ROYAL INSTITUTE
OF TECHNOLOGY

Lecture 15
Power system state estimation

Kun Zhu

2013-05-02

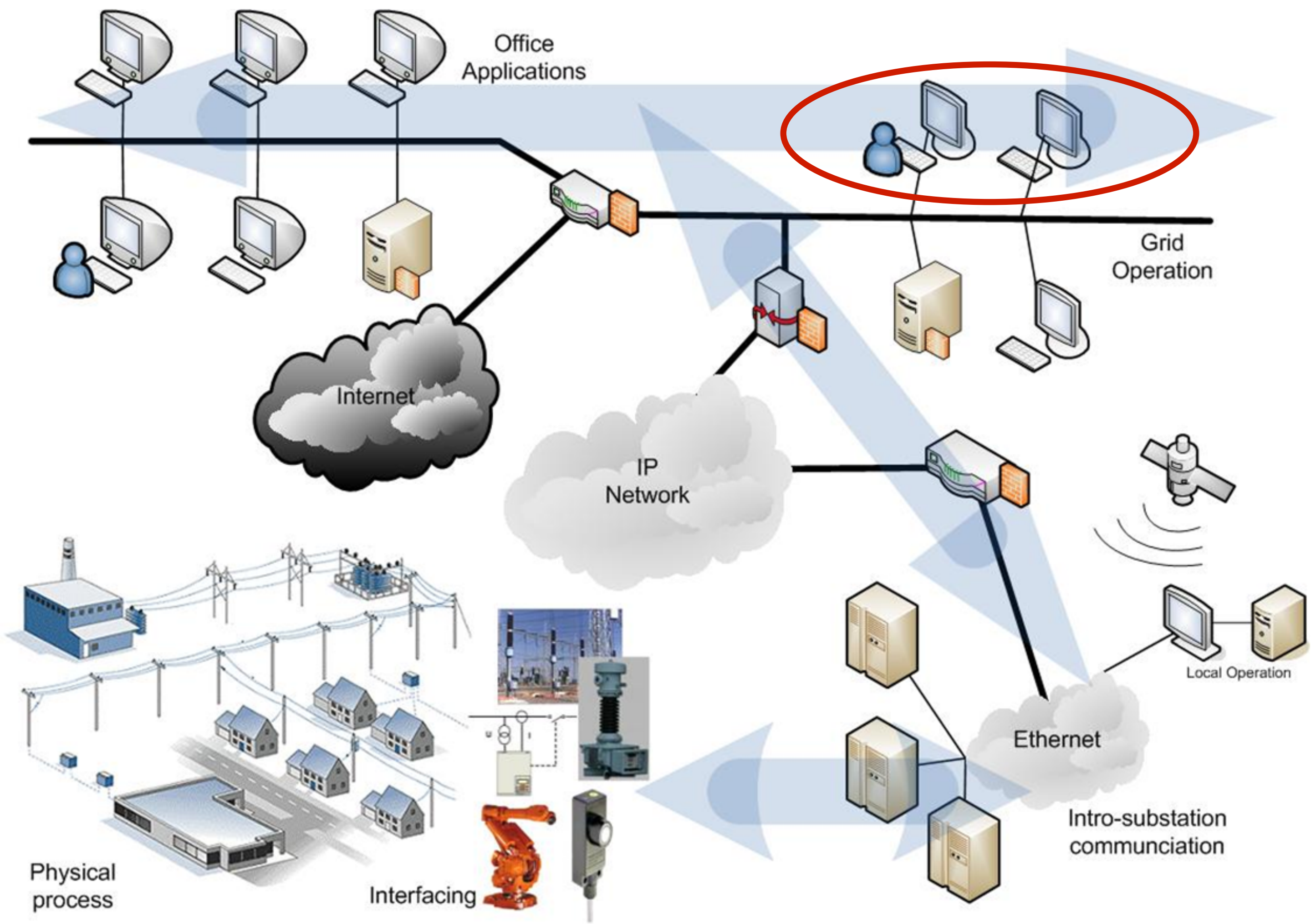
Outline

- State estimation
  - What
  - Why

- Weighted Leaset Square (WLS) algorithms
  - Mathmatics
  - Concepts

