIMO system complications

- Interactions: a single input affects multiple outputs.
- Qualitatively: the more interactions, the harder to control

The relative gain array tries to quantify the degree of interactions!

- Is decentralized control possible? What is a good input-output pairing?

Relative gain array for a 2x2 system

Consider the system

$$\begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix} = \begin{pmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{pmatrix} \begin{pmatrix} U_1 \\ U_2 \end{pmatrix} \qquad G^{-1} = \frac{1}{G_{11}G_{22} - G_{12}G_{21}} \begin{pmatrix} G_{22} & -G_{12} \\ -G_{21} & G_{11} \end{pmatrix}$$

First, assume that $Y_1=0$ is achieved by $U_1=0$ (open loop control). Then

$$Y_2=G_{22}U_2$$

Now, assume that U_1 is chosen so that Y_1 remains at zero (tight control), i.e., that $U_1=-G_{11}^{-1}G_{12}U_2$. Then a change in U_2 gives

$$Y_2=(-G_{21}G_{11}^{-1}G_{12}+G_{22})U_2 := 1/[G^{-1}]_{22}U_2$$

The *relative gain* of open- and closed-loop measures degree of interactions. If $G_{22}[G^{-1}]_{22}\approx 1$, interactions are weak, otherwise interactions are significant.

General RGA

For a general system Y=GU implies that $U=G^{-1}Y$.

Consider influence of change in u_k on y_k, when

1. $y_l=0$, $l \neq k$, by letting $u_l=0$
2. y_l, $l \neq k$, are forced to zero by tight control

Since $y_l=0$, $l \neq k$, $U_j=[G^{-1}]_{jk}Y_k$, or $Y_k=([G^{-1}]_{jk})^{-1}U_j$

The relative gain of $u_j \rightarrow y_k$ is $G_{kj}[G^{-1}]_{jk}$

Definition. The *relative gain array, RGA,* of a square system G is defined as

$$\text{RGA}(G) = G.*(G^{\dagger})^T$$

or, in Matlab notation, RGA(G)= G.*(pinv(G).')

RGA and Pairing

Result: if the decentralized control is stable in each scalar control loop, and RGA(G(iω))=I $\forall\ \omega$, then the closed loop system is also stable

Result: If some diagonal element of RGA(G(0)) is negative and a diagonal controller is used, then either the closed loop system is unstable, or the system becomes stable if any of the scalar control loops break.

Suggest the following rules of thumb:

Rule 1: Find a pairing that makes the diagonal elements of RGA(iω_c)$\approx$ 1 where ω_c is the desired crossover frequency.

Rule 2: Avoid pairings that give negative diagonal elements in RGA(G(0))

RGA for motivating example

Since

$$G(0) = \begin{pmatrix} 1 & k_{12} \\ k_{21} & 1 \end{pmatrix}, \qquad G(0)^{-1} = \frac{1}{1 - k_{12}k_{21}} \begin{pmatrix} 1 & -k_{12} \\ -k_{21} & 1 \end{pmatrix}$$

we have

$$\begin{aligned} \text{RGA}(G(0)) &= G(0).*(G(0)^{\dagger})^T \\ &= \frac{1}{1 - k_{12}k_{21}} \begin{pmatrix} 1 & -k_{12}k_{21} \\ -k_{12}k_{21} & 1 \end{pmatrix} \end{aligned}$$

