2E1252
Control Theory and Practice

Lecture 11:
Actuator saturation and anti wind-up

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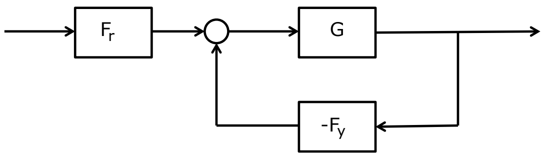
Learning aims

After this lecture, you should

- understand how saturation can cause controller states to "wind up"
- know how to modify a linear observer to account for saturation
- be able to interpret the observer modification in a block diagram
- be able to analyze closed-loop stability using the small-gain theorem
- know how to tune the anti-windup gain for a PID controller

So far…

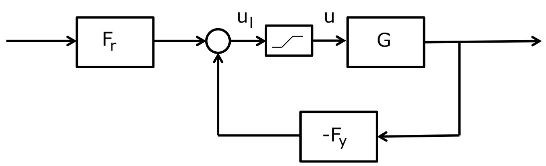
…we have designed **linear controllers** for **linear systems**



(uncertainty models allow us to account for some nonlinearities)

Actuation is typically limited

In practically all control systems, the actuation is limited



The new block represents a **saturation**

$$u = \text{sat}(u_l) = \begin{cases} u_{\min} & \text{if } u_l \leq u_{\min} \\ u_{\max} & \text{if } u_l \geq u_{\max} \\ u_l & \text{otherwise} \end{cases}$$

The impact of limited actuation

Consider the simple servo model:

$$G(s) = \frac{1}{s(s+1)}$$

A controller is

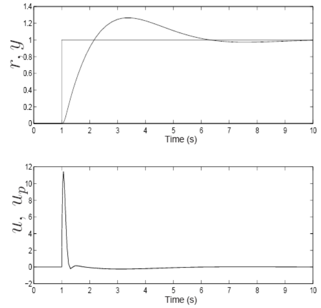
$$F_y(s) = \frac{439s^2 + 710.5s + 316.2}{s^3 + 26.47s^2 + 349.8s - 7.13}$$

which has poles in -13.2444± 13.2255i, and 0.0204

Note: controller is unstable.

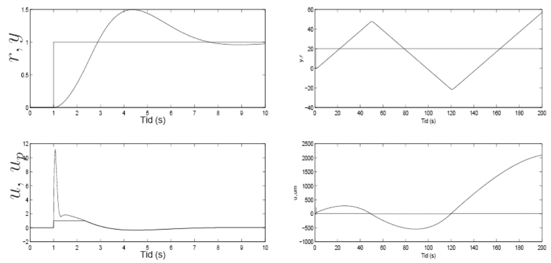
Step response of linear system

A unit step response for the linear closed-loop (assuming no saturation)



Step response with saturated control

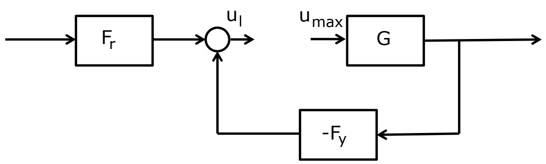
Step responses with actuator saturation for small and large reference change



Slower, larger overshoot　　　unstable

What is wrong?

When actuator is saturated, the system operates in open loop (changes in u_l do not affect G or controller states while saturated)



Constant input make plant and controller states to grow large ("wind up")

– particularly critical when plant or controller is unstable or integrating

Understanding what is wrong

Consider observer-based controllers (e.g. LQG, H-infinity, H-2, etc.)

$$\frac{d}{dt}\hat{x}(t) = A\hat{x}(t) + Bu_l(t) + K(y(t) - \hat{y}(t))$$
$$u_l(t) = -L\hat{x}(t)$$

Controller transfer function:

$$U_l(s) = -F_y(s)Y(s) = -L(sI - A + BL + KC)^{-1}KY(s)$$

Basic idea of observer: "simulate system" and correct when $\hat{y}(t) \neq y(t)$
- simulation part does not reflect reality when input is saturated!

An improvement

Make sure that observer reflects actual system dynamics

$$\frac{d}{dt}\hat{x}(t) = A\hat{x}(t) + Bu(t) + K(y(t) - \hat{y}(t))$$
$$u(t) = \mathrm{sat}(u_l(t))$$
$$u_l(t) = -L\hat{x}(t)$$

Modification known as **observer-based anti-windup**
- avoids that controller states wind up
- often enough to get reasonable performance

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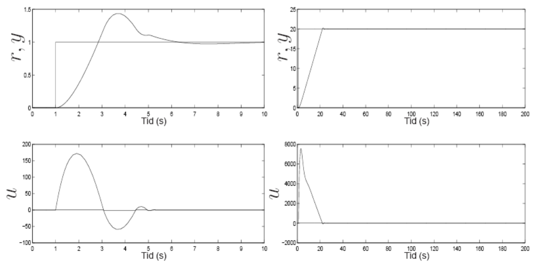
Step responses with modified observer

Step responses with actuator saturation for small and large reference change



Smaller overshoot, no longer unstable for large reference changes.