- Operation challenges

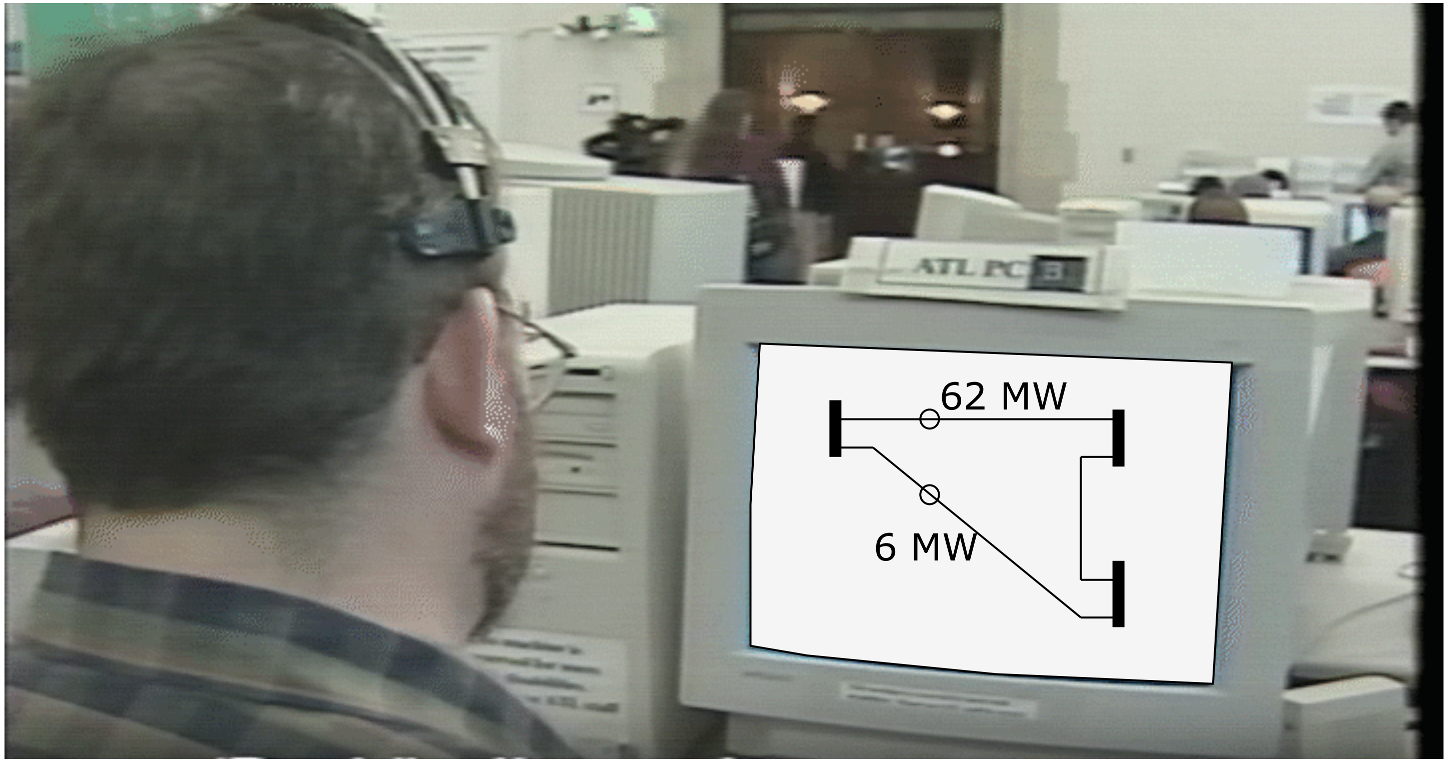
Course road map



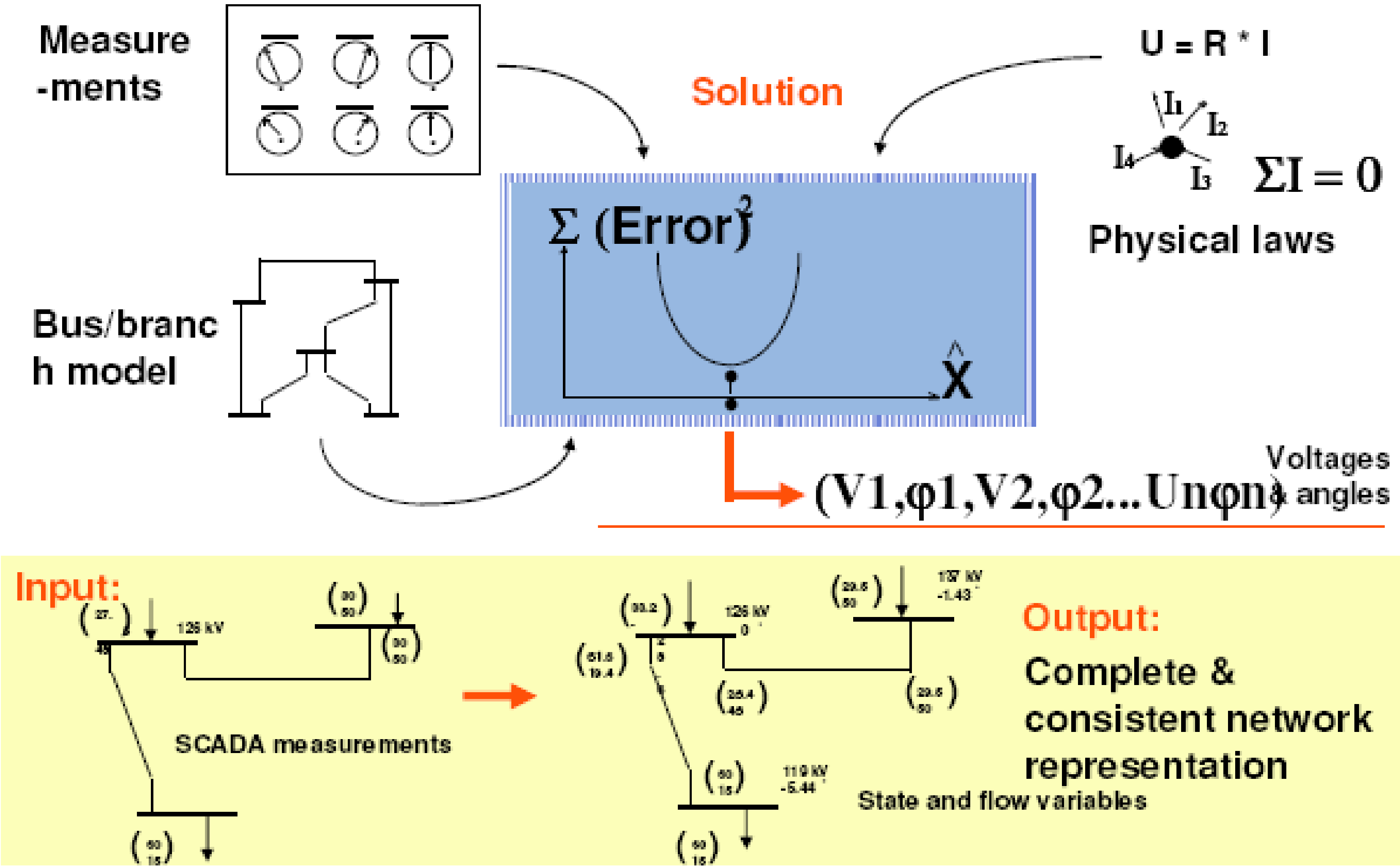
ROYAL INSTITUTE OF TECHNOLOGY

What is state estimation?

How could the operator know the system?



The truth is out here!



