EL2520
Control Theory and Practice

Lecture 14: Summary

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Course aims

The course aims to make the participants aware of ideas and methods in advanced control, especially multivariable feedback systems.

Key ingredients:

- A modern view of SISO control
  - To be able to perform good control designs, trading off performance goals, respecting fundamental limitations, and ensuring robust performance in spite of model uncertainty
- Multivariable control
  - Multivariable systems (poles, zeros, gains and directions), interactions and RGA, optimization-based design methods
- Dealing with hard constraints
  - Anti-windup and model-predictive control

Checklist

The basics

- Matrix manipulations, eigenvalue and singular value computations
- Complex numbers
- Differential equations, state-space models and transfer functions

A modern perspective on SISO control:

- Signal norms, system gains and the small gain theorem
- The closed loop system and the central transfer functions
- Internal stability
- Fundamental limitations due to RHP poles/zeros, time delays
- Reasonable design goals and mapping to loop gain specifications
- Dealing with (multiplicative) system uncertainty

Checklist

Multivariable linear systems

- Transfer matrices and block diagram calculations
- Multivariable poles and zeros
- Gains and directions

Multivariable control design techniques:

- H_2 and H_∞ control: weighting functions and extended system
- LQG: design, optimal control structure and disturbance models
- Glover-McFarlane robustification
- The relative gain array for decentralized control structure design
- Control-order reduction

Checklist

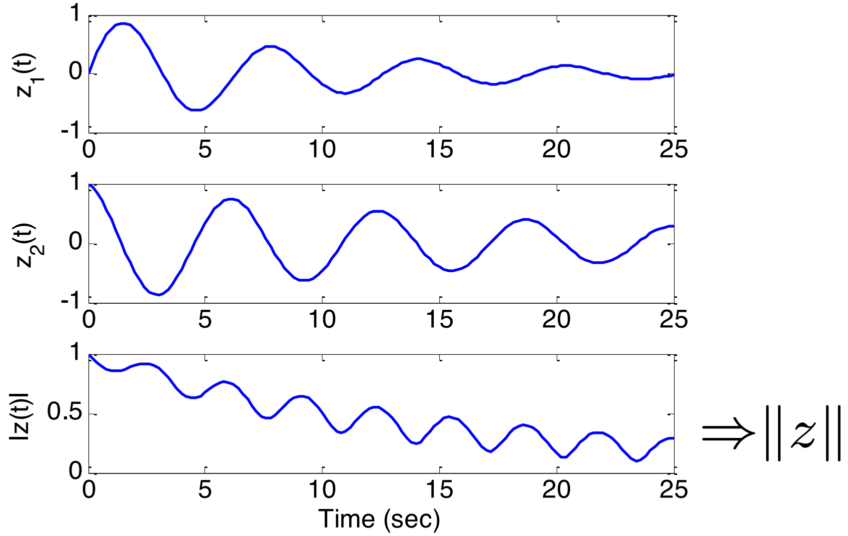
Dealing with hard constraints:

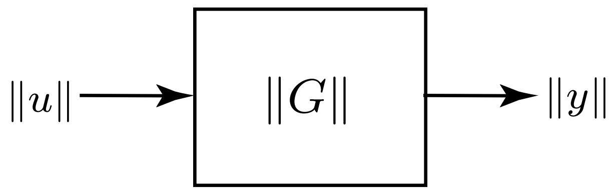
- Sampling of linear systems (continuous→discrete time)
- Anti-windup to deal with actuator saturation
- Model predictive control.

Lecture 1

Signal norms: measure signal size across space (channels) and time

System gains: bounds signal (norm) amplification





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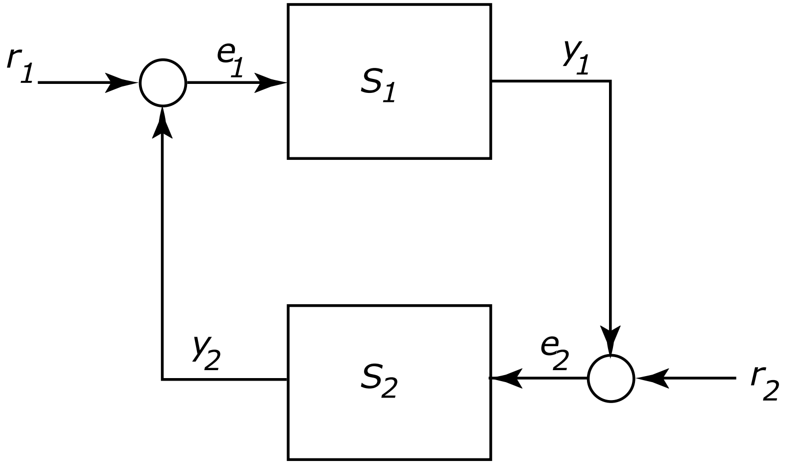
Small gain theorem

