Two-particle excitations

May 8, 2013

We found that to avoid core excitations the one-body operator should be defined in terms of normal products. That is to use $: c^\dagger_\alpha c_\beta :$ instead of $c^\dagger_\alpha c_\beta$. It was due to this that we wrote the two-body operator in normal form also. But in doing so we bypassed what maybe an important physics. And indeed there is an important

$$H = \sum_{\alpha\beta} \langle \alpha | T | \beta \rangle c^\dagger_\alpha c_\beta + \frac{1}{4} \sum_{\alpha\beta\gamma\delta} \langle \alpha\beta | V | \gamma\delta \rangle c^\dagger_\alpha c^\dagger_\beta c_\delta c_\gamma .$$

Converting to normal form one gets,

$$H = \sum_{\alpha\beta} \langle \alpha | T | \beta \rangle (: c^\dagger_\alpha c_\beta : + \overbrace{c^\dagger_\alpha c_\beta}) + \frac{1}{4} \sum_{\alpha\beta\gamma\delta} \langle \alpha\beta | V | \gamma\delta \rangle [: c^\dagger_\alpha c^\dagger_\beta c_\delta c_\gamma :$$
$$+ : c^\dagger_\alpha c_\gamma : \overbrace{c^\dagger_\beta c_\delta} - : c^\dagger_\alpha c_\delta : \overbrace{c^\dagger_\beta c_\gamma} - : c^\dagger_\beta c_\gamma : \overbrace{c^\dagger_\alpha c_\delta} + : c^\dagger_\beta c_\delta : \overbrace{c^\dagger_\alpha c_\gamma}$$
$$+ \overbrace{c^\dagger_\alpha c_\gamma}\, \overbrace{c^\dagger_\beta c_\delta} - \overbrace{c^\dagger_\alpha c_\delta}\, \overbrace{c^\dagger_\beta c_\gamma}]$$

Hartree-Fock potential

After some algebra to be performed,

$$H = E_0 + H_{HF} + \frac{1}{4}\sum_{\alpha\beta\gamma\delta}\langle\alpha\beta|V|\gamma\delta\rangle : c_\alpha^\dagger c_\beta^\dagger c_\delta c_\gamma :$$

where

$$E_0 = \sum_\alpha n_\alpha \langle\alpha|T|\alpha\rangle + \frac{1}{2}\sum_{\alpha\beta}\langle\alpha\beta|V|\alpha\beta\rangle_a \tag{9}$$

This is the kinetic energy of particles in the occupied states plus the interaction between particles placed in any pair of levels of the representation. It is the energy carried by the core, as can also be seen by noticing that $E_0 = \langle 0|H|0\rangle$.

The one-body Hamiltonian is

$$H_{HF} = \sum_{\alpha\beta}\left(\langle\alpha|T|\beta\rangle + \sum_\gamma n_\gamma \langle\alpha\gamma|V|\beta\gamma\rangle_a\right) : c_\alpha^\dagger c_\beta :$$

In this Hamiltonian the levels α and β include all states of the representation. These are the levels that we will occupied by particles which eventually will be added to the

The one-body Hamiltonian is

$$H_{HF} = \sum_{\alpha\beta} \left(\langle \alpha | T | \beta \rangle + \sum_{\gamma} n_{\gamma} \langle \alpha\gamma | V | \beta\gamma \rangle_a \right) : c_{\alpha}^{\dagger} c_{\beta} :$$

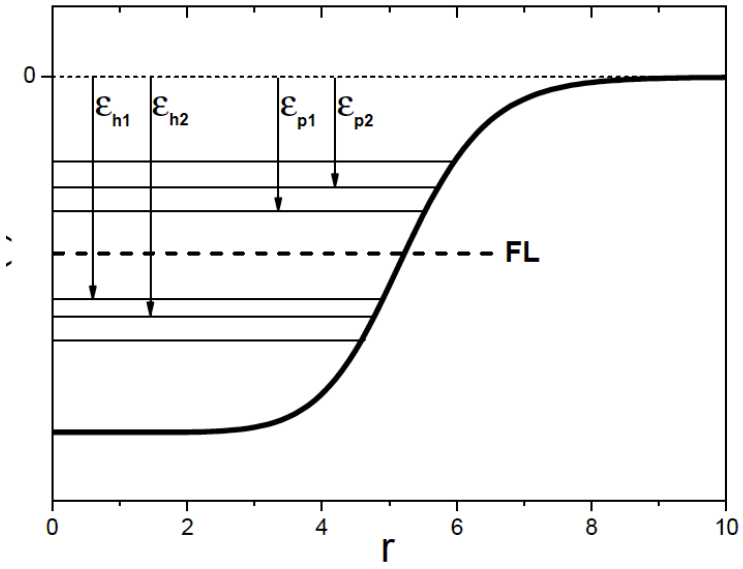
In this Hamiltonian the levels α and β include all states of the representation. These are the levels that we will occupied by particles which eventually will be added to the core. One thus sees that H_{HF} contains the core excitations through the interaction of particles in all occupied states (called $|\gamma\rangle$ in H_{HF}) with the rest of the particles (including those in the core). The Hamiltonian H_{HF}, which is called the Hartree-Fock Hamiltonian, thus corresponds to the core excitation which in the one-body case were assumed to be contained in the renormalized operators.

Random Phase Approximation (RPA)

In this Section we will study the dynamics of the (A+2)- and (A-2)-nuclei, that is of two nucleons added or substracted from the core. For this we will write the Hamiltonian in the Hartre-Fock representation which we will label with greek as well as latin letters. It is,

$$H = \sum_{\alpha} \varepsilon_{\alpha} : c_{\alpha}^{\dagger} c_{\alpha} : + \frac{1}{2} \sum_{\alpha\beta\gamma\delta} \langle \alpha\beta | V | \gamma\delta \rangle : c_{\alpha}^{\dagger} c_{\beta}^{\dagger} c_{\delta} c_{\gamma} : \qquad (7)$$

where ε_{α} is the Hartrre-Fock single-particle energy. The constant energy E_0, Eq. (6), is not included because all eigenvalues of the Hamiltonian (7) will be referred to the core and, therefore, E_0 plays no role.