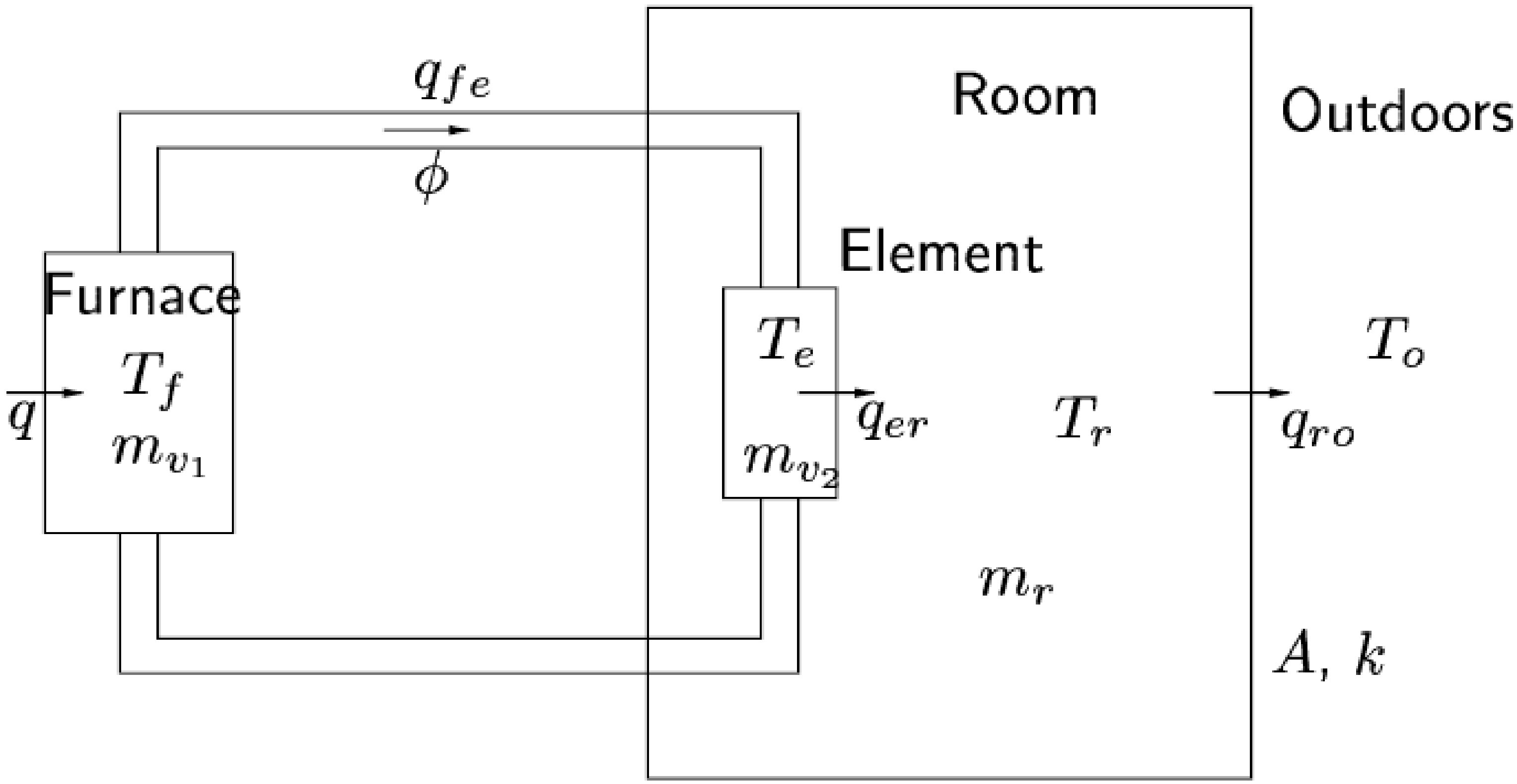
In the unstable case, when $(k_{12}, k_{21}) = (-1, -2)$,

$$\text{RGA}(G(0)) = \begin{pmatrix} -1 & 2 \\ 2 & -1 \end{pmatrix}$$

Compare second RGA-rule!

Other pairing is probably better (why?)

Example: Domestic heating system



RGA and pairing

System

$$\begin{pmatrix} T_f \\ T_r \end{pmatrix} = G \begin{pmatrix} q \\ \phi \end{pmatrix}$$

has

$$\text{RGA}(G(0)) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

and

$$\text{RGA}(G(0.001i)) = \begin{pmatrix} 0.9347 + 0.2507i & 0.0653 + 0.2507i \\ 0.0653 + 0.2507i & 0.9347 + 0.2507i \end{pmatrix}$$

so controlling T_f by q and T_r by ϕ is the best choice.