Power system states: x

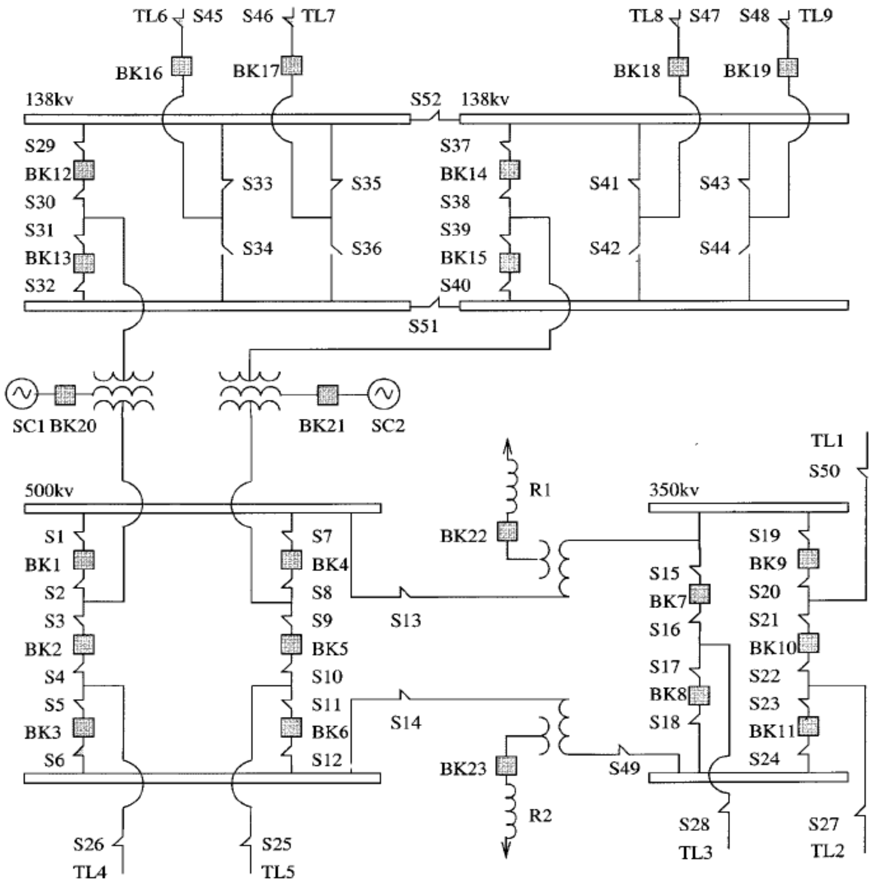
- The power system states are those parameters that can be used to determine all other parameters of the power system

- Node voltage phasor
  - Voltage magnitude V_k
  - Phase angle Θ_k
- Transformer turn ratios
  - Turn ratio magnitude t_{kn}
  - Phase shift angle φ_{kn}
- Complex power flow
  - Active power flow P_{kn}, Pn_k
  - Reactive power flow Q_{kn}, Qn_k

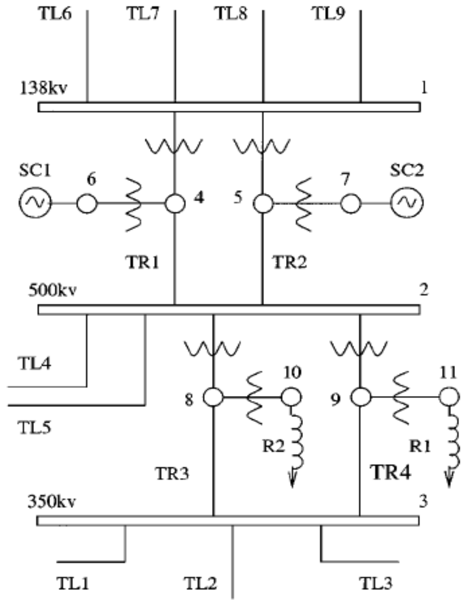
Analog measurements

- Voltage magnitude
- Current flow magnitude & injection
- Active & reactive power
  - Branches & groups of branches
  - Injection at buses
  - In switches
  - In zero impedance branches
  - In branches of unknown impedance
- Transformers
  - Magnitude of turns ratio
  - Phase shift angle of transformer
- Synchronized phasors from Phasor Measurement Unit