Theorem. Consider the interconnection



If $\mathcal{S}_1$ and $\mathcal{S}_2$ are input-output stable and

$$\|\mathcal{S}_1\| \cdot \|\mathcal{S}_2\| < 1$$

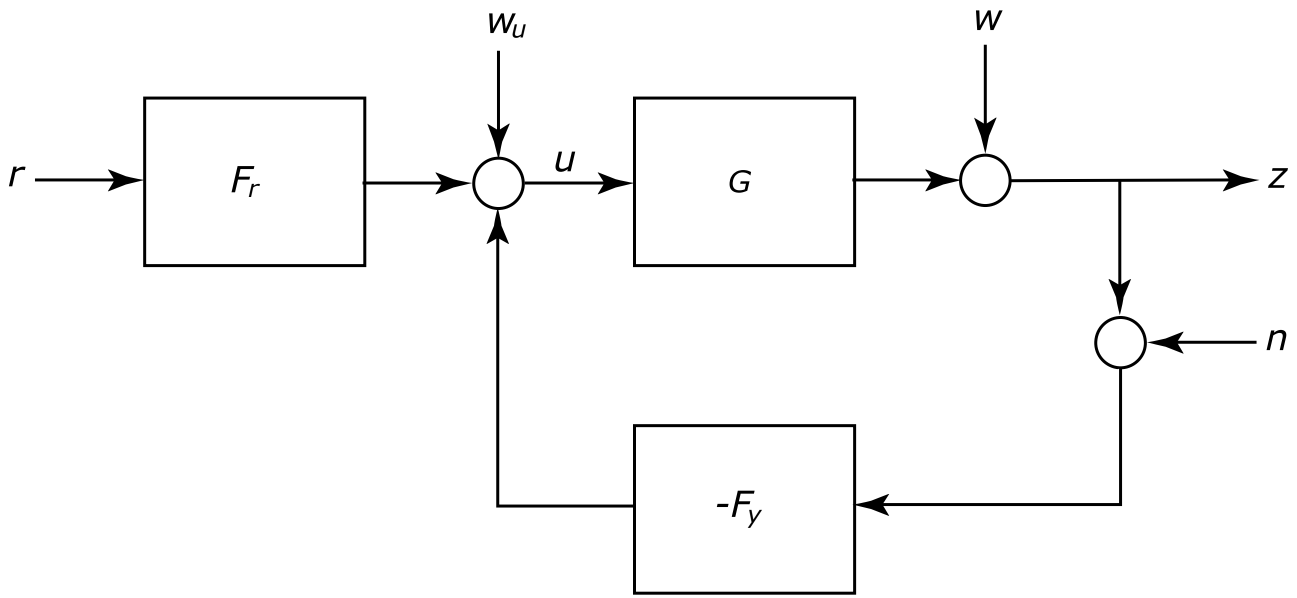
Then, the closed-loop system with r_1, r_2 as inputs and e_1, e_2, y_1, y_2 as outputs is input-output stable.

Lecture 1

Example: Compute the H_∞ norm of the system

$$G(s) = \frac{\omega_0^2}{s^2 + 0.2\omega_0 s + \omega_0^2}$$

Lecture 2 - The closed-loop system



Controller: feedback F_y and feedforward F_r

Disturbances: w, w_u drive system from desired state

Measurement noise: corrupts information about z

Aim: find controller such that z follows r.

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Transfer functions and observations