To obtain the two-particle energies we evaluate the commutator,

$$\begin{aligned}\left[H, c_\alpha^\dagger c_\beta^\dagger\right] =& \sum_i \varepsilon_i \left[: c_i^\dagger c_i :, c_\alpha^\dagger c_\beta^\dagger\right] + \frac{1}{2}\sum_{ijkl}\langle ij|V|kl\rangle \left[: c_i^\dagger c_j^\dagger c_l c_k :, c_\alpha^\dagger c_\beta^\dagger\right] \\ =&(\varepsilon_\alpha + \varepsilon_\beta) c_\alpha^\dagger c_\beta^\dagger + (1 - n_\alpha - n_\beta)\sum_{i<j}\langle ij|V|\alpha\beta\rangle_a c_i^\dagger c_j^\dagger \\ &+ \sum_{i<j}\sum_l \langle ij|V|\beta l\rangle_a : c_i^\dagger c_j^\dagger c_\alpha^\dagger c_l : - \sum_{i<j}\sum_l \langle ij|V|\alpha l\rangle_a : c_i^\dagger c_j^\dagger c_\beta^\dagger c_l :\end{aligned} \tag{8}$$

One sees in this equation that the two-particle creation operators are mixed with three-particle one-hole excitations, that is with core excitation components. In the Random Phase Approximation (RPA) one neglects the core excitations, that is terms of the form $\langle n_2| : c_i^\dagger c_j^\dagger c_\alpha^\dagger c_l : |0\rangle$, because they are supposed to generate states which lie high in the spectrum, thus having little influence over the low-lying two-particle

RPA equation

one gets,

$$\langle n_2|\left[H, c_\alpha^\dagger c_\beta^\dagger\right]|0\rangle = (E_{n_2} - E_0)\langle n_2|c_\alpha^\dagger c_\beta^\dagger|0\rangle$$
$$= (\varepsilon_\alpha + \varepsilon_\beta)\langle n_2|c_\alpha^\dagger c_\beta^\dagger|0\rangle + (1 - n_\alpha - n_\beta)\sum_{i<j}\langle ij|V|\alpha\beta\rangle_a\langle n_2|c_i^\dagger c_j^\dagger|0\rangle$$

which is the RPA equation. The term $1 - n_\alpha - n_\beta$ in the RPA equations shows that one can place two particles above the Fermi level, in which case it is $1 - n_\alpha - n_\beta = 1$, or below it ($1 - n_\alpha - n_\beta$ = -1). These two forms of excitations are mixed to each other, given rise to the so-called RPA correlations. This also implies that within the RPA one evaluates simultaneously the (A+2)- and (A-2)-systems and, therefore, there is an influence of one system upon the other.

With $\omega_{n_2} = E_{n_2} - E_0$ the RPA equation can be written in matrix form as

$$\omega_{n_2} \begin{pmatrix} X_{n_2} \\ Y_{n_2} \end{pmatrix} = \begin{pmatrix} A & B \\ -C & -D \end{pmatrix} \begin{pmatrix} X_{n_2} \\ Y_{n_2} \end{pmatrix} \tag{10}$$

where $X_{n_2}(\alpha\beta) = \langle n_2 | c_\alpha^\dagger c_\beta^\dagger | 0 \rangle$ with α and β particle states and $Y_{n_2}(\alpha\beta) = \langle n_2 | c_\alpha^\dagger c_\beta^\dagger | 0 \rangle$ but with α and β hole states. In the same fashion the indices of A are all particle states and the indices of D are all hole states. Instead in the matrices B and C the indices are mixed. For instance $C(\alpha\beta\gamma\delta) = \langle \gamma\delta | V | \alpha\beta \rangle_a$, where α and β are hole states while γ and δ are particle states. Notice that the minus sign in front of the matrices C and D comes from the factor $1 - n_\alpha - n_\beta$ in Eq. (9). Due to this, the RPA matrix (10) is not Hermitian and, therefore, the energies ω_{n_2} can become complex quantities.

As an example, we evaluate the energy of the a system with one particle orbital (α) and one hole orbital (β). The RPA equation can be written in a 2×2 matrix form as

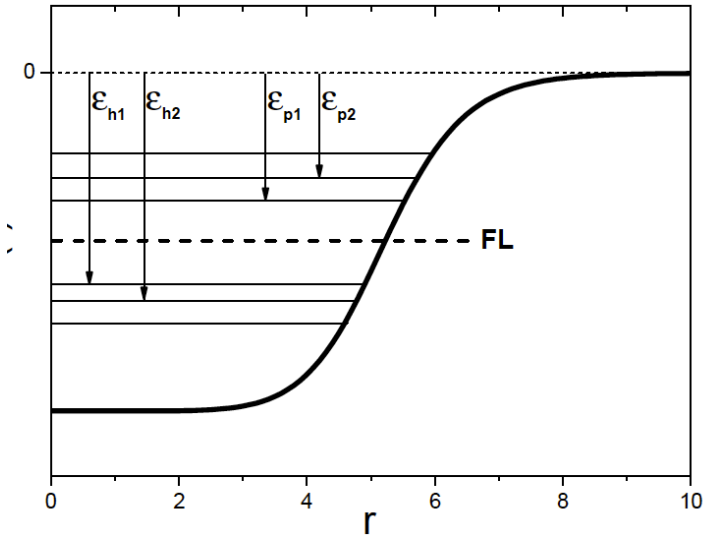
$$\omega_{n_2} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} A & B \\ -C & -D \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}. \tag{7.74}$$

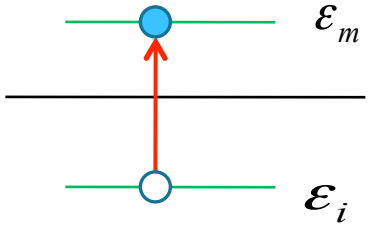
We have $A = 2\varepsilon_\alpha + \langle \alpha\alpha | V | \alpha\alpha \rangle^{JT}$ where JT denote the spin and isospin of the two particle state. $D = -2\varepsilon_\beta + \langle \beta\beta | V | \beta\beta \rangle^{JT}$. $B = C = \langle \alpha\alpha | V | \beta\beta \rangle^{JT}$. In such a case we get two solutions for the eigen values, $\omega_{n_2} = \frac{A-D}{2} \pm \sqrt{\left(\frac{A-D}{2}\right)^2 + AD - C^2}$, corresponding to the particle-particle and hole-hole excitations respectively. For a system with no particle-hole correlation, we get $\omega_{n_2} = A$ for the two-particle state and $\omega_{n_2} = -D$ for the two-hole state.