Network topology processing



Bus breaker model



Bus branch model

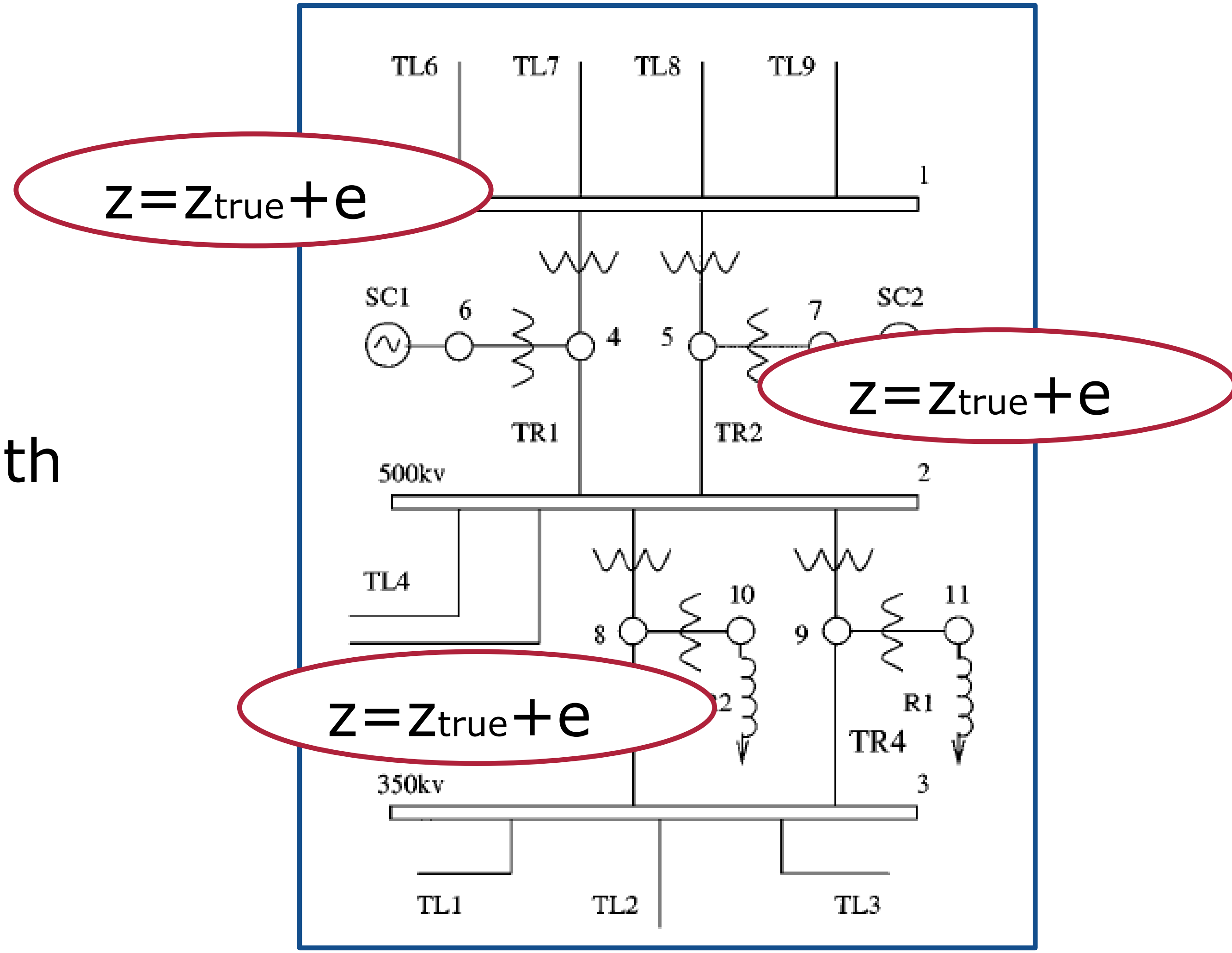
Power system measurements: z

z_{true}: power system truth

$z_{true}=h(x)$

e: measurement error

$e=e_{systematic}+e_{random}$



Measurement model

- How to determine the states (x) given a set of measurements (z)?

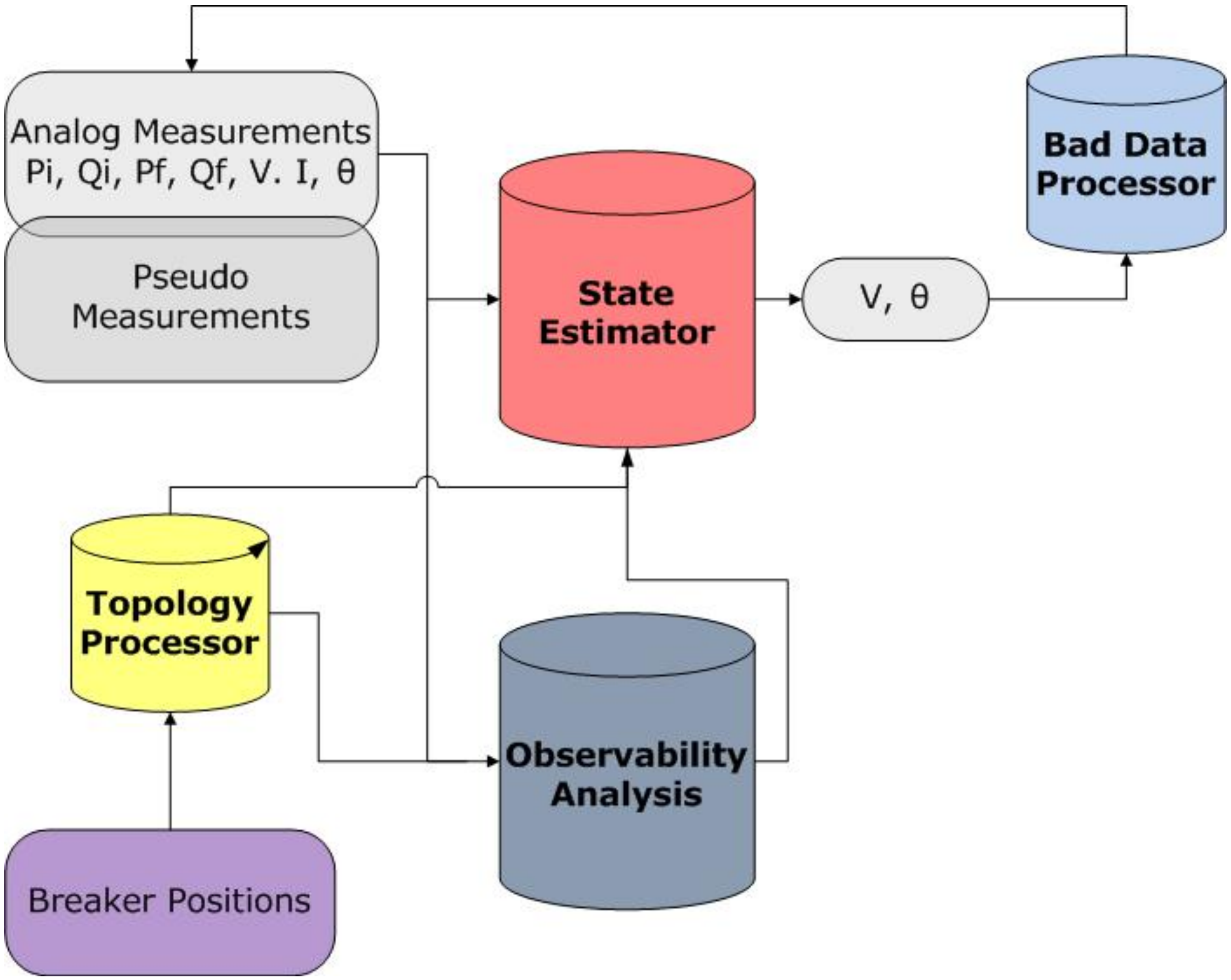
$$z_j = h_j(\boldsymbol{x}) + e_j$$

known *unknown* *unknown*

where

- x is the true state vector $[V_1, V_2, \ldots V_k, \Theta_1, \Theta_2, \ldots \Theta_k]$
- z_j is the jth measurement
- h_j relates the jth measurement to states
- e_j is the measurement error

State estimation process



ROYAL INSTITUTE OF TECHNOLOGY

Why do we need state estimation?

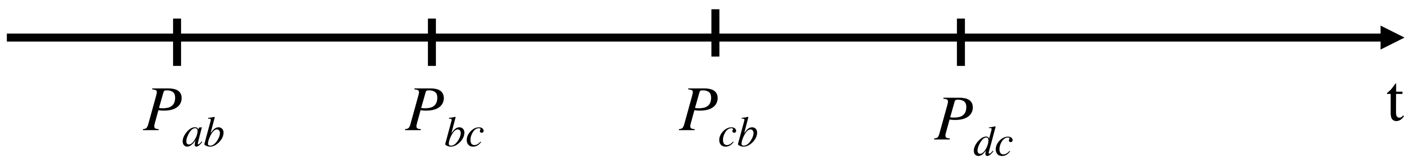
Measurements correctness

- Imperfections in
  - Current & Voltage transformer
  - Transducers
    - A/D conversions
    - Tuning
  - RTU/IED Data storage
  - Rounding in calculations
  - Communication links

- Result in uncertainties in the measurements

Measurement timeliness

- Due to imperfections in SCADA system the measurements will be collected at different points in time, time skew.



