KTH Electrical Engineering

Exam in EG2050 System Planning, 5 June 2013, 8:00–13:00, E34-E36

Allowed aids

In this exam you are allowed to use the following aids:

- Calculator without information relevant to the course.
- One **handwritten**, **single-sided** A4-page with **your own** notes (original, not a copy), which should be handed in together with the answer sheet.

PART I (MANDATORY)

Write all answers on the answer sheet provided. Motivations and calculations do not have to be presented.

Part I can yield 40 points in total. The examinee is guaranteed to pass if the score is at least 33 points. If the result in part I is at least 31 points, then there will be a possibility to complement for passing the exam with the grade E.

Problem 1 (4 p)

Answer the following theoretical questions by choosing *one* alternative, which you find correct.

a) (2 p) The system operator of an electricity market has the following responsibility and authority: I) Physical responsibility that the production and consumption are in balance at every moment, II) Authority to disconnect consumption if it is necessary to maintain safe operation of the system, III) Economically responsible that during a particular trading period (for example one hour) the system is supplied as much energy as the consumption.

1. None of the statements is true.
2. Only I is true.
3. Only II is true.
4. Only III is true.
5. I and II are true but not III.

b) (1 p) We use the notion "ahead trading" to describe all the trading which occurs before the hour of delivery (or any other trading period). Which of the following contracts can be traded in an ahead market?

1. Balance power, i.e., when a balance responsible player is selling any surplus in their balance to the system operator, or when a balance responsible player is buying from the system operator to cover for any deficit in their balance.
2. Firm power, i.e., the customer buys the same amount of energy in each trading period as long as the contract is valid.
3. Regulation power, i.e., when a player at request from the system operator is supplying more power to the system (up-regulation) or when a player at request from the system operator is supplying less power to the system (down-regulation).

c) (1 p) In which of the following electricity markets can the consumers choose which retailer they want?

1. In a vertically integrated electricity market.
2. In a centralised electricity market.
3. In a bilateral electricity market.

Problem 2 (6 p)

The electricity market in Land has perfect competition, perfect information and there are neither transmission nor reservoir limitations. The figure below shows the electricity generation in Land during a day. The variable operation costs of the different power sources are given in table 1.

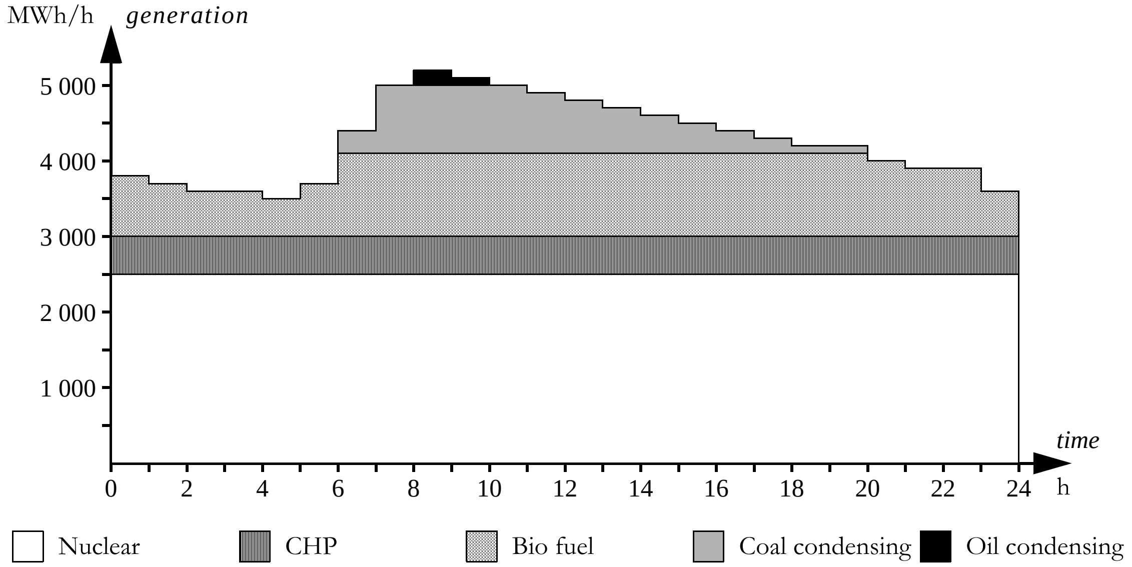


Table 1 Variable costs for the power plants in Land.

Power source	Variable costs [¤/MWh]
Nuclear	100
Combined heat and power (CHP)	250
Bio fuel	300
Coal condensing	350
Oil condensing	450

a) (1 p) Which electricity price is there in Land between 4:00 and 5:00?

b) (1 p) Which electricity price is there in Land between 7:00 and 8:00?

c) (1 p) Which electricity price is there in Land between 8:00 and 9:00?

d) (1 p) Which electricity price is there in Land between 19:00 and 20:00?

e) (2 p) Are there any capacity limitations in the electricity market of Land during this day? Give a brief motivation for your answer!