$$S = \frac{1}{1+GF_y} \qquad (w \to z,\; w_u \to u)$$ sensitivity function

$$T = \frac{GF_y}{1+GF_y} \qquad (n \to z)$$ complementary sensitivity

$$G_c = \frac{GF_r}{1+GF_y} \qquad (r \to z)$$ closed loop system

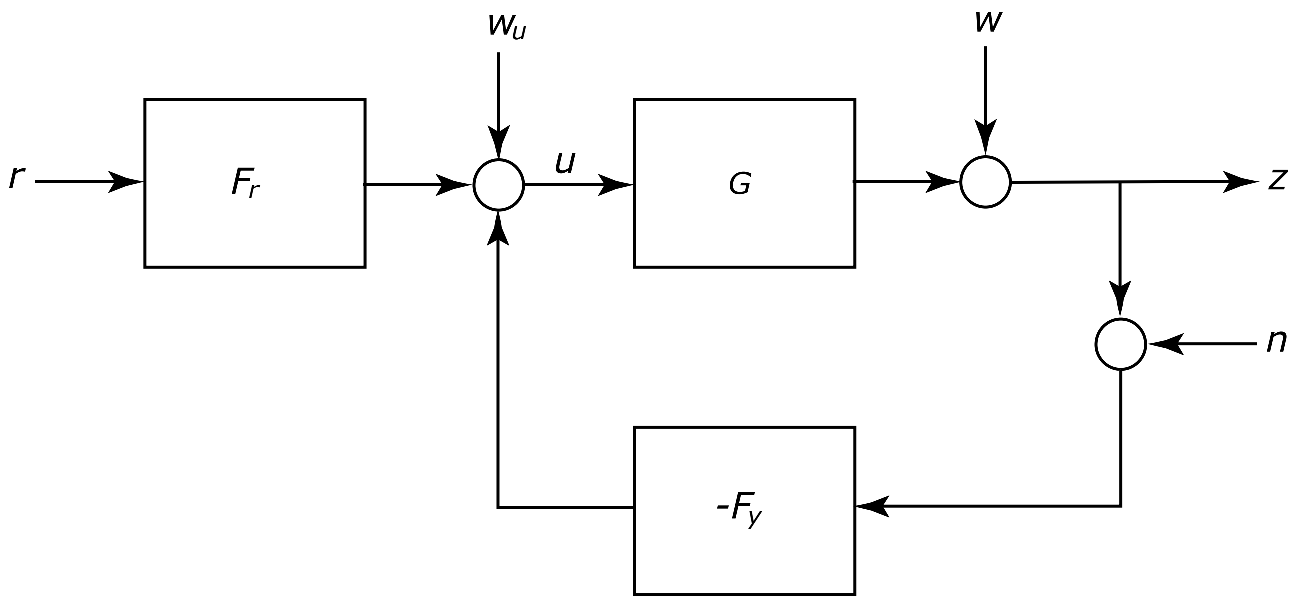
$$SG = \frac{G}{1+GF_y} \qquad (w_u \to z)$$

$$SF_y = \frac{F_y}{1+GF_y} \qquad (n \to u)$$

$$SF_r = \frac{F_r}{1+GF_y} \qquad (r \to u)$$

Observations: need to look at all! Many tradeoffs (e.g. S+T=1)

Lecture 2 – Internal Stability



Definition. The closed loop system above is *internally stable* if it is input-output stable from r, w_u, w, n to all outputs u, z, y.

Theorem. The closed-loop system is stable if and only if

$$S,\ SG,\ SF_y,\ F_r$$

are stable

Lecture 2 – The Sensitivity Functions

The sensitivity function (S):

- Quantifies the disturbance attenuation due to closed-loop control

The complementary sensitivity (T)

- Equals the closed-loop system when controller is error feedback
- Determines robust stability properties

A first trade-off: S+T=1.

Lecture 2

Example (2001-03-08) Let

$$G(s) = \frac{1}{s}\left(\frac{-s+2}{s+2}\right)$$

Determine a controller U(s)=F(s)[R(s)-Y(s)] that achieves

$$G_c(s) = \frac{1}{1+sT}\left(\frac{-s+2}{s+2}\right)$$

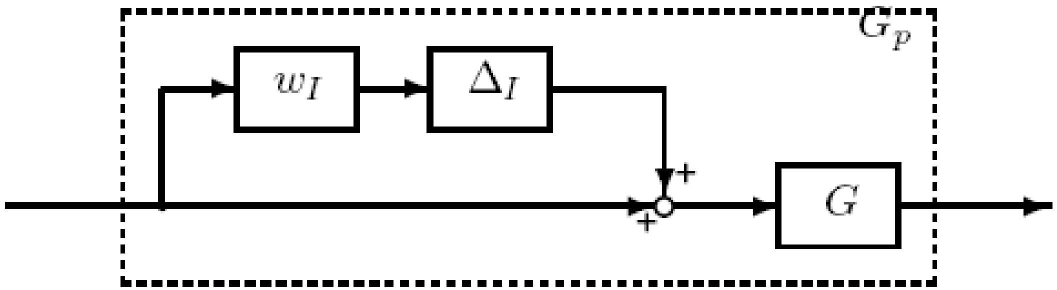
and show that the closed-loop system is internally stable

Lecture 3 - Robustness

Robustness = insensitivity to model errors

Philosophy: specify uncertainty set and guarantee stability and performance for all possible plants.