- If several measurements are missing how long to wait for them?
- Fortunately, not a problem during quansi-steady state.
- State estimation is used for off-line applications

ROYAL INSTITUTE OF TECHNOLOGY

How can the states be estimated?

ROYAL INSTITUTE OF TECHNOLOGY

Approaches

- Minimum variance method
  - Minimize the sum of the squares of the weighted deviations of the state calculated based on measurements from the true state
- Maximum likelihood method
  - Maximizing the probability that the estimate equals to the true state vector ***x***
- Weighted least square method (WLS)
  - Minimize the sum of the weighted squares of the estimated measurements from the true state

$$J(x) = \sum_{i=1}^{m} \frac{(z_i - h_i(x))^2}{R_{ii}}$$

Least square (Wiki)

- "Least squares" means that the overall solution minimizes the sum of the squares of the errors made in the results of every single equation.
- The method of **least squares** is a standard approach to the approximate solution of over determined system, i.e., sets of equations in which there are more equations than unknowns.
- The most important application is in data fitting.
- Carl Friedrich Gauss is credited with developing the fundamentals of the basis for least-squares analysis in **1795**.

WLS state estimation

- **Fred Schweppe** introduced state estimation to power systems in **1968**.
- He defined the state estimator as “a data processing algorithm for converting redundant meter readings and other available information into an estimate of the state of an electric power system”.
- Today, state estimation is an essential part in almost every energy management system throughout the world.

Felix F. Wu, “Power system state estimation: a survey”, International Journal of Electrical Power & Energy Systems, Volume 12, Issue 2, April 1990, Pages 8

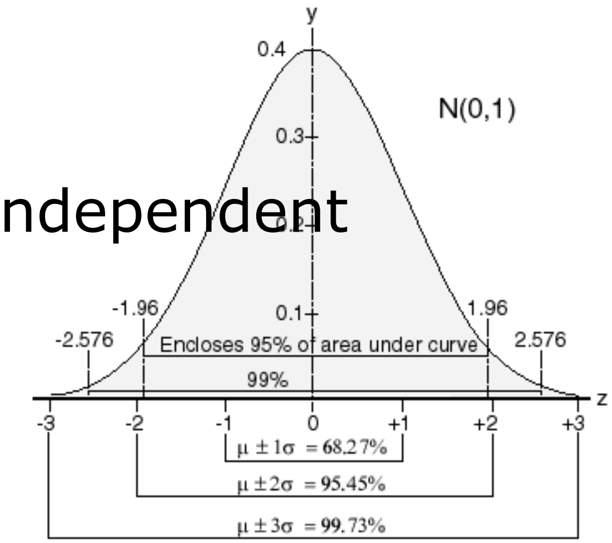
WLS state estimation model

$$z_j = h_j(\boldsymbol{x}) + e_j$$

known *unknown* *unknown*

Error characteristics

- The errors in the measurements is the sum of several stochastic variables
  - CT/VT, Transducer, RTU, Communication…
- The errors is assumed as a Gaussian Distribution with known deviations
  - Expected value $E[e_j] = 0$
  - Known deviation σ_j
- The errors are also assumed to be independent
  - $E[e_i e_j] = 0$



WLS objective function

$$J(x) = \sum_{i=1}^{m} \frac{(z_i - h(x))^2}{\sigma_i^2} = [z - h(x)]^T R^{-1} [z - h(x)]$$

where

i=1,2,…m

$$R = diag\{\sigma_1^2, \sigma_2^2, \sigma_3^2, \ldots \sigma_m^2\} = Cov(e) = E[e \cdot e^T]$$

Solution to above is iterative using newton methods

ROYAL INSTITUTE OF TECHNOLOGY

Newton iteration

- At the minimum, the first-order optimality conditions will have to be satisfied

$$g(x) = \frac{\partial J(x)}{\partial x} = H^T(x) R^{-1} [z - h(x)] = 0$$

$H^T(x) = [\frac{\partial h(x)}{\partial x}]$ is the measurement Jacobian matrix

Newton iteration cont'd

- Expanding the *g(x)* into its Taylor series around state vector x^k

$$g(x) = g(x^k) + G(x^k)(x - x^k) + \ldots\ldots = 0$$

where

$$G(x^k) = \frac{\partial g(x^k)}{\partial x} = H^T(x^k)R^{-1}H(x^k)$$

ROYAL INSTITUTE OF TECHNOLOGY

Newton iteration cont’d

- Neglecting the higher order terms leads to an iterative solutions scheme know as the Gauss-Newton method as :