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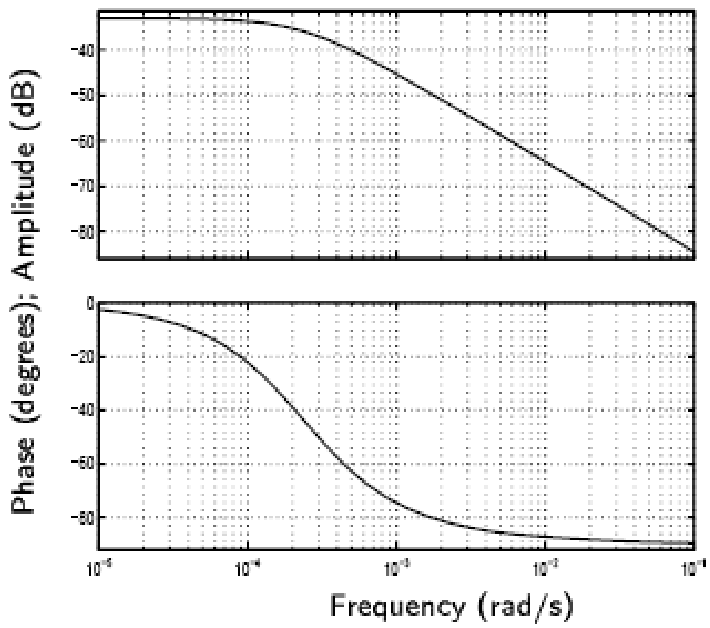
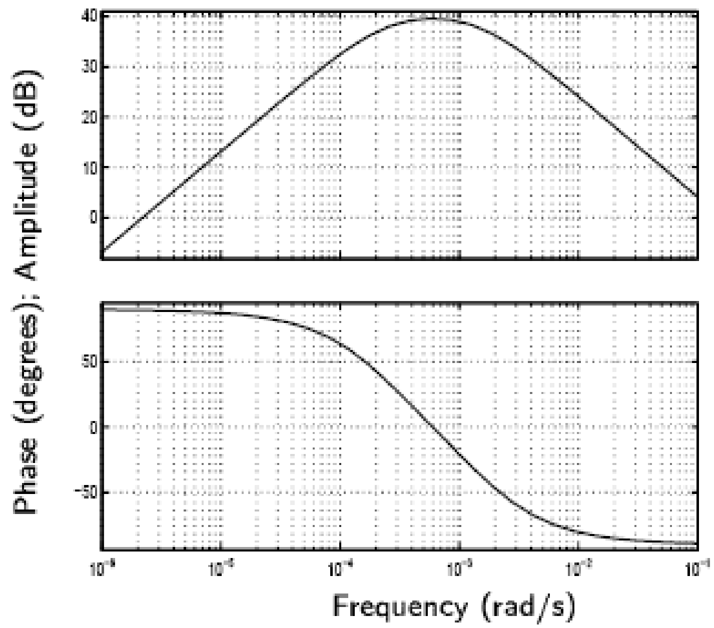
Bode diagrams