We have focused on multiplicative uncertainty



Lecture 3 - Robustness

Robust stability via small-gain theorem

$$|T(i\omega)| \leq |w_I^{-1}(i\omega)| \qquad \forall \omega$$

where

$$|w_I(i\omega)| \geq \left| \frac{G_p(i\omega) - G(i\omega)}{G(i\omega)} \right| \qquad \forall G_p \in \Pi_i$$

Robust performance puts simultaneous bound on S and T.

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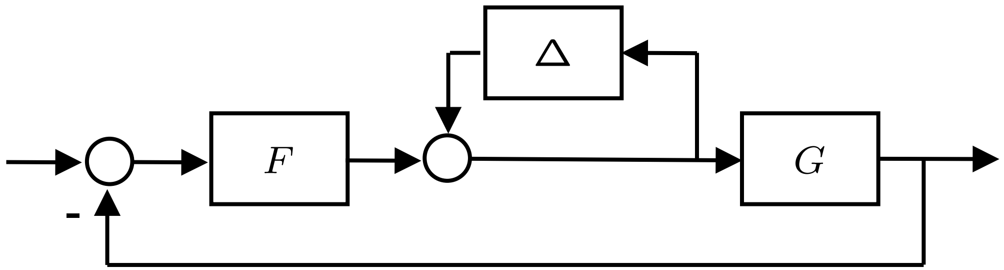
Lecture 3 - Robustness

Example (2003-08-19): Consider the uncertain system

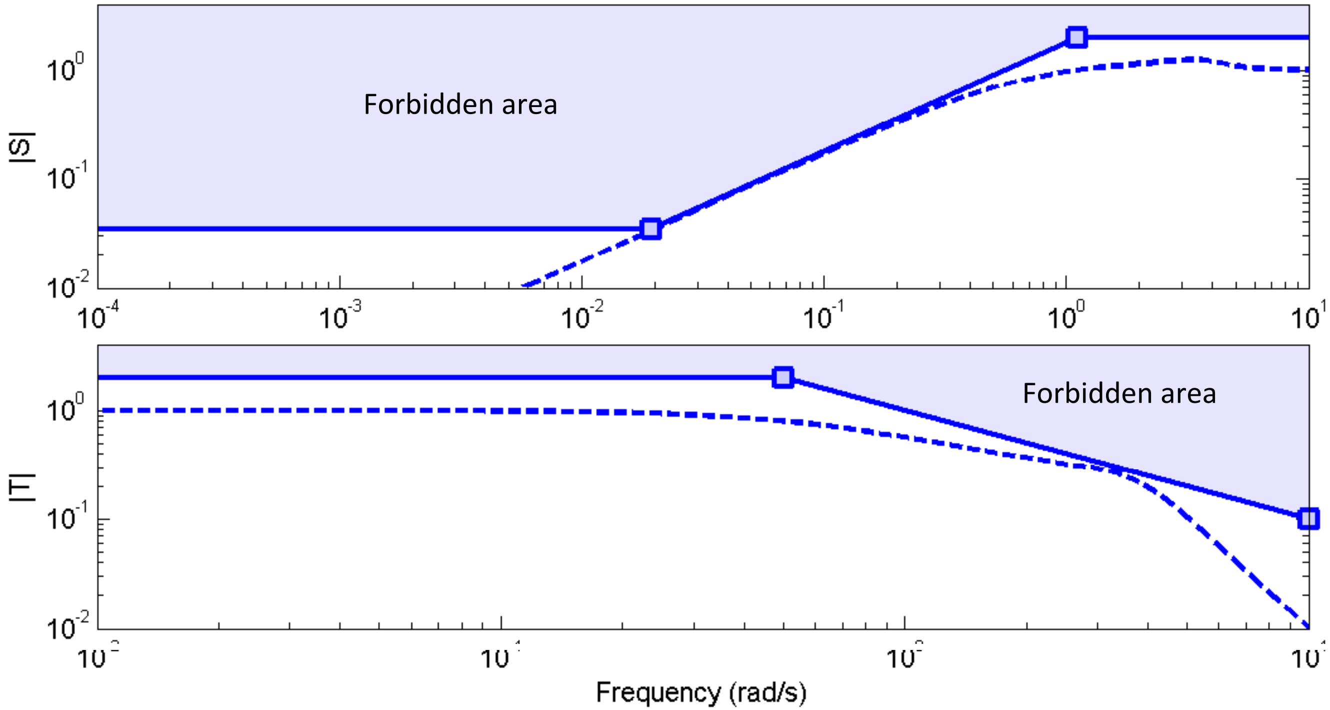


Use the small-gain theorem to derive a robustness criterion

$$\|P\|_\infty \|\Delta\|_\infty < 1$$

for some transfer function P independent of Δ.