$$x^{k+1} = x^k - [G(x^k)]^{-1} \cdot g(x^k)$$

k is the iteration index,

x^k is the solution vector at iteration *k*

ROYAL INSTITUTE OF TECHNOLOGY

Newton iteration IV

- Convergence

$$\max\left(\left|\Delta x^k\right|\right) \leq \xi$$

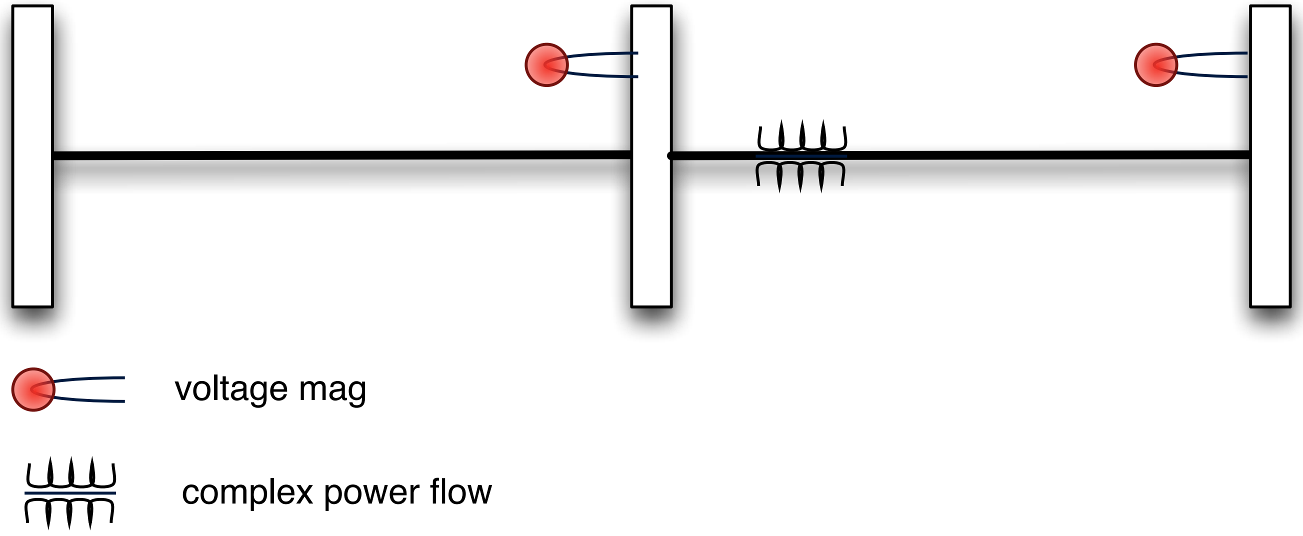
- If not, update

$$x^{k+1} = x^k + \Delta x^k$$

$$k = k + 1$$

Go back to the previous step

Why the measurements are weighted?



$$P_{ij} = f(v_i, v_j, \theta_i, \theta_j)$$

$$Q_{ij} = f(v_i, v_j, \theta_i, \theta_j)$$

ROYAL INSTITUTE OF TECHNOLOGY

Weight

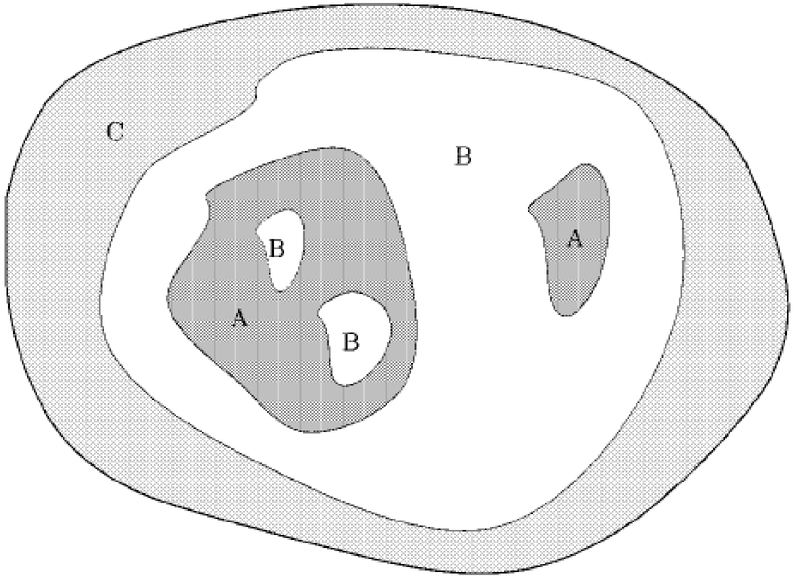
- Weight is introduced to emphasize the trusted measurement while de-emphasize the less trusted ones.

- WLS

$$W_i = \frac{1}{\sigma_i^2}$$

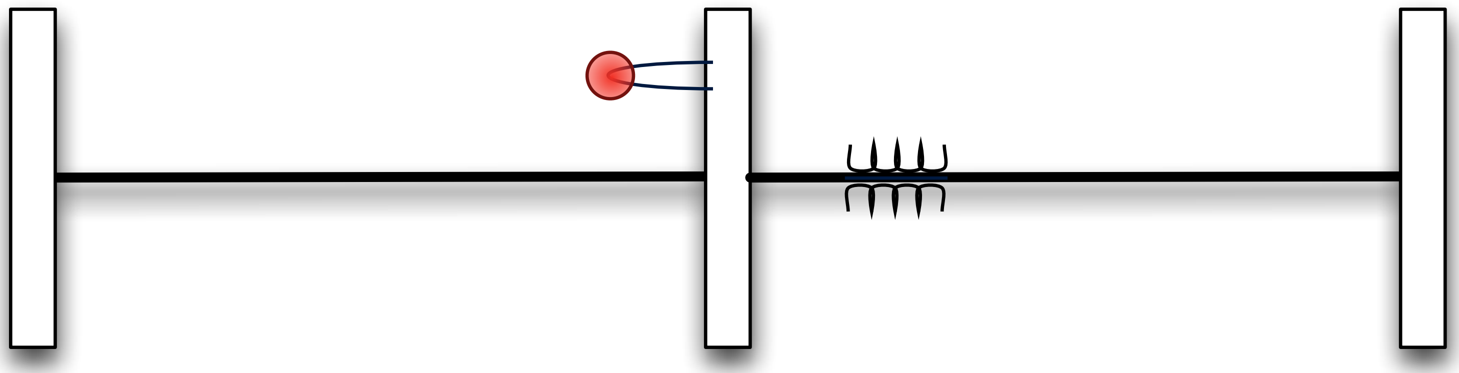
Observability

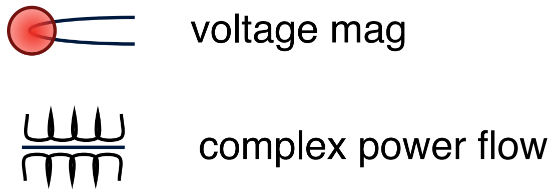
- Based on system topology and location of measurements parts of the power system may be unobservable.
- Unobservable parts of the system can be made observable via data exchange (CIM), pseudo measurements, etc…



A Observable part of the system of interest
B Unobservable part of the system of interest
C Rest of the interconnected system