$G_{T_f q}$

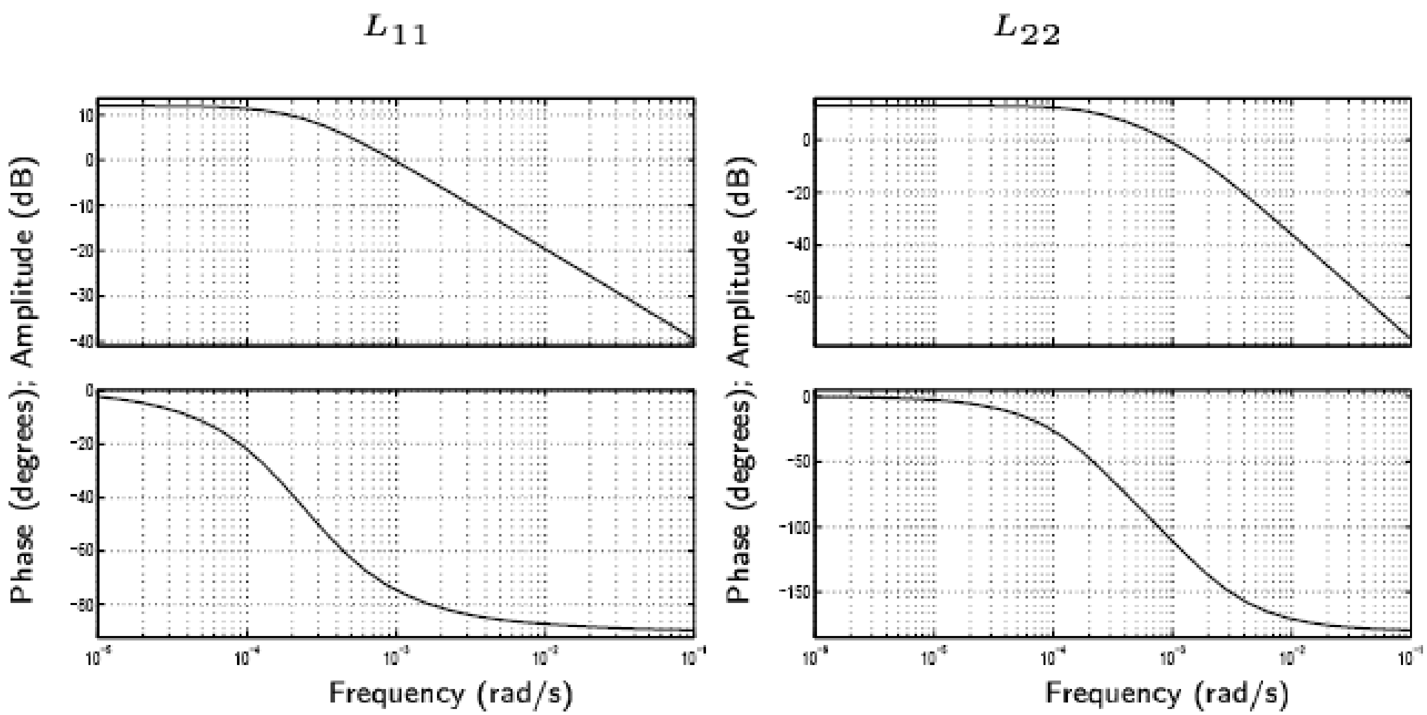


$G_{T_r \phi}$



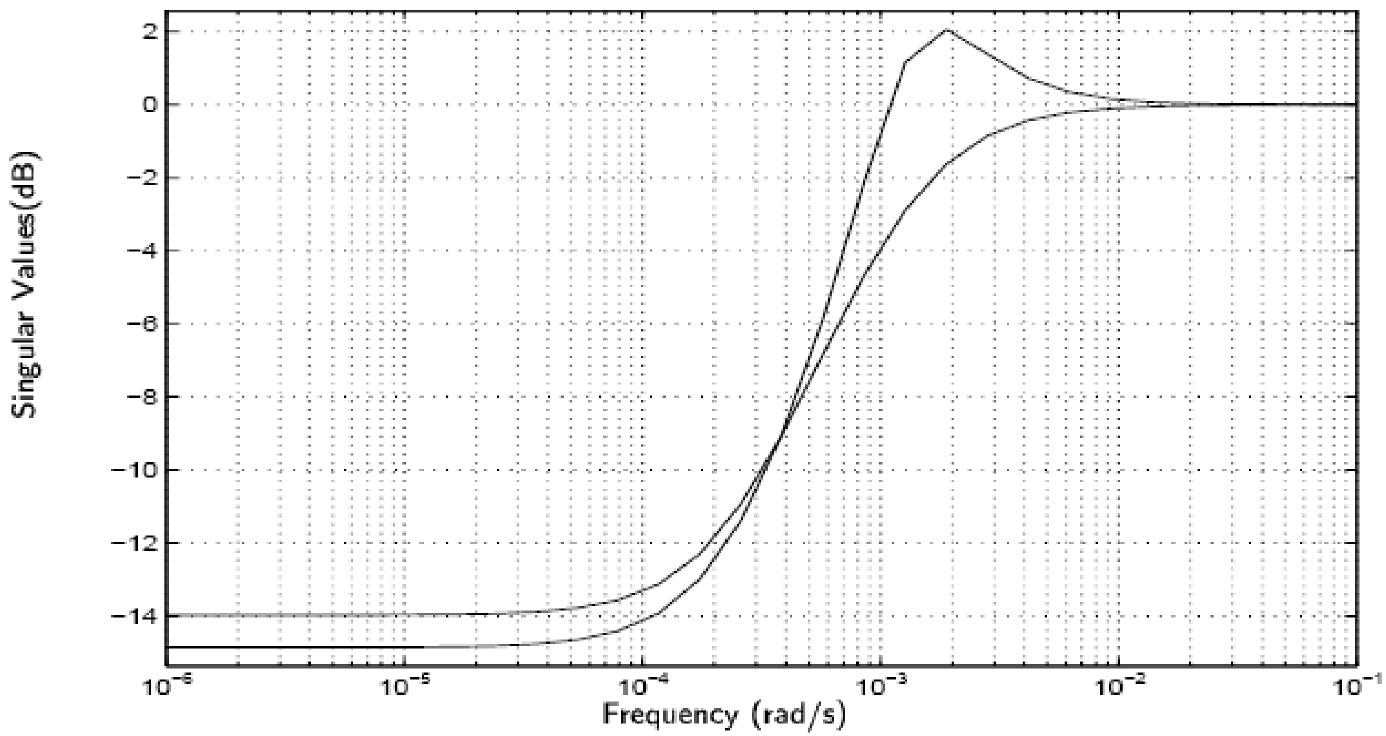
Bode diagrams of individual loops

$F_y = \begin{pmatrix} 178 & 0 \\ 0 & 0.00001/s \end{pmatrix}$ gives cross-over frequencies 