Problem 3 (6 p)

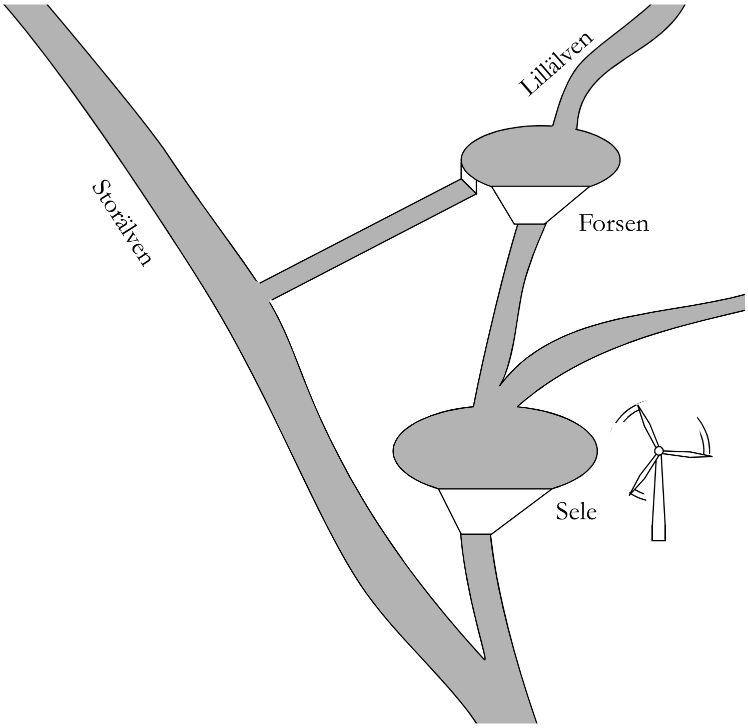
a) (2 p) Consider a power system at a time when the frequency is stable at 49.97 Hz. At this moment the wind power generation in the system is increased by 50 MW. Which of the following alternatives describes what happens in the system?

1. There is a deficit of energy, which results in a voltage decrease in the grid. The control systems of the power plants participating in the primary control responds to the voltage decrease by increasing the electricity generation.
2. There is a deficit of energy, which results in a voltage increase in the grid. The control systems of the power plants participating in the primary control responds to the voltage increase by increasing the electricity generation.
3. There is a surplus of energy, which results in a voltage increase in the grid. The control systems of the power plants participating in the primary control responds to the voltage increase by decreasing the electricity generation.
4. There is a surplus of energy, which results in a frequency decrease in the system. The control systems of the power plants participating in the primary control responds to the frequency decrease by increasing the electricity generation.
5. There is a surplus of energy, which results in a frequency increase in the system. The control systems of the power plants participating in the primary control responds to the frequency increase by decreasing the electricity generation.

b) (2 p) The hydro power plant Fallet has a gain of 200 MW/Hz. The base generation (i.e., the generation when the frequency is exactly 50 Hz) is 60 MW and the installed capacity of the power plant is equal to 90 MW. To avoid damaging the turbines it is not allowed to generate less than 40 MW. How much will the power plant generate when the frequency of the system is 49.94 Hz?

c) (2 p) The hydro power plant Språnget has a gain of 250 MW/Hz. The base generation (i.e., the generation when the frequency is exactly 50 Hz) is 75 MW and the installed capacity of the power plant is equal to 120 MW. To avoid damaging the turbines it is not allowed to generate less than 50 MW. How much will the power plant generate when the frequency of the system is 50.14 Hz?

Problem 4 (12 p)



AB Vattenkraft owns two hydro power plants and a wind farm located as in the figure above. To enable salmon to migrate from Storälven to the exploited part of Lillälven upstreams of Forsen, the environment court has decided that AB Vattenkraft must release a flow of 10 m^3/s in the channel between the reservoir of Forsen and Storälven.

AB Vattenkraft sells power to customers with firm power contracts, but the company also has the possibility to trade at the local power exchange ElKräng. The following symbols have been introduced in a short-term planning problem for these hydro power plants:

Indices for the power plants: Forsen 1, Sele 2.