Tamm-Damkoff Approximation (TDA)

The difference between TDA and RPA is that we use

- The simple particle-hole vacuum |HF> in TDA
- The correlated ground state in the RPA





Tamm-Damkoff Approximation (TDA)

We will concentrate in the shell model in this course, and here one has either two-particle or two-hole excitations, and the (A+2) and (A-2) systems are independent of each other. The shell model cases are actually particular cases of the RPA since one gets them by imposing the condition that only particles can occupied particle states and only holes can occupied hole states. This is called Tamm-Damkoff approximation (TDA).

This approximation implies that the matrices B and C vanish in Eq. (10). The particle- and hole-states decoupled and the RPA equation transforms in two TDA equations, one for particle states, i. e.

$$\omega_{n_2} X_{n_2} = A X_{n_2}$$

and the other one for hole states,

$$-\omega_{n_2} Y_{n_2} = D Y_{n_2}$$

Since the matrices A and D are Hermitians the energies are real, as they should be.

TDA equation

$$\begin{aligned}\langle n_2|\left[H, c_\alpha^\dagger c_\beta^\dagger\right]|0\rangle =&(E_{n_2}-E_0)\langle n_2|c_\alpha^\dagger c_\beta^\dagger|0\rangle \\ =&(\varepsilon_\alpha+\varepsilon_\beta)\langle n_2|c_\alpha^\dagger c_\beta^\dagger|0\rangle+\sum_{i<j}\langle ij|V|\alpha\beta\rangle_a\langle n_2|c_i^\dagger c_j^\dagger|0\rangle\end{aligned} \tag{14}$$

which is the TDA equation. It is also the shell model equation, which we will apply in the next Chapter.

For a system with two particles in one orbital α, we simply have $\omega_{n_2}=2\varepsilon_\alpha+\langle\alpha\alpha|V|\alpha\alpha\rangle^{JT}$ where JT denote the spin and isospin of the two particle state.

For holes

$$\begin{aligned}\langle n_2|\left[H, c_\alpha c_\beta\right]|0\rangle =&(E_{n_2}-E_0)\langle n_2|c_\alpha c_\beta|0\rangle \\ =&-(\varepsilon_\alpha+\varepsilon_\beta)\langle n_2|c_\alpha^\dagger c_\beta^\dagger|0\rangle+\sum_{i<j}\langle ij|V|\alpha\beta\rangle_a\langle n_2|c_i c_j|0\rangle\end{aligned}$$

Two-particle outside a closed core (in jj coupled scheme)

$$H = H_0 + V$$

$$H_0|pq; JM\rangle = \varepsilon_p + \varepsilon_q|pq; JM\rangle$$

$\{|\alpha\rangle = |pq; JM\rangle\}$ form the orthonormal bases

$$\sum_\alpha |\alpha\rangle\langle\alpha| = \hat{I}$$

$$\sum_\beta \langle\alpha|(H_0 + V)|\beta\rangle\langle\beta|n\rangle = E_n\langle\alpha|n\rangle$$

$$\sum_\beta \Big[(\varepsilon_\beta - E_n)\delta_{\alpha\beta} + \langle\alpha|V|\beta\rangle\Big]\langle\beta|n\rangle = 0$$

where

$$|\beta\rangle = |rs; JM\rangle, \quad \varepsilon_\beta = \varepsilon_r + \varepsilon_s$$

The wave function is

$$|n\rangle = \sum_{\beta} \langle \beta | n \rangle | \beta \rangle, \quad \text{or} \quad |n\rangle = \sum_{p \leqslant q} X(pq;n) | pq; J \rangle$$

where $X(pq;n) = \langle pq; JM | n \rangle$ and the Hamiltonian equations are

$$\sum_{r \leqslant s} \Big[(\varepsilon_p + \varepsilon_q - E_n)\delta_{pr}\delta_{qs} + \langle pq; J | V | rs; J \rangle \Big] X(rs;n) = 0$$