0.001 rad/sec, and good phase margins (105^o and 60^o, respectively)

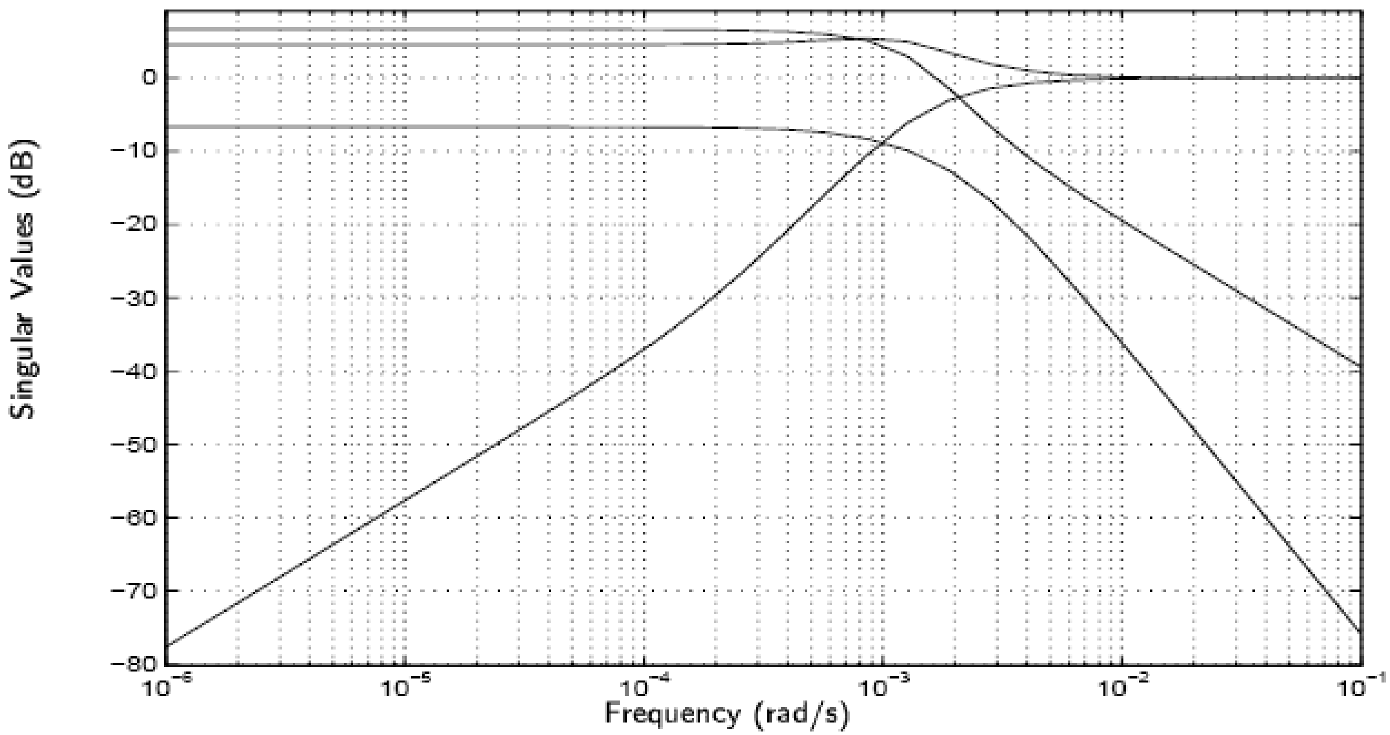


Individual loops OK

Magnitude plot of $1/(1+L_{11})$ and $1/(1+L_{22})$



Sensitivity and complementary



Sensitivities too large!