γ_i = expected future production equivalent for water stored in reservoir i, i = 1, 2,
D_t = contracted load hour t, t = 1, …, 24,
λ_t = expected electricity price at ElKräng hour t, t = 1, …, 24,
λ_{25} = expected electricity price at ElKräng after the end of the planning period,
$M_{i,0}$ = contents of reservoir i at the beginning of the planning period, i = 1, 2,
$M_{i,t}$ = contents of reservoir i at the end of hour t, i = 1, 2, t = 1, …, 24,
$\mu_{i,j}$ = marginal production equivalent in power plant i, segment j, i = 1, 2, j = 1, 2,
p_t = purchase from ElKräng hour t, t = 1, …, 24,
$Q_{i,j,t}$ = discharge in power plant i, segment j, during hour t, i = 1, 2, j = 1, 2, t = 1, …, 24,
r_t = sales to ElKräng hour t, t = 1, …, 24,
$S_{i,t}$ = spillage from reservoir i during hour t, i = 1, 2, t = 1, …, 24,
S_{channel} = water flow through the channel between Forsen and Storälven,
W_t = expected generation of the wind farm in hour t, t = 1, …, 24.

a) (4 p) The maximal discharge in the hydro power plant Forsen is 125 m^3/s and the best efficiency is obtained for the discharge 100 m^3/s. The maximal production equivalent of the power plant is 0.25 MWh/HE and the production equivalent at maximal discharge is 0.244 MWh/HE. Assume that we need a piecewise linear model of electricity generation as function of the discharge in Forsen. The model should have two segments and the breakpoint between them should be located at the best efficiency. Calculate the following parameters:

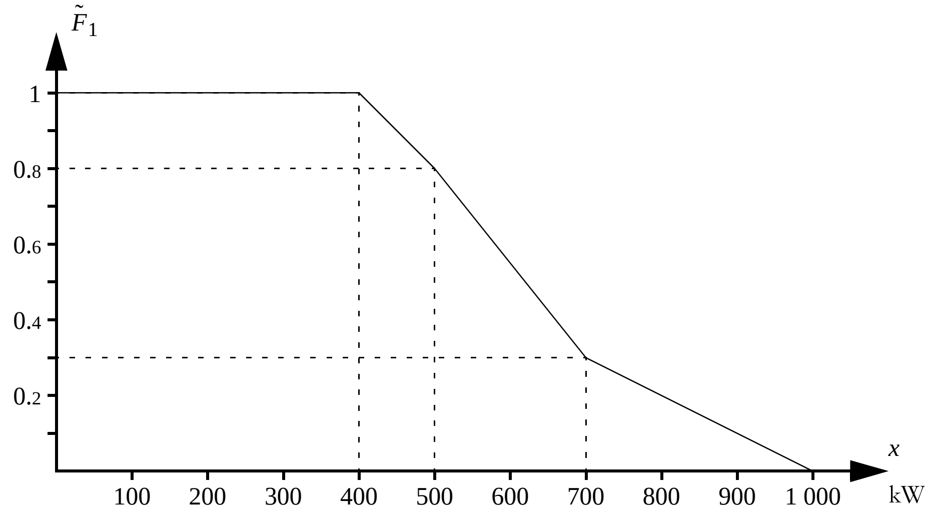
$\mu_{1,j}$ = marginal production equivalent in Forsen, segment *j*,
$\overline{Q}_{1,j}$ = maximal discharge in Forsen, segment *j*.

b) (4 p) Formulate the objective function if the aim of the planning problem is to maximise the income of sold electricity at ElKräng plus the value of stored water minus the costs of purchasing electricity from ElKräng. Use the symbols defined above.

c) (4 p) Formulate the load balance constraint in the short-term planning problem of the company. Use the symbols defined above.

Problem 5 (12 p)

Ekibuga is a town in East Africa. The town is not connected to a national grid, but has a local system of its own. The local grid is supplied by a hydro power plant and a diesel generator set. The hydro power plant does not have a reservoir, but the water flow is always sufficient to generate the installed capacity (800 kW) and the risk for outages in the power plant is negligible. The diesel generator set has a capacity of 200 kW, the availability is 80% and the operation cost is 5 ¤/kWh.



a) (3 p) Using probabilistic production cost simulation it is calculated that the expected hydro power generation of this system is 625 kWh/h and that the unserved energy is 4 kWh/h. How large is the expected operation cost?

b) (2 p) Use probabilistic production cost simulation to calculate the risk of power deficit in the system.

c) (2 p) Assume that the power system in Ekibuga is simulated using a more advanced model, which takes into account the losses and the risk of outages in the distribution grid. Monte Carlo simulation is then applied in order to use the advanced model. 2 000 scenarios are generated in the Monte Carlo simulation and power deficit appears in 132 of these. Which estimate of *LOLP* is obtained from this simulation?

d) (3 p) Assume that the Monte Carlo simulation should be improved by applying control variates. The simplified model in this simulation corresponds to the model used in probabilistic production cost simulation. The results of the same 2 000 scenarios as in part c are that in 84 scenarios there is power deficit in both the multi-area model and the simplified model. In 48 scenarios there is only power deficit in the multi-area model. Which estimate of *LOLP* is obtained from this simulation?

e) (2 p) The system index *EENS* is a measure of I) The probability that at least one consumer in the system is disconnected, II) The mean energy which during a certain time (for example one year) is expected not to be delivered due to capacity limitations, III) The probability of power deficit multiplied by the average load.