Observability example





$$P_{ij} = f(v_i, v_j, \theta_i, \theta_j)$$

$$Q_{ij} = f(v_i, v_j, \theta_i, \theta_j)$$

Observability criterion

Necessary but not sufficient condition

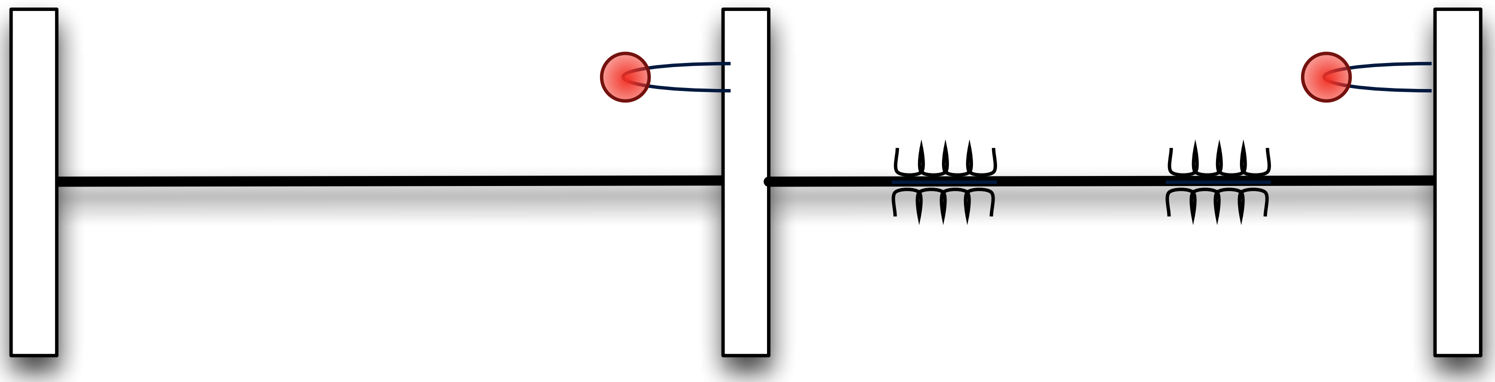
$$m \geq n$$

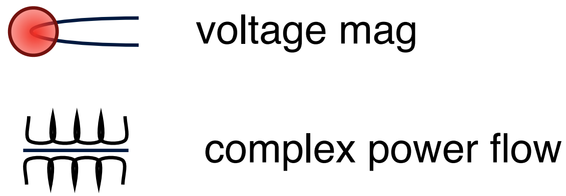
m: Number of measurements

n: Number of states

Is the system's oservability guranteed in this case?

Observability example II



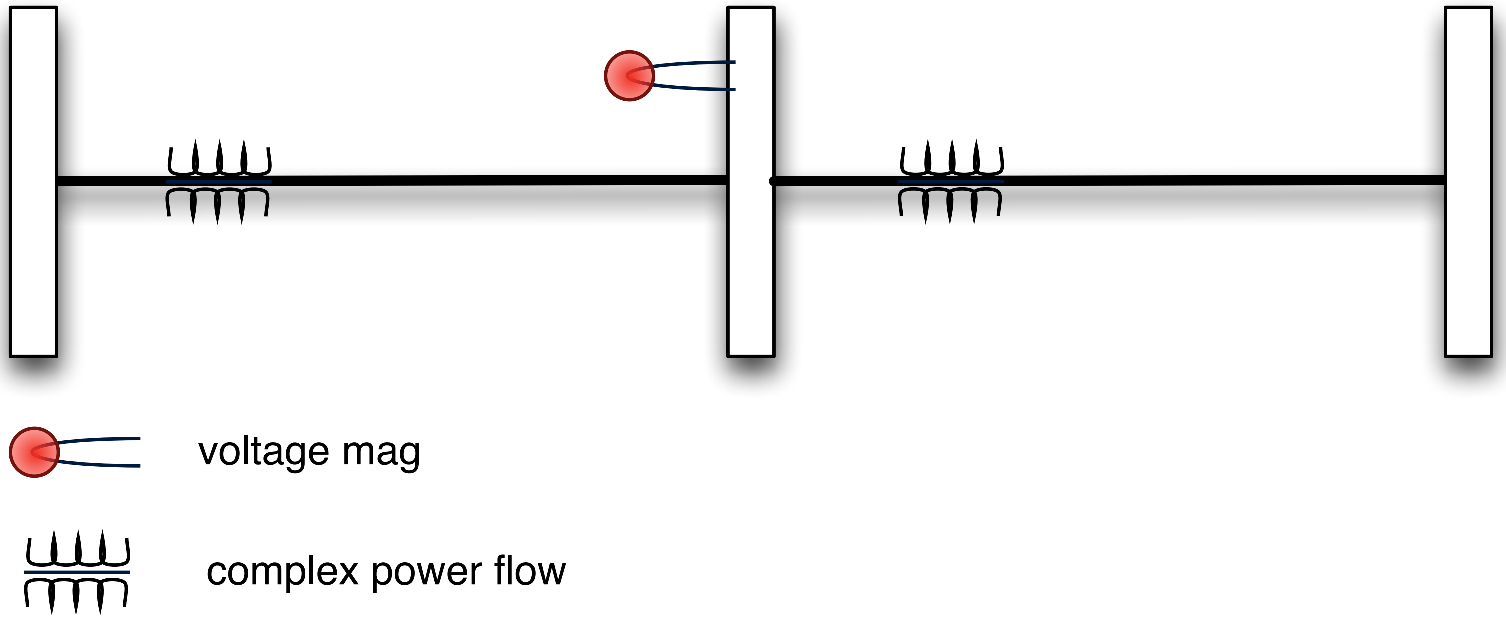


$$P_{ij} = f(v_i, v_j, \theta_i, \theta_j)$$

$$Q_{ij} = f(v_i, v_j, \theta_i, \theta_j)$$

ROYAL INSTITUTE OF TECHNOLOGY

n of *m* measurements has to be independent



$$P_{ij} = f(v_i, v_j, \theta_i, \theta_j)$$

$$Q_{ij} = f(v_i, v_j, \theta_i, \theta_j)$$

Summary of assumptions

- Quansi-steady system
  - No large variations of states over time
  - State estimator will be suspended when large disturbance happens.
- Errors in measurements are
  - Gaussian in nature with known deviation
  - Independent
- Strong assumption
  - Power system topology model is correct

Refinements

- Refinements of method for State Estimation has been the objective of much research.

- Reduce the numerical calculation complexity in order to speed up the execution.