Lecture 4– Limiations and Conflicts



$$|S(i\omega)| \leq |w_S(i\omega)|$$
$$|T(i\omega)| \leq |w_T(i\omega)|$$

Can we choose weights w_S, w_T ("forbidden areas") freely?

– No, there are many constraints and limitations!

Lecture 4 – Limitations and conflicts

- Fundamental limitations in control systems design
  - S+T=1 (both can't be small at the same time)
  - Can't attenuate disturbances at all frequencies

- Limiting factors:
  - Unstable poles
  - Non-minimum phase zeros
  - Time delays
  - Control authority

- Reasonable specifications, and rules-of-thumb!

Lecture 4 – Rules of thumb

RHP zeros limit bandwidth (of S)

$$\omega_{BS} \leq \frac{z}{2}$$

Time-delays impose a similar bound

$$\omega_{BS} \leq \frac{1}{T}$$

RHP poles require a minimum bandwidth (for T)

$$\omega_{0T} \geq 2p$$

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Lecture 4

Example (2004-04-20): Given the unstable system

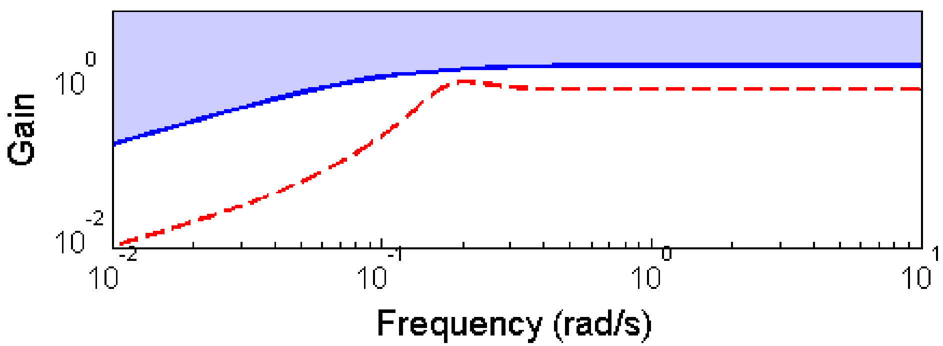
$$G(s) = \frac{1}{s-2}$$

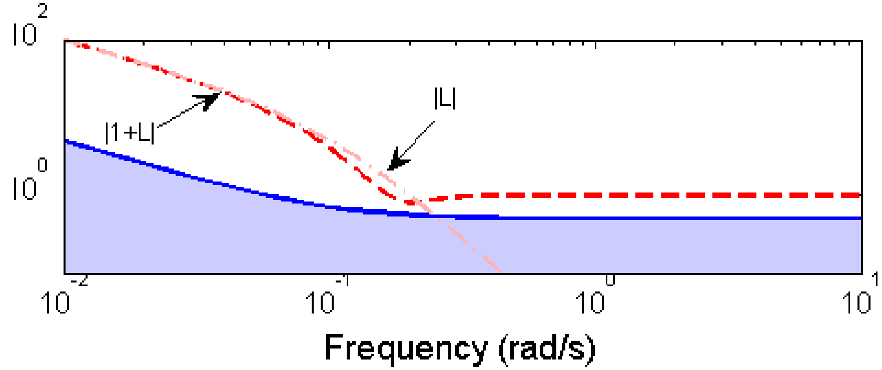
a) Give a reasonable specification for the smallest bandwidth of the complementary sensitivity (based on the rule-of-thumb)
b) Determine a proportional controller that satisfies the bandwidth constraints in (a)
c) Compute $\|T\|_\infty$ and $\|S\|_\infty$ for the controller in (b)

Lecture 4 – Classical loop shaping

In certain frequency ranges, there is a good mapping between constraints on S and T onto requirements on loop gain L.

$$|S(i\omega)| \leq |W_S^{-1}(i\omega)| \Leftrightarrow |1 + L(i\omega)| \geq |W_S(i\omega)|$$





Problematic area is around cross-over frequency

- Poor man's solution is to derive requirements on phase and amplitude margin

Lecture 5 – Multivariable linear systems

Poles, zeros and gains. Gains and directions.

Theorem. The *pole polynomial* of a system with transfer matrix G(s) is the common denominator of all minors of G(s). The poles of G(s) are the roots of the pole polynomial.