1. None of the statements is true.
2. Only I is true.
3. Only II is true.
4. Only III is true.
5. II and III are true but not I.

PART II (FOR HIGHER GRADES)

All introduced symbols must be defined. Solutions should include sufficient detail that the argument and calculations can be easily followed.

The answer to each problem must begin on a new sheet, but answers to different parts of the same problem (a, b, c, etc.) can be written on the same sheet. The fields *Namn* (Name), *Blad nr* (Sheet number) and *Uppgift nr* (Problem number) must be filled out on every sheet.

Part II gives a total of 60 points, but this part will only be marked if the candidate has obtained at least 33 points in part I. Then the results of parts I and II and the bonus points will be added together to determine the examination grade (A, B, C, D, E).

Problem 6 (10 p)

The power exchange ElKräng allows the following types of bids:

- **Sell bid.** A sell bid is valid for a specific hour and comprises a specific volume in MWh and a price interval in SEK/MWh. If the electricity price in the corresponding hour is higher than the highest price in the bid then the bid is accepted in whole. If the electricity price however is lower than the least price in the bid then the bid is rejected. If the electricity price is within the specified price range, then a linear relation is assumed between the electricity price and the quantity that is accepted. (If the price for example is in the middle of the price range then half of the bid is accepted.)
- **Purchase bid.** A purchase bid is valid for a specific hour and comprises a specific volume in MWh and a price interval in SEK/MWh. If the electricity price in the corresponding hour is lower than the least price in the bid then the bid is accepted in whole. If the electricity price however is higher than the highest price in the bid then the bid is rejected. If the electricity price is within the specified price range, then a linear relation is assumed between the electricity price and the quantity that is accepted. (If the price for example is in the middle of the price range then half of the bid is accepted.)
- **Convertible block bid.** A convertible block bid is a special type of sell bid, which is valid for several hours. The bid comprises a specific volume in MWh, a base price in SEK/MWh and an alternative price interval in SEK/MWh. If the average price during the hours for which the bid is comprising is higher than the base price in the bid then the bid is accepted in whole. Otherwise, the bid is converted into normal sell bids for each hour included in the block bid. The price interval of these sell bids is equal to the alternative price range of the block bid.[1]

The electricity price at ElKräng is calculated by combining sell and purchase bids into a supply and a demand curve for each hour. The electricity price for the corresponding hour is set by the intersection between the curves. Then the price for each hour has been calculated this way, it is checked if the conditions of the block bids are fulfilled. If that is the case, the block bids are included in the beginning of the supply curve (i.e., the bid price of the block bids is assumed to be zero in this calculation) and the price calculation is remade. After this computation, where is a risk that the prices have changed so that the conditions for some block bids are no longer fulfilled. In that case these block bids are converted to sell bids and the price calculation is remade again.

Table 2 shows the bids to ElKräng for two hours. What are the resulting electricity prices for these two hours?

1. The idea of the convertible block bids is to allow producers to either distribute the start-up cost of a power plant on a longer time period or to cover the start-up cost in one single hour.

Table 2 Bids to ElKräng.

Bid	Volume [MWh]		Price interval [SEK/MWh]	
	Hour 1	Hour 2	Hour 1	Hour 2
Sell bid 1	1 500	1 500	450–600	450–600
Sell bid 2	800	800	520–680	520–680
Sell bid 3	500	500	540–640	540–640
Sell bid 4	200	200	1 000–1 100	1 000–1 100
Purchase bid 1	1 100	1 210	8 000–10 000	8 000–10 000
Purchase bid 2	1 500	1 670	8 000–10 000	8 000–10 000
Purchase bid 3	350	400	425–525	425–525
Block bid 1	200 MWh/h		Base price: 850 Alternative price interval: 1 000–1 100	
Block bid 2	500 MWh/h		Base price: 950 Alternative price interval: 1 100–1 200	

Problem 7 (10 p)

The power system in Rike is divided in two parts. There are large amounts of hydro power in the northern part of the system, but the main consumption centres are in the southern part. There are several parallel AC transmission lines between the two parts. The maximal flow on these lines is 4 000 MW – if this limit is exceeded the power system becomes unstable and there is a risk for extensive blackouts in the entire or parts of the system. Therefore, in order to avoid this, the maximal flow is set to a value less than 4 000 MW at nominal frequency; the unused transmission capacity is reserved for power flows which are due to the primary control. Riksnät, which is the system operator in Rike, must at last 8:00 notify the players in the electricity market about how much transmission from north to south that can be allowed for each hour the next day.

