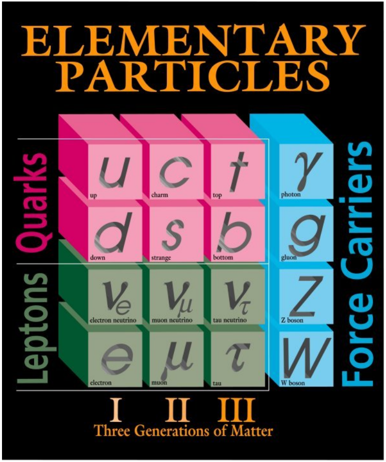
Applications of nuclear physics in neutrino physics

Emanuel Ydrefors

Outline of lecture

- Brief introduction to neutrinos
- Nuclear beta decay
- Neutrino-nucleus scattering



Fermilab 95-759

The neutrino

- One of the particles in the Standard Model
- Three known flavors (+ their antiparticles): $\nu_{\mathrm{e}}, \nu_{\mu}, \nu_{\tau}$.
- Produced in weak interactions such as nuclear beta decay.
- They have small but non-zero masses: $\sum m_{\nu} \lesssim 0.31$ eV. This shows that the Standard Model is incomplete!
- Properties of the neutrino are studied by using "nuclear laboratories".

The neutrino: Important information carriers

- Neutrinos interact very weakly with matter.
- Therefore, neutrinos produced in e.g. supernovae can be studied with detectors placed on Earth.
- Such studies are important for astrophysical applications like supernova modelling.

Nuclear beta decay

Neutron decay:

$$n \longrightarrow p + e^- + \bar{\nu}_e, \quad \tau \approx 15 \text{ min} \tag{1}$$

Nuclear beta decay:

β^- decay:

$$(A, Z) \longrightarrow (A, Z+1) + e^- + \bar{\nu}_e \tag{2}$$

β^+ decay:

$$(A, Z) \longrightarrow (A, Z-1) + e^+ + \nu_e \tag{3}$$

Electron capture:

$$(A, Z) + e^- \longrightarrow (A, Z-1) + \nu_e \tag{4}$$

Calculation of rates for allowed beta decay

- $\Delta J = 0, 1$ and $\pi_i \pi_f = 1$
- The half-life can be written in the form

$$f_0 t_{1/2} = \frac{\kappa}{(B_\mathrm{F} + B_\mathrm{GT})}, \tag{5}$$

where $\kappa = 6147$s, and

$$B_\mathrm{F} = \frac{g_\mathrm{V}^2}{2J_i + 1} |\mathcal{M}_\mathrm{F}|^2, \quad B_\mathrm{GT} = \frac{g_\mathrm{A}^2}{2J_i + 1} |\mathcal{M}_\mathrm{GT}|^2 \tag{6}$$

- Plain values: $g_\mathrm{V} = 1.0$ and $g_\mathrm{A} = 1.26$. But, often $g_\mathrm{A}^\mathrm{eff} = 1.0$ is used in applications.

Nuclear matrix elements for allowed beta decay

- Fermi matrix element

$$\mathcal{M}_{\mathrm{F}} = (J_f||\mathbf{1}||J_i) = \sum_{ab}(a\|\mathbf{1}||b)(J_f||[c_a^\dagger \tilde{c}_b]_0\|J_i) \tag{7}$$

- Gamow-Teller matrix element

$$\mathcal{M}_{\mathrm{GT}} = (J_f\|\boldsymbol{\sigma}||J_i) = \frac{1}{\sqrt{3}}\sum_{ab}(a\|\boldsymbol{\sigma}||b)(J_f||[c_a^\dagger \tilde{c}_b]_1\|J_i) \tag{8}$$

- Conventions: $a = (n_a, l_a, j_a)$, $\alpha = (a, m_\alpha)$, and $\tilde{c}_a = (-1)^{j_a+m_\alpha} c_{-\alpha}$.
- The reduced transition densities $(J_f\|[c_a^\dagger \tilde{c}_b]_L\|J_i)$ contain the nuclear structure dependence and are computed from the adopted nuclear model (e.g. Shell Model, TDA, RPA, ...)
- The single-particle matrix elements depend only on the single-particle states (i.e. the mean field)

Neutrino-nucleus scattering

- Neutrinos and their properties are studied by using neutrino scatterings off nuclei.
- Charged-current (anti-)neutrino-nucleus scattering