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Analysis: no wind-up in saturation

When in saturation, $u(t) = u_{\lim}$ is constant and controller dynamics is

$$\frac{d}{dt}\hat{x}(t) = A\hat{x}(t) + Bu(t) + K(y(t) - \hat{y}(t)) =$$
$$= (A - KC)\hat{x}(t) + Bu_{\lim} + Ky(t)$$

Stability properties given by A-KC, which is typically stable

Taking Laplace transforms, we find

$$U_l(s) = -L(sI - A + KC)^{-1}KY(s) - K(sI - A + KC)^{-1}B\frac{u_{\lim}}{s}$$

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Interpretation: feedback from u-u_l

To relate modified controller to original, re-write observer as

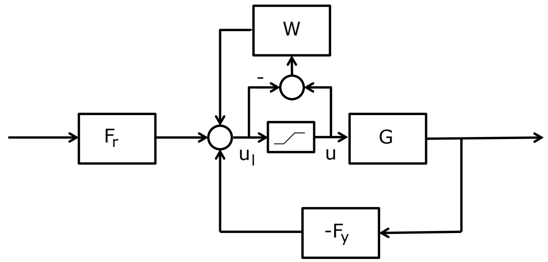
$$\frac{d}{dt}\hat{x}(t) = A\hat{x}(t) + Bu(t) + K(y(t) - \hat{y}(t)) =$$
$$= (A - KC)\hat{x}(t) + B(u_l(t) + u(t) - u_l(t)) + K(y(t) - \hat{y}(t)) =$$
$$= (A - BL - KC)\hat{x}(t) + Ky(t) + B(u(t) - u_l(t))$$

Taking Laplace transforms, we find

$$U_l(s) = -L(sI - A + BL + KC)^{-1}KY(s)$$
$$- L(sI - A + BL + KC)^{-1}B(U(s) - U_l(s)) =$$
$$= -F_y(s)Y(s) + W(s)(U(s) - U_l(s))$$

The linear control law plus compensation from $u(t) - u_l(t)$

Interpretation in block diagram



Basis for many heuristic techniques for anti-windup (more later…)

What about stability?

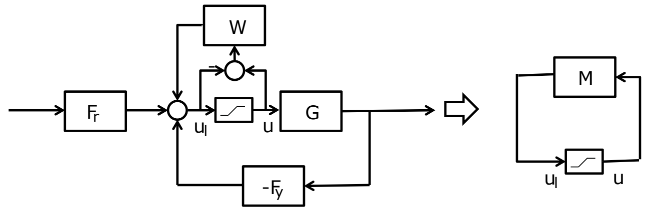
We have shown that

- in absence of saturation, the closed-loop system is stable
- when input remains in saturation, the controller is stable

but **no** stability guarantees when control moves in and out of saturation!

Global stability can sometimes be ensured using small-gain theorem

A small-gain analysis



To find the linear system M, note that

$$u_l = W(u - u_l) - GF_y u \Rightarrow (I + W)u_l = (W - GF_y)u := Mu$$

Since gain of saturation nonlinearity is one (cf. Lecture 1), so if

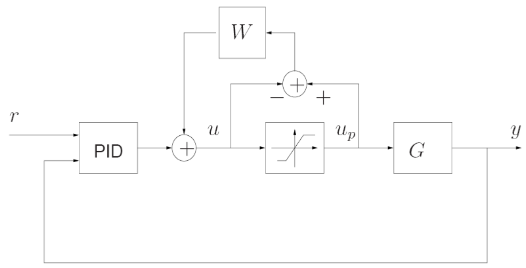
$$\|M\|_\infty < 1$$

closed-loop stability is ensured

Design guidelines

1. Design observer-based controller using technique of choice
2. Modify observer to reflect the presence of saturation nonlinearity
3. Attempt to establish stability using small gain theorem, simulate
4. If unsatisfactory, re-design controller with higher penalty on control

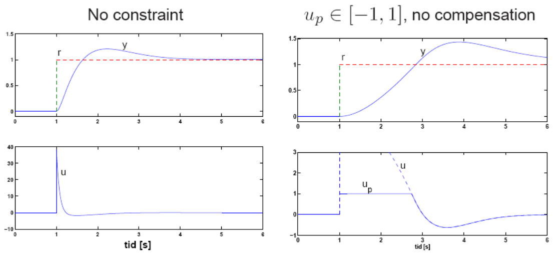
Application to PID control



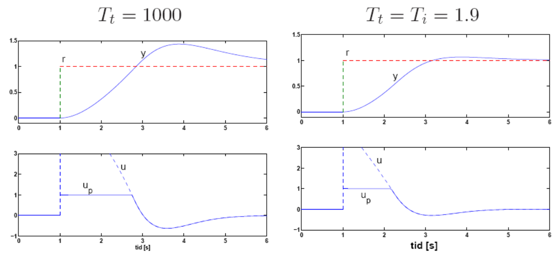
Common choice: $W(s) = \dfrac{1}{sT_t}$ (tracking anti-reset windup)

Rule-of-thumb: $T_t = \sqrt{T_i T_d}$ (T_i integral time, T_d derivative time of PID)

DC Servo under PID control



Servo: PID+anti-windup



Summary

The problem with limited actuation:

- New phenomena, not predicted by linear control theory
  - controller, plant states "wind up" (grow large), feedback loop open
- Observer-based anti-windup
  - make estimator reflect actual process dynamics
  - ensures that controller states are stable in saturation
  - interpretation as feedback from "saturation error"
  - global stability analysis via small-gain theorem
- Anti-windup for PID control
  - same structure, heuristic compensator, rules-of-thumbs