The primary control in Rike is divided in a normal operation reserve, which is used to manage normal variations in for example load and wind power generation, and a disturbance reserve, which is used to manage outages in larger power plants. The normal operation reserve is available in the frequency range 49.9–50.1 Hz and has a total gain of 2 400 MW/Hz in northern Rike and 600 MW/Hz in southern Rike..

The first step when Riksnät is computing the hourly transmission capacity that is available for the electricity market is to determine how much transmission capacity that might be required by the normal operation reserve. A difficulty in this calculation is that Riksnät does not know the exact conditions for a given hour. Table 3 shows the forecasts for the trading period between 10:00 and 11:00 next day. The table shows the size of increase and decrease that the normal operation reserve is expected to manage in the two parts of Rike. Assume that all of these deviations are independent. How much transmission capacity must be reserved for the normal operation reserve?

Hint: The necessary reserve is different when the interconnection is used ot transfer electric ity from north to south compared to from south to north.

Table 3 The forecast of Riksnät for the trading period 10–11.

	Northern Rike		Southern Rike	
	Maximal decrease	Maximal increase	Maximal decrease	Maximal increase
Load [MW]	20	20	195	195
Wind power generation [MW]	10	15	35	50

Problem 8 (20 p)

Stads energi AB owns the combined heat and power plant Flisinge with three blocks. Flisinge prodeces heat, which is delivered to a district heating system, as well as electricity, which is sold to the power exchange ElKräng. The generation cost as function of the heat generation is shown in the figure on the next page (some points from these curves are also given in table 4). The relation between heat generation and electricity generation is constant for each block, i.e., each MWh heat generated yields a specific amount of MWh electricity. This relation, as well as other data for the three blocks, are given in table 5. There is also an electric boiler in the power plant, i.e., it is possible to use electricity to generate heat for the district heating system. At maximal loading, the electric boiler consumers 25 MW. The efficiency of the electric boiler is 99%. Finally, there is also an accumulator tank, where it is possible to store thermal energy (as water at a temperature of 98 °C). The maximal storage capacity is 800 MWh heat. The heat losses in the accumulator tank are negligible in a short-term planning problem, and it can be assumed that there is no limitation in how fast the tank can be emptied and filled respectively.

Each hour there a specific amount of MWh heat is consumed in the district heating system. This heat load depend mostly on the outdoor temperature. A forecast for the heat load is given in The heat load depend mostly on the outdoor temperature. A forecast for the heat load is given in table 6, which also shows the expected prices at ElKräng.

Stads energi AB wants to operate the power plants so that they maximises the income of sold electricity minus the generation costs, while each hour delivering enough heat to the district heating system. Formulate the planning problem of the company as a MILP problem. The accumulator tank holds 500 MWh at the start of the planning period and should hold at least as much at the end of the planning period. At the beginning of the planning period, blocks I and II will be committed, whereas block III will not be committed.

Use the notation in table 7 for the parameters (it is however permitted to add further symbols if you consider it necessary).

NOTICE! The following is required to get full score for this problem:

- The symbols for the optimisation variables must be clearly defined.
- The optimisation problem should be formulated so that it is easy to determine what the objective function is, which constraints there are and which limits there are.
- The possible values for all indices should be clearly stated for each equation.

Table 4 Generation cost in SEK/h for the three blocks in Flisinge.

Block	Heat generation [MWh/h]								
	21	25	40	50	60	65	75	90	105
I	–	15 000	21 250	26 250	30 750	33 000	37 000	–	–
II	–	–	21 000	25 300	29 600	31 750	36 050	42 500	48 350
III	13 515	15 375	22 350	27 000	31 100	–	–	–	–

Table 5 Additional data for Flisinge.

	Block I	Block II	Block III
Maximal heat generation [MWh/h]	75	105	60
Minimal heat generation when committed [MWh/h]	25	40	21
Electricity generation per MWh heat generation [MWh]	0,41	0,38	0,40
Start-up cost [SEK]	60 000	85 000	50 000