Conclusion: individual loops ok, interactions ruin performance!

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What now?

It turns out that one can lower $\overline{\sigma}(S(i\omega))$ for low frequencies by increasing the loop-gain L_{11} at low frequencies.

But no systematic method exists for tuning of decentralized controllers!

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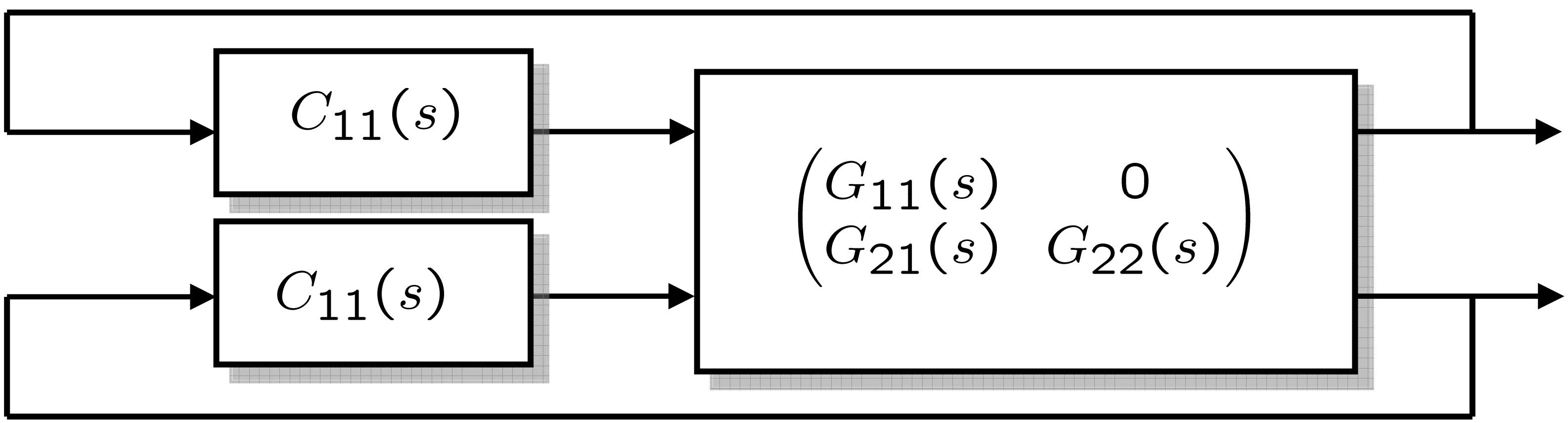
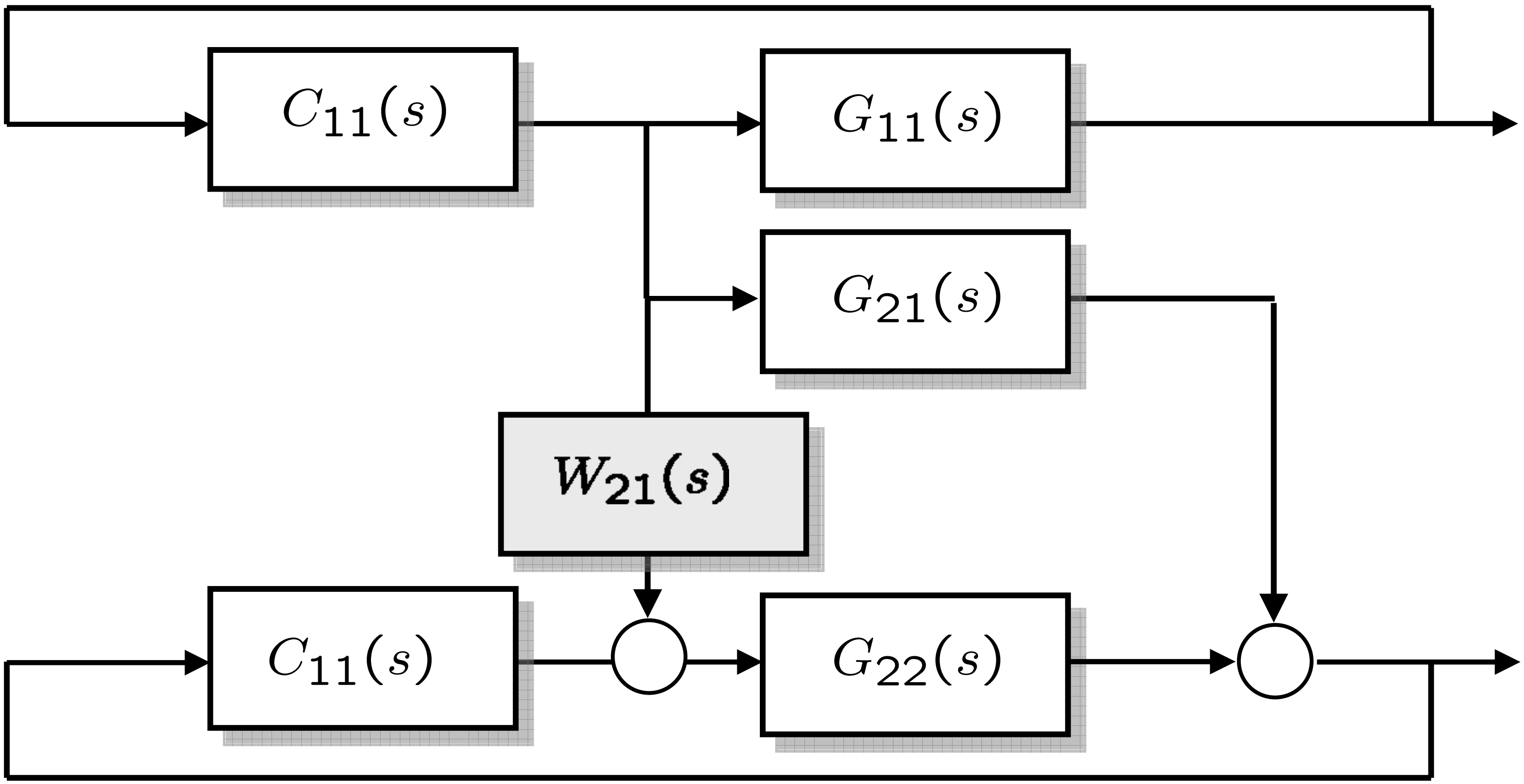
Compensating for interactions

