Lecture 15
Power system state estimation

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Outline

• State estimation
  - What
  - Why

• Weighted Least Square (WLS) algorithms
  - Mathematics
  - Concepts

• Operation challenges
Course road map
What is state estimation?
How could the operator know the system?
The truth is out here!

Input: SCADA measurements

Output: Complete & consistent network representation

Voltagess angles

\[ U = R \cdot I \]

\[ \Sigma I = 0 \]

Bus/branch model

Measurements

Solution

\[ \Sigma (Error)^2 \]

\[ \hat{X} \]

\[ (V1, \phi 1, V2, \phi 2... U_{nq}) \]
Power system states: $x$

- The power system states are those parameters that can be used to determine all other parameters of the power system.

- Node voltage phasor
  - Voltage magnitude $V_k$
  - Phase angle $\Theta_k$

- Transformer turn ratios
  - Turn ratio magnitude $t_{kn}$
  - Phase shift angle $\phi_{kn}$

- Complex power flow
  - Active power flow $P_{kn}$, $P_{n_k}$
  - Reactive power flow $Q_{kn}$, $Q_{n_k}$
Analog measurements

- Voltage magnitude
- Current flow magnitude & injection
- Active & reactive power
  - Branches & groups of branches
  - Injection at buses
  - In switches
  - In zero impedance branches
  - In branches of unknown impedance
- Transformers
  - Magnitude of turns ratio
  - Phase shift angle of transformer
- Synchronized phasors from Phasor Measurement Unit
Network topology processing

Bus breaker model

Bus branch model
Power system measurements: $z$

$z = z_{\text{true}} + e$

$z_{\text{true}}$: power system truth
$z_{\text{true}} = h(x)$

e: measurement error
e = e_{\text{systematic}} + e_{\text{random}}
Measurement model

- How to determine the states ($x$) given a set of measurements ($z$)?

$$z_j = h_j(x) + e_j$$

known  unknown  unknown

where

- $x$ is the true state vector $[V_1, V_2, ..., V_k, \Theta_1, \Theta_2, ..., \Theta_k]$
- $z_j$ is the jth measurement
- $h_j$ relates the jth measurement to states
- $e_j$ is the measurement error
State estimation process

- Analog Measurements: $P_i, Q_i, P_f, Q_f, V, I, \theta$
- Pseudo Measurements
- State Estimator
  - $V, \theta$
- Bad Data Processor
- Topology Processor
- Observability Analysis
- Breaker Positions
Why do we need state estimation?
Measurements correctness

• Imperfections in
  - Current & Voltage transformer
  - Transducers
    • A/D conversions
    • Tuning
  - RTU/IED Data storage
  - Rounding in calculations
  - Communication links

• Result in uncertainties in the measurements
Measurement timeliness

- Due to imperfections in SCADA system the measurements will be collected at different points in time, time skew.

\[ P_{ab} \quad P_{bc} \quad P_{cb} \quad P_{dc} \quad t \]

- If several measurements are missing how long to wait for them?
- Fortunately, not a problem during quansi-steady state.
- State estimation is used for off-line applications
How can the states be estimated?
Approaches

• Minimum variance method
  - Minimize the sum of the squares of the weighted deviations of the state calculated based on measurements from the true state

• Maximum likelihood method
  - Maximizing the probability that the estimate equals to the true state vector $\mathbf{x}$

• Weighted least square method (WLS)
  - Minimize the sum of the weighted squares of the estimated measurements from the true state

\[
J(x) = \sum_{i=1}^{m} \frac{(z_i - h_i(x))^2}{R_{ii}}
\]
• "Least squares" means that the overall solution minimizes the sum of the squares of the errors made in the results of every single equation.

• The method of least squares is a standard approach to the approximate solution of over determined system, i.e., sets of equations in which there are more equations than unknowns.

• The most important application is in data fitting.

• Carl Friedrich Gauss is credited with developing the fundamentals of the basis for least-squares analysis in 1795.
Fred Schweppe introduced state estimation to power systems in 1968.

He defined the state estimator as “a data processing algorithm for converting redundant meter readings and other available information into an estimate of the state of an electric power system”.

Today, state estimation is an essential part in almost every energy management system throughout the world.

WLS state estimation model

\[ z_j = h_j(x) + e_j \]

known \quad unknown \quad unknown
Error characteristics

• The errors in the measurements is the sum of several stochastic variables
  - CT/VT, Transducer, RTU, Communication...
• The errors is assumed as a Gaussian Distribution with known deviations
  - Expected value $E[e_j] = 0$
  - Known deviation $\sigma_j$
• The errors are also assumed to be independent
  - $E[e_i e_j] = 0$
WLS objective function

\[ J(x) = \sum_{i=1}^{m} \frac{(z_i - h(x))^2}{\sigma_i^2} = [z - h(x)]^T R^{-1} [z - h(x)] \]

where
i = 1, 2, ... m

\[ R = \text{diag}\{\sigma_1^2, \sigma_2^2, \sigma_3^2, ..., \sigma_m^2\} = \text{Cov}(e) = E[e \cdot e^T] \]

Solution to above is iterative using newton methods
Newton iteration

- At the minimum, the first-order optimality conditions will have to be satisfied

\[ g(x) = \frac{\partial J(x)}{\partial x} = H^T(x)R^{-1}[z - h(x)] = 0 \]

\[ H^T(x) = \left[ \frac{\partial h(x)}{\partial x} \right] \] is the measurement Jacobian matrix
Newton iteration cont’d