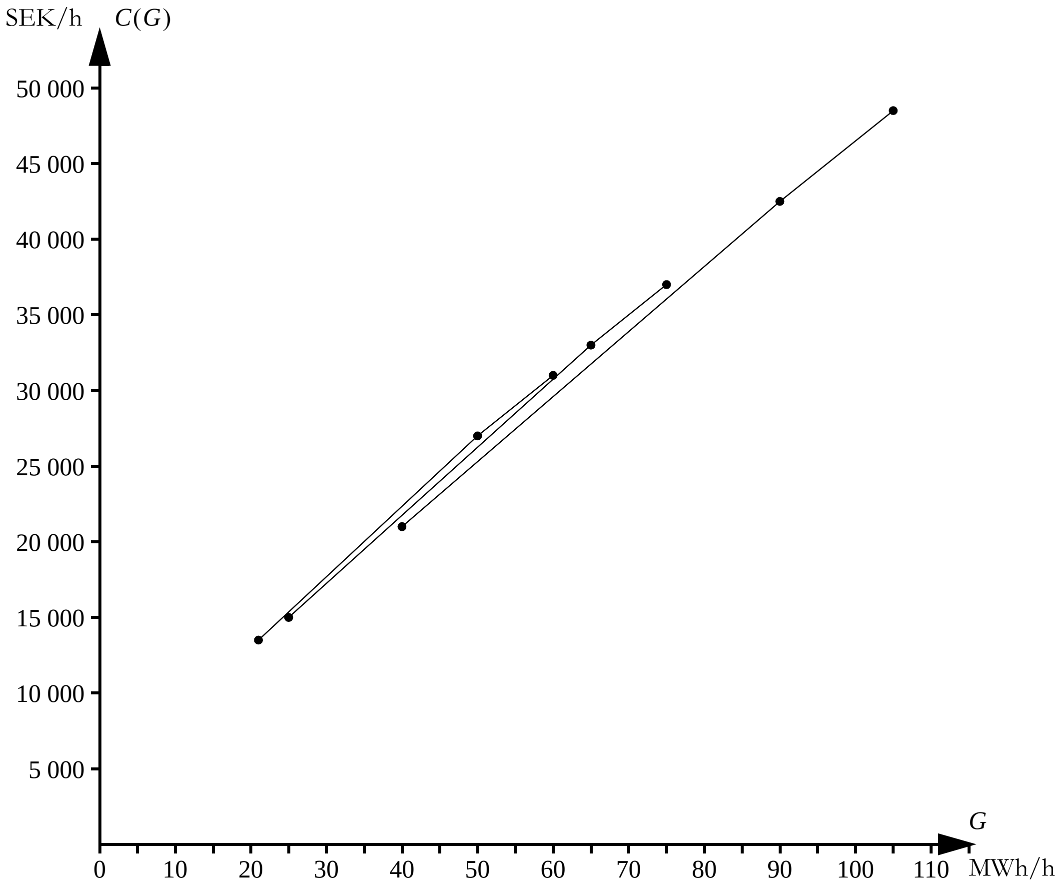


Table 6 Forecasts for the next day.

Hour	0-1	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10	10-11	11-12
Price at ElKräng [SEK/MWh]	290	280	260	250	250	270	335	380	390	380	375	380
Heat load [MWh]	180	180	180	180	180	180	160	160	160	140	140	140
Hour	12-13	13-14	14-15	15-16	16-17	17-18	18-19	19-20	20-21	21-22	22-23	23-24
Price at ElKräng [SEK/MWh]	380	380	380	370	365	370	365	355	345	345	335	315
Heat load [MWh]	150	160	170	180	190	200	200	200	200	200	210	210

Tabell 7 Notation for the planning problem of Stads energi AB.

Symbol	Explanation	Value
$\overline{H}$	Maximal electricity consumption in the electric boiler	25
η_H	Efficiency in the electric boiler	0.99
$\overline{M}$	Maximal contents of the accumulator tank	800
M_0	Start contents of the accumulator tank	500
$\underline{M}_{24}$	Least allowed contents of the accumulator tank at the end of the planning period	500
$\overline{G}_g$	Maximal heat generation in block g	See table 5
$\underline{G}_g$	Minimal heat generation when block g is committed	See table 5
c_g	Obtained electricity per MWh generated heat in block g	See table 5
λ_t	Electricity price in hour t	See table 6
D_t	Heat load in hour t	See table 6

Problem 9 (20 p)

To determine the value of an investment it is necessary to find out how the electricity market is affected by the investment. This requires at least two simulations: one simulation of the system before the investment is made and then a simulation of how the system behaves after the investment.

Two cases are described below, where there is a need to determine if it is profitable to invest in expanded transmission capacity. Describe which simulation methods that at all are applicable in the two cases. Then choose the simulation method that you find most appropriate and briefly describe step by step how to perform the simulation and describe how the simulation results can be used to determine the profitability of the investment.

NOTICE! The following is required to get full score for this problem:

- There should be a clear and correct motivation if a simulation method is considered inappropriate for a certain case.
- There should be a clear and reasonable motivation for the choice of simulation method in each case.
- All assumptions must be described and justified.

a) (10 p) Öarna is an autonomous group of islands in the Atlantic Ocean. The islands are small and sparsely populated, and there is therefore no common power system, but each island has its own grid. The electricity market of Öarna is not large enough to make it meaningful to restructure the electricity market and introduce competition; therefore, Öarna still has a vertically integrated electricity market, where a government owned utility operates all generation, transmission and distribution.