Theorem. The *zero polynomial* of G(s) is the greatest common divisor of the maximal minors of G(s), normed so that they have the pole polynomial of G(s) as denominator. The *zeros* of the G(s) are the roots of its zero polynomial.

Theorem. The gain of the linear system G(s) is given by

$$\|G\|_\infty = \sup_\omega |G(i\omega)| = \sup_\omega \overline{\sigma}(G(i\omega))$$

Lecture 5

Example (2001-09-01): Consider the system

$$G(s) = \begin{pmatrix} \frac{4}{s+4} & \frac{16}{s+8} \\ \frac{16}{s+8} & \frac{4}{s+4} \end{pmatrix}$$

Determine the poles and zeros of G(s).

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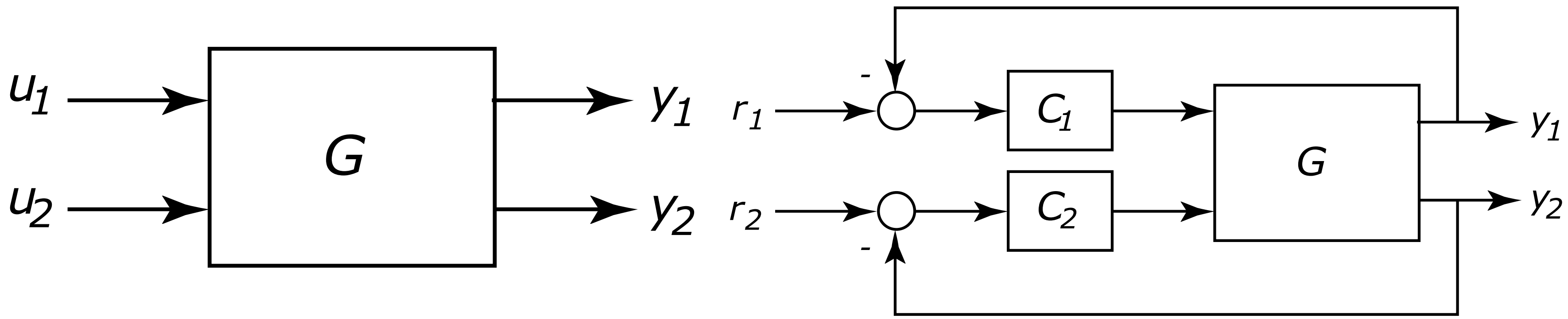
Lecture 6 – Directionality and decoupling

Centralized vs. decentralized control:



Interactions: single input affects multiple outputs

Qualitatively: the more interactions, the harder to control

- The relative gain array tries to quantify the degree of interactions

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The relative gain array

Definition. the *relative gain array, RGA,* of a square system is defined as

$$\mathsf{RGA}(G) = G. * (G^{\dagger})^T$$

or, in Matlab notation, RGA(G)= G.*pinv(G.')

Lecture 6

Example (2001-09-01) Consider

$$G(s) = \begin{pmatrix} \frac{4}{s+4} & \frac{16}{s+8} \\ \frac{10}{s+8} & \frac{4}{s+4} \end{pmatrix}$$