How to deal compensate for non-negligible interactions?

Example: (lower triangular transfer matrix)



Compensating for interactions



Feedforward to cancel interactions: $W_{21}(s) = -G_{21}(s)/G_{22}(s)$

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Compensating for interactions

In our example, we use

$$U(s) = F(s)C(s) = \begin{pmatrix} 1 & 0 \\ -G_{21}/G_{22} & 1 \end{pmatrix} \begin{pmatrix} C_{11} & 0 \\ 0 & C_{22} \end{pmatrix}$$

where F is designed so that the system “seen by” C is diagonal

$$\tilde{G}(s) = G(s)F(s) = \begin{pmatrix} G_{11} & 0 \\ G_{21} & G_{22} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ F_{21} & 1 \end{pmatrix} = I$$

This idea of decoupling can be generalized.

Decoupled control

More general decoupling scheme. Introduce

$$\tilde{U} = W_1^{-1}U, \qquad \tilde{Y} = W_2 Y \qquad (\,\tilde{Y}(s) = W_2(s)G(s)W_1(s)\tilde{U}(s)\,)$$

so that $\tilde{G}(s) = W_2(s)G(s)W_1(s)$ becomes as diagonal as possible.

Design decentralized controller $\tilde{F}_y(s)$ for $\tilde{G}(s)$ and implement

$$F_y(s) = W_1(s)\tilde{F}_y(s)W_2(s)$$

Summary

A reductionist's approach to MIMO control

- Decentralized control – use multiple SISO controllers
- RGA – measures degree of interaction
  - Is system ammendable to decentralized control?
  - How to pair inputs and outputs
- Decoupled control:
  - pre/post compensate diagonal controller to reduce interactions