The different grids of Öarna are mostly supplied by electricity from different fossil fuelled power plants (combined cycle gas turbines and diesel generator sets). However, on the largest island there are some wind power plants since a while. A problem with these wind power plants is that their generation sometimes is larger than what can be accommodated in the local grid, which means that some wind power must be spilled. Therefore, the power company now considers to build transmission lines connecting the local grids into a common power system. The distances between the islands are not that long and the capacity of the lines will be more than sufficient to transfer the surplus generation from one island to the others.

b) (10 p) The power system in Rike is divided in two parts. There are large amounts of hydro power in the northern part of the system, but the main consumption centres are in the southern part. There are several parallel AC transmission lines between the two parts. Earlier the capacity of these lines have been sufficient to manage the export from northern Rike to southern Rike. However, a lot of wind power has been built in Rike (especially in the northern part of the country) during the recent years. As a consequence, there are often large differences in the electricity price between the northern and southern parts of the country, which has caused complaints from the consumers (among them many large industries) in southern Rike. If the expansion of wind power in northern Rike continues, there will also be a risk that wind power or hydro power must be spilled. For these reasons, Riksnät is thinking about building a new HVDC interconnection, which would increase the transmission capacity from north to south by 20%. The reason why an HVDC link is preferred instead of another AC line is that Riksnät believes it can be built faster and that the operation costs will be lower (for example due to lower losses and increased controllability of the power system).

KTH Electrical Engineering

Answer sheet for part I

Name: ....

Personal number: ....

Problem 1

a) Alternative .... is correct.

b) Alternative .... is correct.

c) Alternative .... is correct.

Problem 2

a) .... ¤/MWh **b)** .... ¤/MWh

c) .... ¤/MWh **d)** .... ¤/MWh

e) ....

....

Problem 3

a) Alternative .... is correct.

b) .... MW **c)** .... MW

Problem 4

a) $\mu_{1,1}$ .... MWh/HE $\mu_{1,2}$ .... MWh/HE

$\overline{Q}_{1,1}$ .... HE $\overline{Q}_{1,2}$ .... HE

b) ....

c) ....

Problem 5

a) .... ¤/h **b)** .... %

c) .... % **d)** .... %

d) Alternative .... is correct.

Suggested solution for exam i EG2050 System Planning, 5 June 2013.

Problem 1

a) 5, **b)** 2, **c)** 3.

Problem 2

a) The electricity price is in this our set by the bio fuel generation ⇒ 300 ¤/MWh.

b) The electricity price is in this our set by the coal condensing generation ⇒ 350 ¤/MWh.

c) The electricity price is in this our set by the oil condensing generation ⇒ 450 ¤/MWh.

d) The electricity price is in this our set by the coal condensing generation ⇒ 350 ¤/MWh.

e) Yes. (If there were no capacity limitations there would be the same electricity price during the entire day. In this case neither the bio fuelled power plants nor the coal condensing units have sufficient capacity to follow the load variations during the day.)

Problem 3

a) 5.

b) The relation between frequency and generation yields that the power plant generates

$$G = G_0 - R(f - f_0) = 60 - 200(49.94 - 50) = 72 \text{ MW}.$$

c) The relation between frequency and generation yields that the power plant should generate

$$G = G_0 - R(f - f_0) = 75 - 250(50.14 - 50) = 40 \text{ MW},$$

which is less than the least allowed generation. The power plant will then generate as little as possible, i.e., 50 MW.

Problem 4

a) The following data are given in the problem text:

$\hat{Q}_1$ = discharge in Forsen at best efficiency = 100,
$\bar{Q}_1$ = maximal discharge in Forsen = 125,
$\gamma_1(\hat{Q}_1)$ = production equivalent at best efficiency in Forsen = 0.244,
$\gamma_1(\bar{Q}_1)$ = production equivalent at maximal discharge in Forsen = 0.25.

To calculate the marginal production equivalents, we need the generation at best efficiency as well as maximal discharge. According to the definition we have

$$H_1(Q_1) = \gamma_1(Q_1) \cdot Q_1 = \begin{cases} 25 & Q_1 = 100, \\ 30.5 & Q_1 = 125. \end{cases}$$

The marginal production equivalents can now be calculated according to

$$\mu_{1,1} = \frac{\hat{H}_1}{\hat{Q}_1}$$

and

$$\mu_{1,2} = \frac{\bar{H}_1 - \hat{H}_1}{\bar{Q}_1 - \hat{Q}_1},$$

which results in the following linear models of the power plant:

$$\mu_{1,j} = \text{marginal production equivalent in Forsen, segment } j = \begin{cases} 0.25 & j = 1, \\ 0.22 & j = 2, \end{cases}$$

$$\bar{Q}_{1,j} = \text{maximal discharge in Forsen, segment } j = \begin{cases} 100 & j = 1, \\ 25 & j = 2. \end{cases}$$

b) maximise $\sum_{t=1}^{24} \lambda_t(r_t - p_t) + \lambda_{25}((\gamma_1 + \gamma_2)M_{1,24} + \gamma_2 M_{2,24})$.

c) $\sum_{i=1}^{2} \sum_{j=1}^{2} \mu_{i,j} Q_{i,j,t} + W_t + p_t = D_t + r_t$.