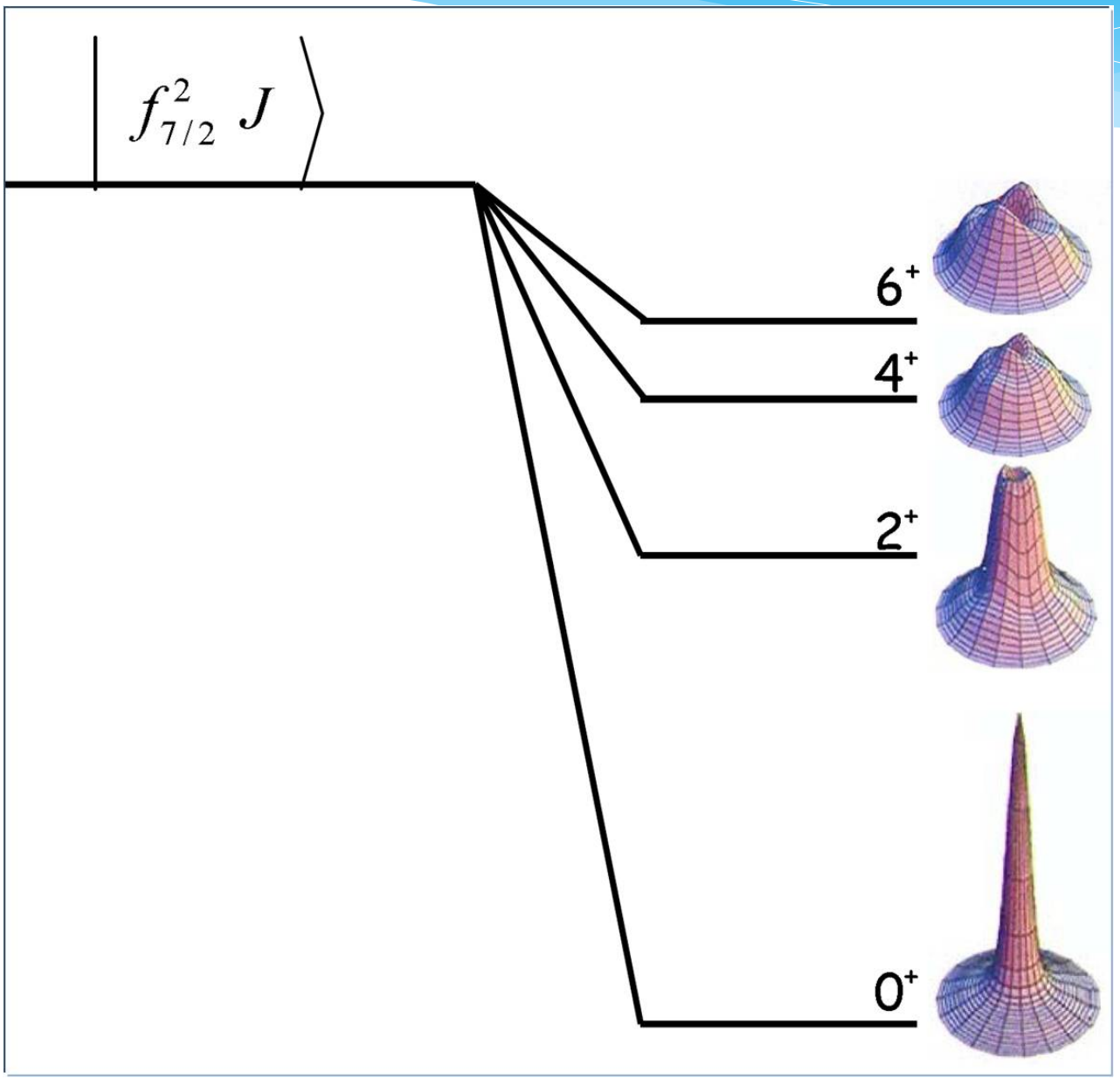
notice that this is M independent.

The two-body interaction

$\langle pq; J | V | rs; J \rangle$

Two particles in a single j shell

The *J=0 pairing interaction* is the dominant component of the nuclear interaction.



Seniority
The number of unpaired nucleons

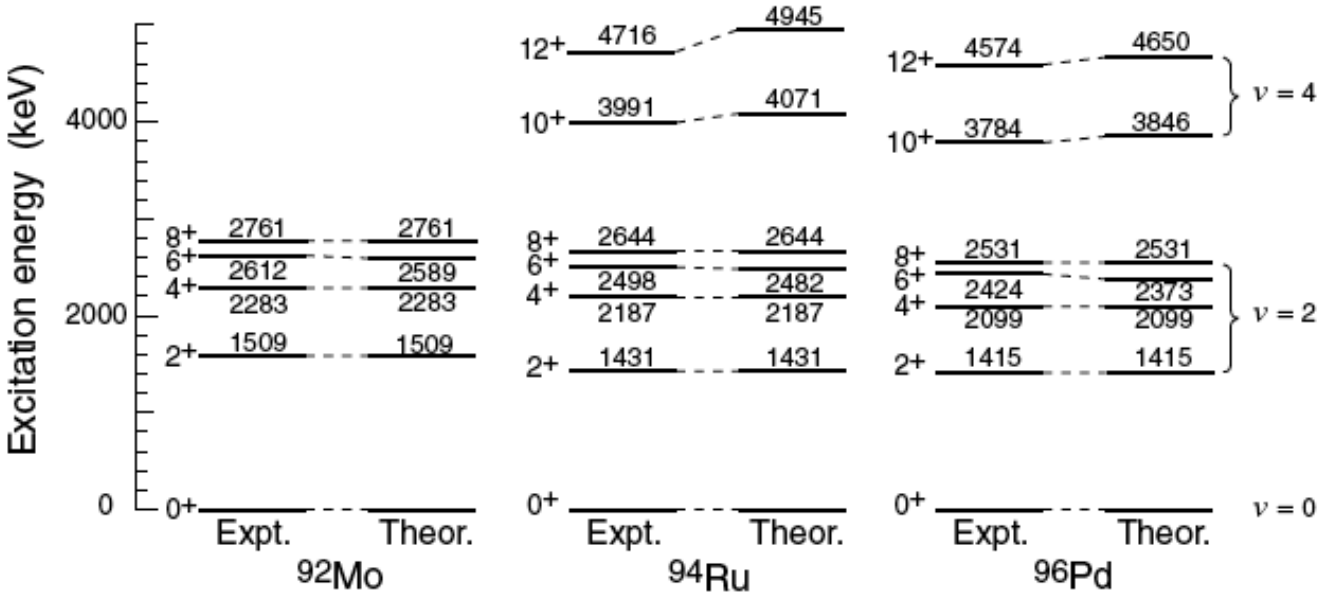


FIG. 1. Low-energy spectra of $N = 50$ isotones fitted with a seniority conserving interaction for $1g_{9/2}$-shell configurations.