• Expanding the $g(x)$ into its Taylor series around state vector $x^k$

$$g(x) = g(x^k) + G(x^k)(x - x^k) + ..... = 0$$

where

$$G(x^k) = \frac{\partial g(x^k)}{\partial x} = H^T(x^k)R^{-1}H(x^k)$$
Newton iteration cont’d

- Neglecting the higher order terms leads to an iterative solutions scheme known as the Gauss-Newton method as:

\[ x^{k+1} = x^k - [G(x^k)]^{-1} \cdot g(x^k) \]

- \( k \) is the iteration index,
- \( x^k \) is the solution vector at iteration \( k \)
Newton iteration IV

- Convergence

\[ \max(|\Delta x^k|) \leq \xi \]

- If not, update

\[
x^{k+1} = x^k + \Delta x^k
\]

\[ k = k + 1 \]

Go back to the previous step
Why the measurements are weighted?

\[ P_{ij} = f(v_i, v_j, \theta_i, \theta_j) \]

\[ Q_{ij} = f(v_i, v_j, \theta_i, \theta_j) \]
Weight

- Weight is introduced to emphasize the trusted measurement while de-emphasize the less trusted ones.

- WLS

\[ W_i = \frac{1}{\sigma_i^2} \]
Observability

- Based on system topology and location of measurements parts of the power system may be unobservable.

- Unobservable parts of the system can be made observable via data exchange (CIM), pseudo measurements, etc...

A  Observable part of the system of interest
B  Unobservable part of the system of interest
C  Rest of the interconnected system
Observability example

\[ P_{ij} = f(v_i, v_j, \theta_i, \theta_j) \]
\[ Q_{ij} = f(v_i, v_j, \theta_i, \theta_j) \]
Observability criterion

Necessary but not sufficient condition

\[ m \geq n \]

\( m \): Number of measurements
\( n \): Number of states

Is the system’s observability guaranteed in this case?
Observability example II

\begin{align*}
P_{ij} &= f(v_i, v_j, \theta_i, \theta_j) \\
Q_{ij} &= f(v_i, v_j, \theta_i, \theta_j)
\end{align*}
$n$ of $m$ measurements has to be independent

\[ P_{ij} = f(v_i, v_j, \theta_i, \theta_j) \]
\[ Q_{ij} = f(v_i, v_j, \theta_i, \theta_j) \]
Summary of assumptions

• Quansi-steady system
  - No large variations of states over time
  - State estimator will be suspended when large disturbance happens.

• Errors in measurements are
  - Gaussian in nature with known deviation
  - Independent

• Strong assumption
  - Power system topology model is correct
Refinements

• Refinements of method for State Estimation has been the objective of much research.

• Reduce the numerical calculation complexity in order to speed up the execution.
  - 3000 bus node in 1-2 seconds

• Improve estimation robustness
  - less affected by erroneous input
Bad Data Detection
Data quality

• Analog measurement error
• Parameter error
• Topological error
  - Discrete measurement error
  - Model error
Bad Data Detection (Analog)

• Putting the measurements up to a set of logical test (Kirchhoff's laws) before they are input into the State Estimator

• Calculating $J(x)$ and comparing with a predetermined limit. If the value exceeds the limit, we can assume there is “something” wrong in the measurements (Chi-square)
Bad Data Detection (Analog)

- Once the system state has been identified, i.e. we have an estimate of $x$

- We can use the estimate to calculate

$$r_i = z_i - h_i(\hat{x})$$

- If there is any residual in this calculation that "stands-out" this is an indication that particular measurement is incorrect
Largest normalized residual

\[ r_i^N = \frac{z_i - h(\hat{x})}{\sigma_i} \]

- The normalized residual follows the standard normal distribution

\[ N(0, \sigma^2) \]

\[ \mu \pm \sigma \ (68.26\%) \]

\[ \mu \pm 3\sigma \ (99.74\%) \]
Limitations

- The critical measurements have zero residuals, hereby they can not be detected

\[
\hat{z}_i = H_i (H^T H) H^T R^{-1} z = Kz \\
\]

\[
r = \hat{z} - z = (1 - K) z = Sz
\]

\[
S = \begin{bmatrix}
t & t & t & t & t & t \\
0 & 0 & 0 & 0 & 0 & 0 \\
. & . & . & 0 & . & . \\
. & t & . & . & . & . \\
0 & 0 & 0 & 0 & 0 & t \\
\end{bmatrix}
\]
Critical measurement example

\[ P_{ij} = f(v_i, v_j, \theta_i, \theta_j) \]
\[ Q_{ij} = f(v_i, v_j, \theta_i, \theta_j) \]
Parameter and structural processing

\[ z_i = h_i(x) + e_i \Rightarrow z_i = f_i(p) + e_i \]

• Similar theory can be applied
1. Bad data processing and elimination given redundant measurements

2. Topology processing:
   create bus/branch model (similar to Y matrix)

3. Observability analysis:
   all the states in the observable islands have unique solutions

4. Parameter and structural processing
Challenges

- Perception of the process is from the measurements whose quality is, to large extent, out of our control.

- The quality of estimates relies on the input whose uncertainty is highly dependent on the ICT infrastructure.

- Power system model can contain errors.