and the two potential pairings for decentralized P-control

a) $u_1 = K_{11}(r_1 - y_1) \quad u_2 = K_{22}(r_2 - y_2)$

b) $u_1 = K_{12}(r_2 - y_2) \quad u_2 = K_{21}(r_1 - y_1)$

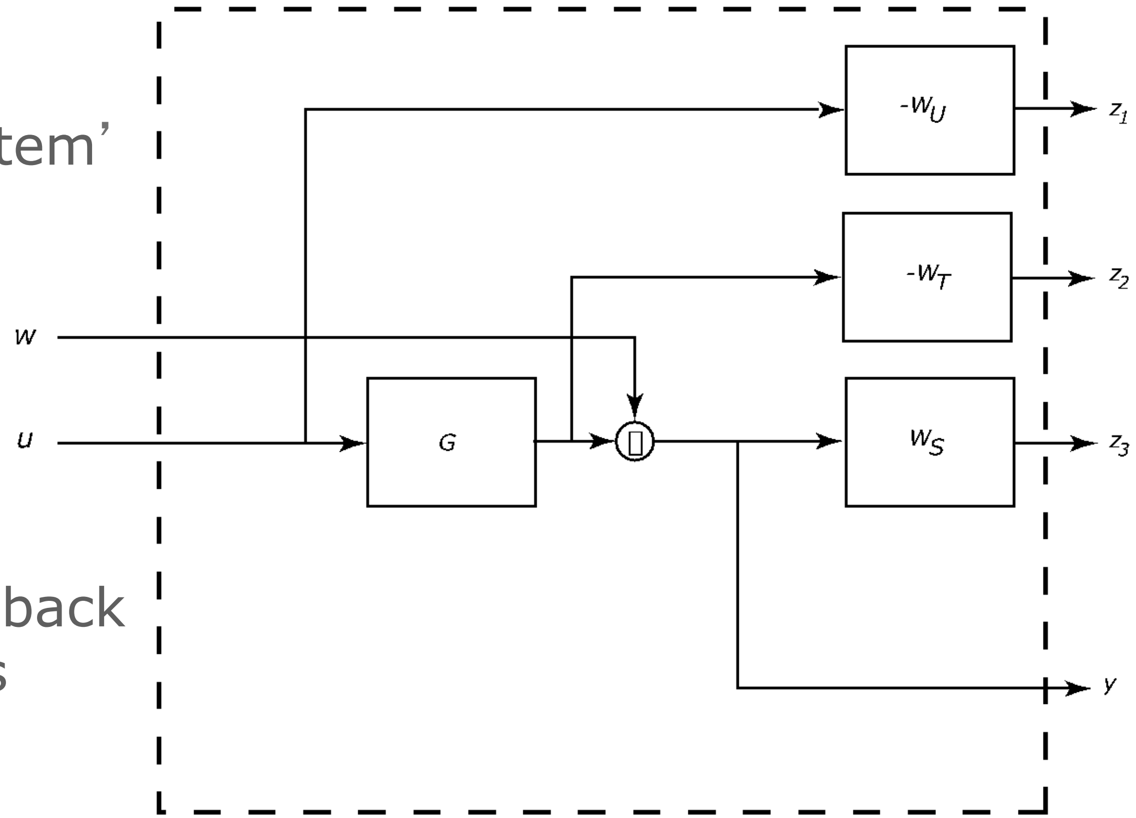
Use RGA to decide which alternative is to prefer, if the desired crossover frequency is around 3 rad/sec.

Lecture 7 – H_∞ optimal control

Specification: optimize over all stabilizing controllers to achieve

$$\left\| \begin{pmatrix} W_S S \\ W_T T \\ W_U S F_y \end{pmatrix} \right\| \leq 1$$

Based on 'extended system'



Optimal controller:

- estimator+linear feedback from estimated states

Lecture 8 – Linear quadratic control

Objective: minimize the cost

$$J = \mathbf{E}\left\{\lim_{T\to\infty}\int_0^T [z^T Q_1 z + u^T Q_2 u]\, dt\right\}$$

Optimal solution:

- State feedback and linear observer
- Feedback and observer gains by solving Riccati equations
- LQG controller can have very poor robustness margins
  - Need to introduce artificial noise into system (LTR)

Lecture 8

Example (2004-03-12): Given the linear system

$$\dot{y}(t) = ay(t) + u(t)$$

where a is a constant. Determine the gain of the P-controller

$$u(t) = -K_c y(t)$$

so that the criterion

$$J = \int_0^\infty (\alpha y^2(t) + u^2(t))\, dt$$

is minimized for arbitrary initial values y(0). Where is the closed-loop system pole located when $\alpha \rightarrow 0$

Lecture 9 – H_2 and H_∞ control

H_2-optimal

$$\min_{F_y} \|G_{ec}\|_2^2 = \min_{F_y} \int_{-\infty}^{\infty} \sum_i \sigma_j^2(G_{ec}(i\omega))\, d\omega$$

H_∞-optimal

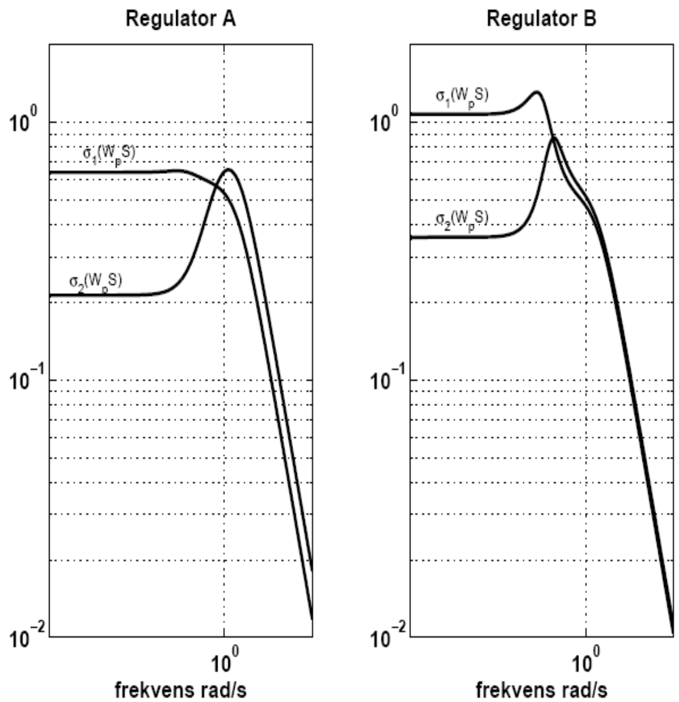
$$\min_{F_y} \|G_{ec}\|_\infty = \min_{F_y} \sup_\omega \overline{\sigma}(G_{ec}(i\omega))$$