Configuration mixing

Two nucleons in p1/2 and g9/2 shells

There are two basis states for 0^+

$$|\alpha\rangle = |(p_{1/2})^2; 0^+\rangle \quad \text{and} \quad |\beta\rangle = |(g_{9/2})^2; 0^+\rangle$$

$$\begin{vmatrix} 2\varepsilon_1 + \langle\alpha|V|\alpha\rangle - E_n & \langle\alpha|V|\beta\rangle \\ \langle\beta|V|\alpha\rangle & 2\varepsilon_2 + \langle\beta|V|\beta\rangle - E_n \end{vmatrix} = 0$$

$$(2\varepsilon_1 + \langle\alpha|V|\alpha\rangle - E_n)(2\varepsilon_2 + \langle\beta|V|\beta\rangle - E_n) - \langle\alpha|V|\beta\rangle^2 = 0$$

calling $V_{\alpha\beta} = \langle\alpha|V|\beta\rangle$ one gets

$$E_n^2 - E_n(2\varepsilon_1 + 2\varepsilon_2 + V_{\alpha\alpha} + V_{\beta\beta}) + (2\varepsilon_1 + V_{\alpha\alpha})(2\varepsilon_2 + V_{\beta\beta}) - V_{\alpha\beta}^2 = 0$$

$$E_n = \varepsilon_1 + \varepsilon_2 + \frac{V_{\alpha\alpha} + V_{\beta\beta}}{2} \pm \left[\left(\varepsilon_1 - \varepsilon_2 + \frac{V_{\alpha\alpha} - V_{\beta\beta}}{2} \right)^2 + V_{\alpha\beta}^2 \right]^{1/2}$$

For the wave functions

$$X(\alpha;n) = \langle \alpha | n \rangle = \langle p^2_{1/2}; 0^+ | n \rangle; \quad X(\beta;n) = \langle \beta | n \rangle = \langle g^2_{9/2}; 0^+ | n \rangle$$

$$\begin{cases} (\varepsilon_\alpha - E_n + V_{\alpha\alpha})X(\alpha;n) + V_{\alpha\beta}X(\beta;n) = 0 \\ V_{\alpha\beta}X(\alpha;n) + (\varepsilon_\beta - E_n + V_{\beta\beta})X(\beta;n) = 0 \end{cases}$$

since we have obtained the energies E_n such that the determinant is 0, it is

$$\begin{cases} (\varepsilon_\alpha - E_n + V_{\alpha\alpha})X(\alpha;n) = -V_{\alpha\beta}X(\beta;n) \\ X^2(\alpha;n) + X^2(\beta;n) = 1 \end{cases}$$

Separable Force

An interaction which is often used in nuclear physics is the separable force given by

$$\langle pq; J|V|rs; J\rangle = -Gf(pq; J)f(rs; J)$$

$$\sum_{r\leqslant s}\Big[(\varepsilon_p + \varepsilon_q - E_n)\delta_{pr}\delta_{qs} - Gf(pq; J)f(rs; J)\Big]X(rs; n) = 0$$

$$X(pq; n) = G\frac{f(pq; J)}{\varepsilon_p + \varepsilon_q - E_n}\sum_{r\leqslant s} f(rs; J)X(rs; n)$$

multiplying by $\sum_{p\leqslant q} f(pq; J)$ one gets

$$\sum_{p\leqslant q} f(pq; J)X(pq; n) = G\sum_{p\leqslant q}\frac{f^2(pq; J)}{\varepsilon_p + \varepsilon_q - E_n}\sum_{r\leqslant s} f(rs; J)X(rs; n)$$

$$G\sum_{p\leqslant q}\frac{f^2(pq; J)}{\varepsilon_p + \varepsilon_q - E_n} = 1$$

$$G\left(\frac{f^2(\alpha;0^+)}{2\varepsilon_1-E_n}+\frac{f^2(\beta;0^+)}{2\varepsilon_2-E_n}\right)=1$$

$$G=\left(\frac{f^2(\alpha;0^+)}{2\varepsilon_1-E_n}+\frac{f^2(\beta;0^+)}{2\varepsilon_2-E_n}\right)^{-1}$$

The pairing force in nuclear physics is used for the states 0^+ as

$$f(pq;0^+) = f(pp;0^+) = \sqrt{2j_p+1}$$

For the states in ^{90}Zr one has

$$f(\alpha;0^+) = f(p^2_{1/2};0^+) = \sqrt{2}; \quad f(\beta;0^+) = f(g^2_{9/2};0^+) = \sqrt{10}$$

$$G = \left(\frac{2}{2\varepsilon_1 - E_{0^+_1}} + \frac{10}{2\varepsilon_2 - E_{0^+_1}} \right)^{-1}$$

and

$$X(\alpha;n) = G\frac{\sqrt{2}}{2\varepsilon_1 - E_n}, \quad X(\beta;n) = G\frac{\sqrt{10}}{2\varepsilon_2 - E_n}$$

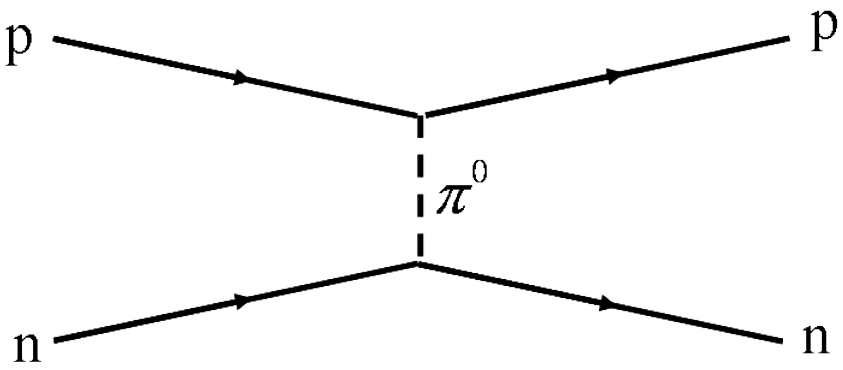
NN interaction and LST coupling

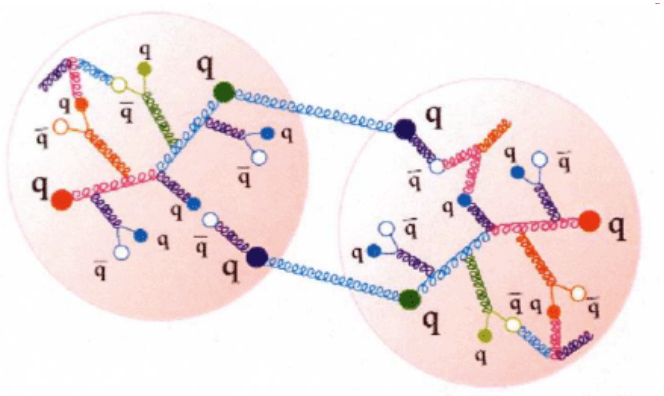
Philosophical issue: What are the relevant degrees of freedom since it is pretty complicated inside a nucleon?