  - 3000 bus node in 1-2 seconds

- Improve estimation robustness

  - less affected by erroneous input

ROYAL INSTITUTE OF TECHNOLOGY

Bad Data Detection

Data quality

- Analog measurement error
- Parameter error
- Topological error
  - Discrete measurement error
  - Model error

Bad Data Detection (Analog)

- Putting the measurements up to a set of logical test (Kirchhoff's laws) before they are input into the State Estimator
- Calculating $J(x)$ and comparing with a pre-determined limit. If the value exceeds the limit, we can assume there is “something” wrong in the measurements (Chi-square)

ROYAL INSTITUTE OF TECHNOLOGY

Bad Data Detection (Analog)

- Once the system state has been identified, i.e. we have an estimate of $\boldsymbol{x}$
- We can use the estimate to calculate

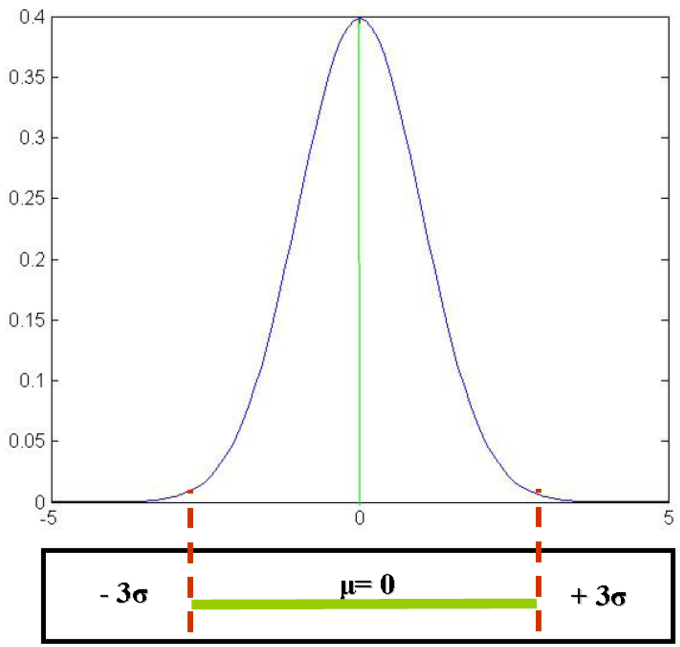
$$r_i = z_i - h_i(\hat{x})$$

- If there is any residual in this calculation that “stands-out” this is an indication that particular measurement is incorrect

Largest normalized residual

$$r_i^N = \frac{z_i - h(\hat{x})}{\sigma_i}$$

- The normalized residual follows the standard normal distribution



N(0,σ²)

μ ±σ (68.26%)

μ±3σ (99.74%)

Limitations

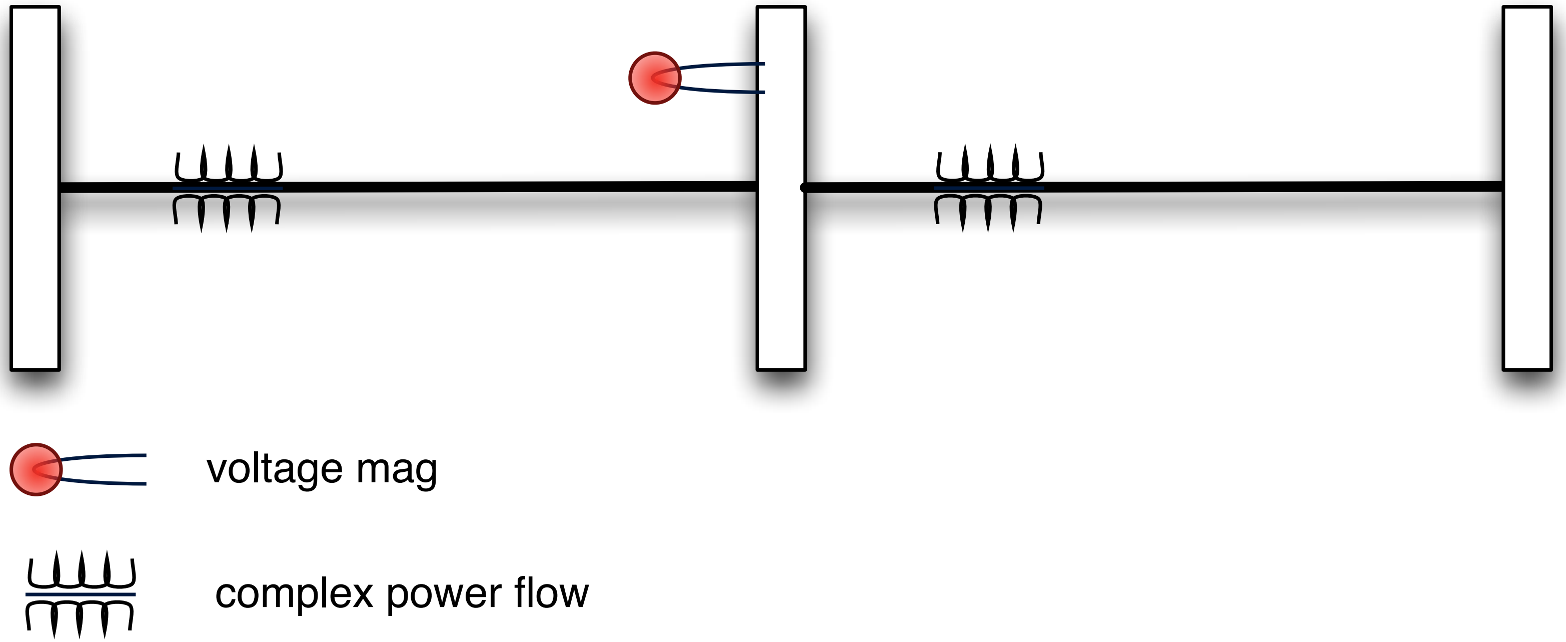
- The critical measurements have zero residuals, hereby they can not be detected

$$\hat{z}_i = H_i (H^T H) H^T R^{-1} z = Kz$$

$$r = \hat{z} - z = (1 - K) z = Sz$$

$$S = \begin{bmatrix} t & t & t & t & t \\ 0 & 0 & 0 & 0 & 0 \\ . & . & . & 0 & . \\ . & t & . & . & . \\ 0 & 0 & 0 & 0 & t \end{bmatrix}$$

Critical measurement example



$$P_{ij} = f(v_i, v_j, \theta_i, \theta_j)$$

$$Q_{ij} = f(v_i, v_j, \theta_i, \theta_j)$$

Parameter and structural processing

$$z_i = h_i(x) + e_i \Rightarrow z_i = f_i(p) + e_i$$

- Similar theory can be applied

State estimation functions

1. Bad data processing and elimination given redundant measurements

2. Topology processing:

   create bus/branch model (similar to Y matrix)

3. Observebility analysis:

   all the states in the observable islands have unique solutions

4. Parameter and structural processing

Challenges

- Perception of the process is from the measurements whose quality is, to large extent, out of our control
- The quality of estimates relies on the input whose uncertainty is highly dependent on the ICT infrastructure
- Power system model can contain errors