Computed from state-space description of G_{ec}

- Solution is observer + static feedback from observed states
  - H_2: all singular values at all frequencies;
  - H_∞: maximum singular value at worst frequency

Lecture 9

Example (2004-03-12): For a linear system with two inputs and outputs, two alternative controllers have been designed that minimize weighted sensitivity $W_p S$. The singular value plots for the two designs are shown below. Which one is based on H_2 minimization, and which is based on H_∞ minimization? Motivate!



Lecture 10 – Robust loop shaping

Glover Mc-Farlane: design additional "robustifying controller"

Solves the problem: find controller that stabilizes

$$G(s) = (M(s) + \Delta_M(s))^{-1}(N(s) + \Delta_N(s))$$

for all uncertainties

$$\|\Delta_M(s)\ \Delta_N(s)\|_\infty \leq \epsilon$$

Dynamic controller of high order (plant+nominal controller)

Lecture 10 – Implementation aspects

Digital implementation: need a discrete-time controller

- Approximate derivatives by differences
- Simplest method: Euler's method (but more advanced exist)

Desirable to reduce model order

- Balanced realization
- Can judge what states to eliminate by Hankel singular values

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Lecture 11 – Dealing with hard constraints

Ad-hoc schemes for common nonlinearities:

- Anti-windup to deal with actuator saturation
- Important in practice, but rather limited theory

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Lecture 12,13 – Model Predictive Control

Structured way of dealing with control and state constraints

1. Predict how state evolves (as function of future controls)

$$x_k = A^k x_0 + \sum_{j=0}^{k-1} A^{k-1-j} B u_j$$

2. Determine optimal control by minimizing criterion

$$\begin{array}{ll} \text{minimize} & \sum_{i=0}^{N_p-1} [(x_i - x_i^{\text{ref}})^T Q_1 (x_i - x_i^{\text{ref}}) + (u_i - u_i^{\text{ref}})^T Q_2 (u_i - u_i^{\text{ref}})] \\ & + (x_{N_p} - x_{N_p}^{\text{ref}})^T S (x_{N_p} - x_{N_p}^{\text{ref}}) \\ \text{subject to} & u_{\text{min}} \leq u_i \leq u_{\text{max}} \\ & y_{\text{min}} \leq C x_i \leq y_{\text{max}} \end{array}$$

3. Implement first control, return to 1 at next sampling instant

Can be solved via efficient optimization (quadratic programming)

- an emerging technology with many applications!

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About the exam

When? Saturday 130525 at 09-14

You need to sign-up at the Lab/Exam booking system
(via home page) to reserve a seat for the exam

If you missed the deadline, you can make the exam if there is room.
(show up the 25th and see if there is sufficient space).

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About the exam

You may bring the following items to the exam

1. The course book
2. The book from the basic course
3. Copies of the lecture slides
4. Mathematical handbook
5. Calculator

You *may not* bring exercise materials or old exams

You *may not* add notes (by hand or otherwise) to the material

About the exam

Written exam in the spirit of previous years' exams

- This year's exam will be offered in english only.
  (but you may answer in Swedish)

Note that the course content has shifted over the years:

- Nonlinear control is no longer part of the course
- Robustness and model-predictive control have become more central

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Beyond the exam

Learn more in our advanced courses!

- Modeling of dynamical systems
- Nonlinear control
- Hybrid and embedded systems

Put your skills to the test: make your master thesis with us!

- See the list on the web, or come talk to us!

Contribute to frontline research: enroll in our PhD program!

- An exciting career – come and talk to us!

Course evaluation and feedback

1. What is the strongest point(s) of the course?

2. What is the weakest point(s) of the course?

3. Which are the (say, top-3) most important changes we should do in the course until next offering? (comment on lectures, labs, exercises)

4. Which part has been hardest to understand?

Course evaluation system is open now (on Social)!