Answer: It depends on the energy scale!

Nuclear Physics: MeV

The atomic nucleus consists of protons and neutrons (two types of baryons) bound by the nuclear force (also known as the residual strong force). The baryons are further composed of subatomic fundamental particles known as quarks. The residual strong force is a minor residuum of the strong interaction which binds quarks together to form protons and neutrons. At low energies, the two nucleons "see" each other as structure-less point particles.





Basic properties of the NN interaction

Properties of nuclear forces :

✧Nuclear forces are finite range forces. For a distance of the order of 1 fm they are quite strong. *Short-range repulsion ("hard core")*

✧These forces show the property of saturation. It means each nucleon interacts only with its immediate neighbours. Volume and binding energies of nuclei are proportional to the mass number A.

The distance *b is* found empirically to be of order *b=1.4fm. V(r) is maximally attractive inside 1 fm while for* very short distances the nucleon-nucleon interaction becomes repulsive.

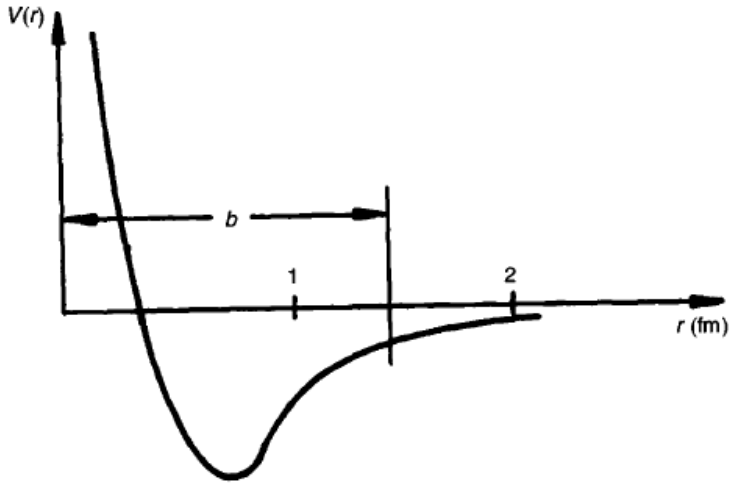


Fig. 13.1. Schematic illustration of the radial dependence of the nucleon–nucleon interaction.

A brief history of NN interactions

1935 – Yukawa (meson theory or Meson Hypothesis)

1950's – Full One-Pion-Exchange potential (OPEP)

--Hamada-Jonston

1960's – non-relativistic One-Boson-Exchange potential (OBEP) (pions, Many pions, scalar mesons, 782(ω), 770(ρ), 600(σ))

1970's – fully relativistic OBEPs

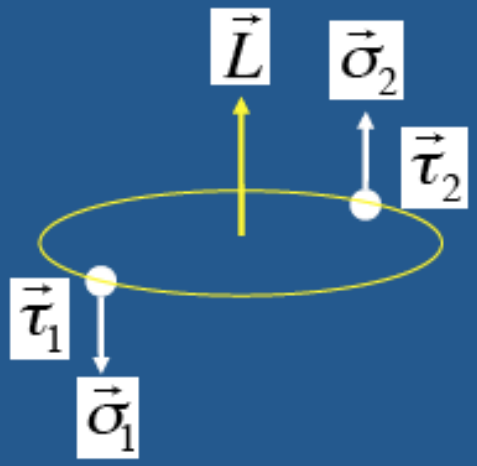
-- 2-pion exchange

-- Paris, Bonn potential

1990's – High-precision Nijmegen, Argonne V18, Reid93, Bonn potentials

1990-2000's – Chiral or Effective Field Theory potentials (2 and 3 body), Lattice QCD

N-N quantum states



$$\vec{L} = \vec{r} \times \vec{p}$$ orbital ang. momentum

$$\vec{S} = \frac{\hbar}{2}\left(\vec{\sigma}_1 + \vec{\sigma}_2\right)$$ spin of N-N pair

$$\vec{J} = \vec{L} + \vec{S}$$ total ang. momentum

$$\vec{T} = \frac{1}{2}\left(\vec{\tau}_1 + \vec{\tau}_2\right)$$ isospin of N-N pair

Spectroscopic notation: $^{(2S+1)}L_J$ use S,P,D,... for L=0,1,2,...

N-N state vector: $\left|\Psi(1,2)\right\rangle = \left|LS;JM_J\right\rangle \otimes \left|T,T_z\right\rangle$

The total spin is either $S = 1$ (triplet) or $S = 0$ (singlet), whose wave functions take the form (problem 13.2)

$$\chi_m^{S=1} = \begin{cases} \alpha(1)\alpha(2) & , m = 1 \\ \beta(1)\beta(2) & , m = -1 \\ (1/\sqrt{2})\,[\alpha(1)\beta(2) + \beta(1)\alpha(2)] & , m = 0 \end{cases}$$

$$\chi_0^{S=0} = \frac{1}{\sqrt{2}}\,[\alpha(1)\beta(2) - \beta(1)\alpha(2)]$$

It is evident that the triplet wave function is symmetric in the spin variables while the singlet wave function is antisymmetric. Thus, for identical particles, even L must be combined with $S = 0$ and odd L with $S = 1$. These wave

Antisymmetric two-particle wave functions

For identical nucleons, i.e. either protons or neutrons, the Pauli exclusion principle requires that a many-nucleon wave function be antisymmetric in all particle coordinates. thus if the space and spin variables of any two protons or any two neutrons are interchanged, the wave function must reverse its sign.