$$(A,Z) + \nu_{\mathrm{e}} \longrightarrow (A,Z+1) + e^- \tag{9}$$

$$(A,Z) + \bar{\nu}_{\mathrm{e}} \longrightarrow (A,Z-1) + e^+ \tag{10}$$

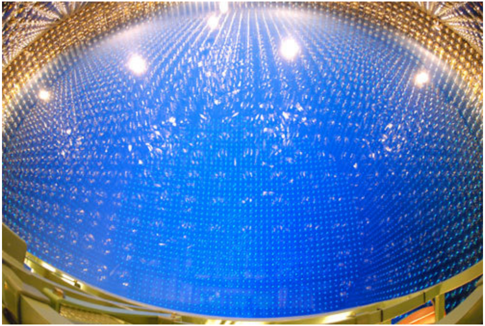
- Neutral-current scattering

$$(A,Z) + \nu \longrightarrow (A,Z)^* + \nu \tag{11}$$

$$(A,Z) + \bar{\nu} \longrightarrow (A,Z)^* + \bar{\nu} \tag{12}$$

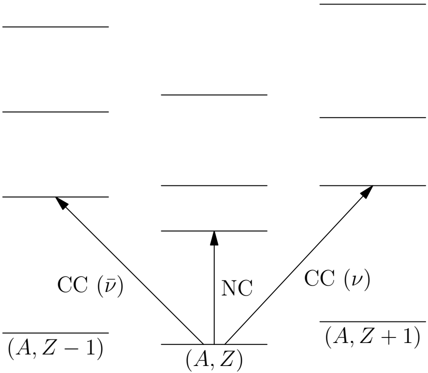
- The neutrinos can be from artificial sources or from astrophysical ones (the Sun, Supernovae)
- In this lecture: Neutrino energies in the range $E \approx 0 - 80$ MeV

Neutrino detection

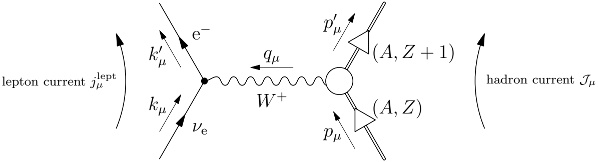


- Neutrino interactions are rare so large detectors are needed!
- The figure shows the Super-Kamiokande detector (Japan) which consists of 50 000 tons of water.

Transitions involved in neutrino scattering

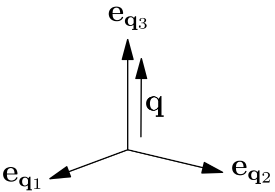


Basic formalism for the ν-nucleus scattering



- $Q^2 = -q_\mu q^\mu \ll M_W^2 \implies \langle f|H_{\text{eff}}|i\rangle = \frac{G}{\sqrt{2}} \int \mathrm{d}^3x l_\mu \mathrm{e}^{-\mathrm{i}\mathbf{q}\cdot\mathbf{x}} \langle f|\mathcal{J}^\mu|i\rangle$
- Here $a^\mu = (a_0, \mathbf{a})$ and $a_\mu b^\mu = a_0 b_0 - \mathbf{a}\cdot\mathbf{b}$.
- Nuclear-structure dependence contained in $\langle f|\mathcal{J}^\mu|i\rangle$

Spherical tensors



Define the spherical components of the unit vectors as

$$\mathbf{e}_{\pm 1} = \mp \frac{1}{\sqrt{2}}(\mathbf{e}_{\mathbf{q}1} \pm \mathbf{e}_{\mathbf{q}2}), \quad \mathbf{e}_0 = \mathbf{e}_{\mathbf{q}3} \tag{13}$$

Neutrino cross section

- Multipole expansion of $\langle f|H_{\text{eff}}|i\rangle$ by using

$$e^{i\mathbf{q}\cdot\mathbf{x}} = \sum_l i^l \sqrt{4\pi(2l+1)} j_l(|\mathbf{q}||\mathbf{x}|) Y_{l0}(\theta,\phi) \tag{14}$$

- After a rather long derivation ...

$$\frac{d^2\sigma}{d\Omega d\omega} = \frac{G^2|k_f|\epsilon_f}{\pi\,(2J_i+1)} F(Z,\epsilon_f)\left(\sum_{J=0}^{\infty}\sigma_{\text{CL}}^J + \sum_{J=1}^{\infty}\sigma_{\text{T}}^J\right),$$

- Total cross section $\sigma(E_i)$ obtained by integrating over the angles and summing up the contributions from all final nuclear states.
- Number of detected neutrinos $n \sim \langle\sigma\rangle$
- For supernova neutrinos $\langle\sigma_\nu\rangle = \frac{1}{T_\nu^3 F_2(\alpha_\nu)} \int \frac{\mathrm{d}E_\nu E_\nu^2 \sigma(E_\nu)}{1+\exp(E_\nu/T_\nu - \alpha_\nu)}$

Nuclear matrix elements

- The nuclear-structure dependence contained in nuclear matrix elements of one-body operators

$$(J_f \| T_J | J_i) = \frac{1}{\sqrt{2J+1}} \sum_{ab} (a \| T_J \| b)(J_f \| [c_a^\dagger \tilde{c}_b]_J \| J_i) \tag{15}$$

- Most important operators are of the forms $F_\mathrm{V}(q) j_0(qx)$, $F_\mathrm{A}(q) j_0(qx) \boldsymbol{\sigma}$, $F_\mathrm{A}(q)[j_1(qx) \boldsymbol{Y}_1 \boldsymbol{\sigma}]_{0^-,1^-,2^-}$
- In the limit $q \to 0$, $j_0(qx) \to 1$ and $j_1(qx) \to 0$.

Summary

- Nuclear-structure calculations are important for many applications in neutrino physics and nuclear astrophysics
- Neutrino-nucleus scattering can be used to probe properties of the neutrino. However, accurate knowledge about the nuclear structure of the relevant target are needed.