Isospin symmetry requires that the wave function reverse its-sign upon an odd permutation of all coordinates (i.e. space, spin and isospin) of any two nucleons.

This property is strongly connected with the symmetry of the two-particle wave function $|1\,2\rangle$. Since nucleons are fermions, they have to be totally antisymmetric. For example, if we take a product wave function built out of ordinary space, a spin and an isospin part

$$\langle \mathbf{r}_1 s_1 t_1, \mathbf{r}_2 s_2 t_2 \mid 12 \rangle = \varphi(\mathbf{r}_1, \mathbf{r}_2)\chi(s_1, s_2)\zeta(t_1, t_2)$$

we have four combinations compatible with the Pauli principle

φ	χ	abbreviation	ζ
even	singlet	es	+
even	triplet	et	−
odd	singlet	os	−
odd	triplet	ot	+

Table 13.1. *Possible states defined by internal spin S, orbital angular momentum L, total angular momentum J and parity π applicable to the NP (neutron–proton) and NN and PP systems, respectively. In the last column, the corresponding isospin is given. Only those states having $L \leq 3$ are indicated.*

	S	L	J^P	Symmetry	Notation	Isospin T
NP only	1	0	1^+	symmetric	$^3S_1$	0
	1	2	$1^+, 2^+, 3^+$	in	$^3D_{1,2,3}$	
	0	1	1^-	spin + position	$^1P_1$	
	0	3	3^-		$^1F_3$	
NN PP and NP	1	1	$0^-, 1^-, 2^-$	antisymmetric	$^3P_{0,1,2}$	1
	1	3	$2^-, 3^-, 4^-$		$^3F_{2,3,4}$	
	0	0	0^+	in	$^1S_0$	
	0	2	2^+	spin + position	$^1D_2$	

The deuteron and low-energy nucleon-nucleon scattering data

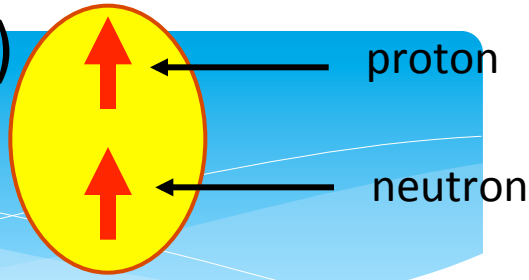
In the 1940s and early 1950s information about the nucleon–nucleon interaction came largely from studying the simplest non-trivial nucleus, the deuteron, denoted d or ^{2}H, consisting of a neutron and a proton. For the deuteron the most important properties, known since the 1930s are the following

binding energy	$E_B = 2.25$ MeV
spin, parity	$J^\pi = 1^+$
isospin	$T = 0$
magnetic moment	$\mu = 0.8574$ n.m. $= \mu_p + \mu_n - 0.0222$ n.m.
quadrupole moment	$Q = 2.82 \times 10^{-3}$ barn

Much more information about the nucleon-nucleon interaction has been obtained from the scattering of proton and neutron projectiles against protons and neutrons.

Deuteron : ground state J = 1 (Total spin S=1)

The deuteron is the only bound state of 2 nucleons, with isospin $T = 0$, spin-parity $J^{\pi} = 1^{+}$, and binding energy E_B=2.225 MeV. For two spin $\frac{1}{2}$ nucleons, only total spins $S = 0, 1$ are allowed. Then the orbital angular momentum is restricted to $J - 1 < l < J + 1$, i.e., $l = 0$, 1 or 2. Since the parity is $\pi = (-)^{l} = +$, only $l = 0$ and $l = 2$ are allowed; this also implies that we have $S = 1$.



$$\psi_d = a\left|{}^3S_1\right\rangle + b\left|{}^3D_1\right\rangle$$

Relative motion : S wave (L=0) + D wave (L=2)

Tensor force does mix

$$V_T = (\tau_1\tau_2)\,(\,[\sigma_1\sigma_2]^{(2)}\,Y^{(2)}(\Omega)\,)\,Z(r)$$

contributes only to **S=1** states

relative motion

The tensor force is crucial to bind the deuteron. Without tensor force, deuteron is unbound.

No S wave to S wave coupling by tensor force because of Y_2 spherical harmonics

magnet

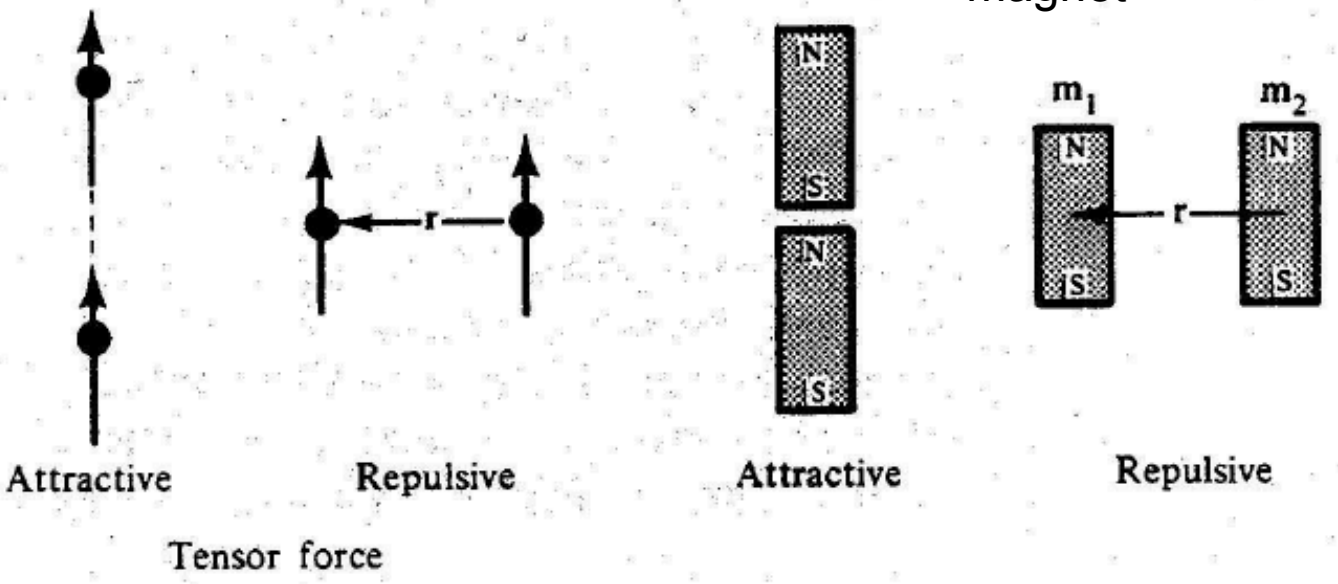


Fig. 14.11. The tensor force in the deuteron is attractive in the cigar-shaped configuration and repulsive in the disk-shaped one. Two bar magnets provide a classical example of a tensor force.

$$\psi_d = a\left|{}^3S_1\right\rangle + b\left|{}^3D_1\right\rangle$$

$$H = -\frac{\hbar^2}{M}\frac{1}{r}\frac{d^2}{dr^2}r + \frac{\hbar^2}{M}\frac{L^2}{r^2} + V_C(r) + V_T(r)S_{12}$$

we find the radial equations

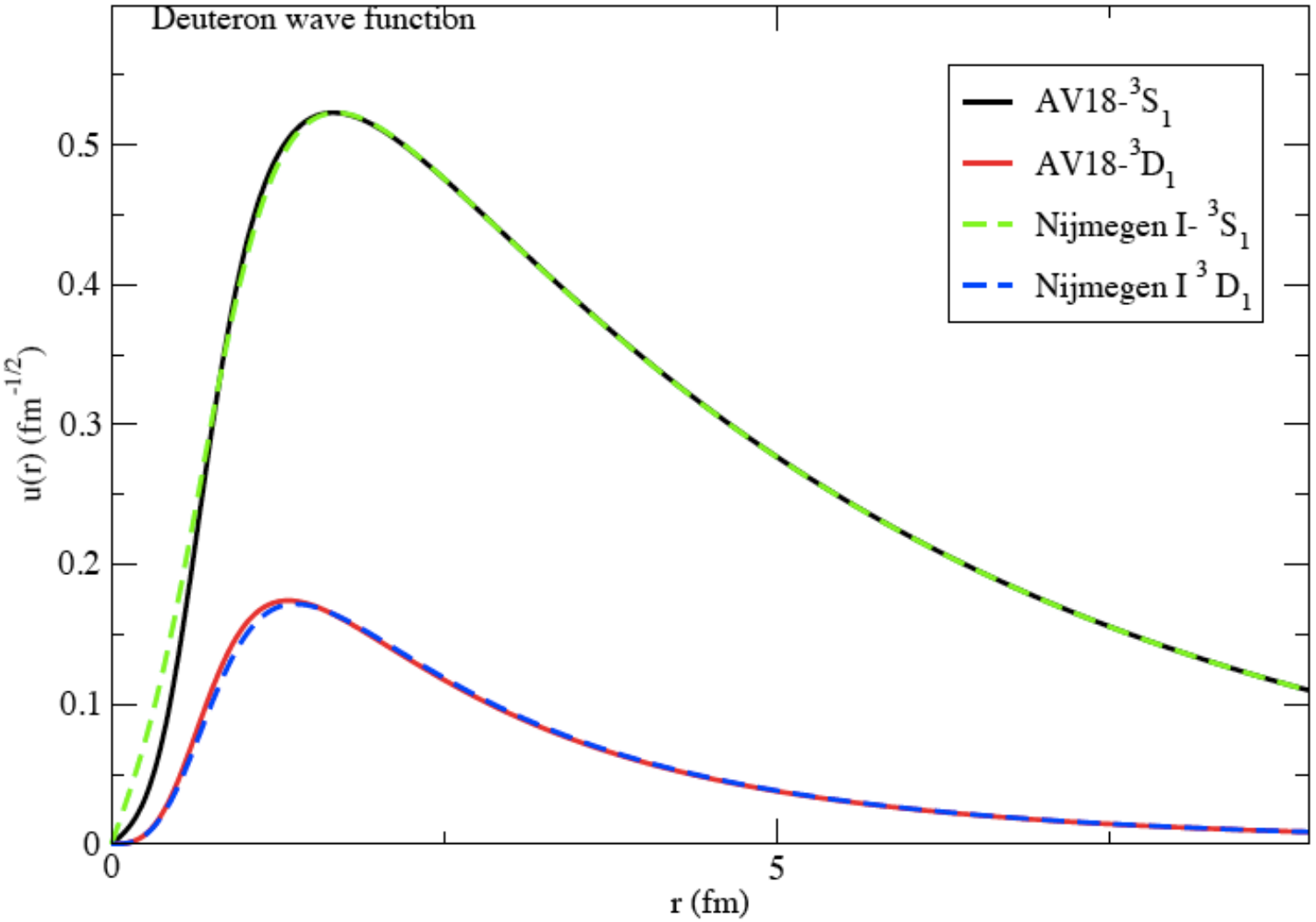
$$\left[\frac{\hbar^2}{M}\frac{d^2}{dr^2} + E - V_c(r)\right]u_S = \sqrt{8}V_T(r)u_D$$

$$\left[\frac{\hbar^2}{M}\left(\frac{d^2}{dr^2} - \frac{6}{r^2}\right) + E + 2V_T(r) - V_c(r)\right]u_D = \sqrt{8}V_T(r)u_S$$

These equations can be solved numerically.

http://www.phy.anl.gov/theory/research/av18/index.html

http://nn-online.org/NN/



http://www.phy.anl.gov/theory/movie-run.html

Anybody has a better solution?