Problem 5

a) The most straightforward solution is to directly compute the expected generation in the diesel generator set:

$$EG_2 = 0{,}8 \int_{800}^{1\,000} \tilde{F}_1(x)dx = 0{,}8 \cdot 0{,}2/2 \cdot 200 = 16 \text{ kWh/h}.$$

Alternatively, the expectation value of the load can be computed by

$$EENS_0 = \int_0^{\infty} \tilde{F}_0(x)dx = 1 \cdot 400 + (1 + 0{,}8)/2 \cdot 100 + (0{,}8 + 0{,}3)/2 \cdot 200 + 0{,}3/2 \cdot 300 = $$
$$= 645 \text{ kWh/h}.$$

which means that $EG_1 + EG_2 + EENS_2 = 645$. As $EG_1 = 625$ kWh/h and $EENS_2 = 4$ kWh/h we get $EG_2 = 16$ kWh/h.

Hence, the expected operation cost is $ETOC = 5EG_2 = 80$ ¤/h.

b) The risk of power deficit is given by $\tilde{F}_2(1\,000) = 0.8\tilde{F}_1(1\,000) + 0.2\tilde{F}_1(800) = 0 + 0.2 \cdot 0.2 = 0.04$. The risk of power deficit is thus 4%.

c) $m_{LOLO} = \frac{1}{n}\sum_{i=1}^{n} lolo_i = 132/2\,000 = 6.6\%$.

d) $m_{LOLO} = m_{(LOLO-\tilde{LOLO})} + \mu_{\tilde{LOLO}} = \frac{1}{n}\sum_{i=1}^{n}(lolo_i - \tilde{lolo}_i) + 0.04 = \frac{1}{2\,000}48 + 0.04 = 6.4\%$.

a) 3.

Problem 6

Assume that the electricity price is higher than 525 SEK/MWh during both hours. In this case, purchase bids 1 and 2 will b e accepted, whereas purchase bid 3 is rejected; consequently, the demand will be2 600 MWh in the first hour and 2 880 MWh in the second.

Assume that the electricity price in the first hour, λ_1, is in the interval 600–640 SEK/MWh. This means that the entire sell bid 1 is accepted, whereas sell bids 2 and 3 are partially accepted and sell bid 4 is rejected, which yields the following equation for the price:

$$1\,500 + \frac{\lambda_1 - 520}{680 - 520}800 + \frac{\lambda_1 - 540}{640 - 540}500 + 0 = 2\,600.$$

Thus, we get the electricity price 640 SEK/MWh in the first hour.

In the second hour, sell bids 1–3 are needed entirely, and the price will be determined by the share of sell bid 4 that is used:

$$1\,500 + 800 + 500 + \frac{\lambda_2 - 1\,000}{1\,100 - 1\,000}200 = 2\,880.$$

Thus, we get the electricity price 1 040 SEK/MWh in the second hour.

Now it is possible to check if the requirements of the block bids are fulfilled. The average price of the two hours is 840 SEK/MWh, which means that neither of the block bids is accepted entirely. We should then convert the block bids to normal sell bids. These sell bids are not of interest in the first hour, as the price interval is way above the above computed electricity price of 640 SEK/MWh. However, in the second hour will a part of the sell bid from block bid 1 be accepted, as the price interval overlaps with sell bid 4 (which was price-setting for the second hour). In this case we get the equation

$$1\,500 + 800 + 500 + \frac{\lambda_2 - 1\,000}{1\,100 - 1\,000}200 + \frac{\lambda_2 - 1\,000}{1\,100 - 1\,000}200 + 0 = 2\,880,$$

which yields $\lambda_2 = 1\,020$ SEK/MWh.

Thus, we can conclude that the electricity price will be 640 SEK/MWh in the first hour and 1 020 SEK/MWh in the second hour.

Problem 7

In this problem we need to investigate how the net load (i.e., load – wind power generation) can change in the two areas and the consequences of these changes for the transmission between northern and southern Rike. The largest changes in net load are obtained when the load and wind power generation are deviating in different directions, for example when the load is decreasing at the same time as the wind power generation is increasing.

Consider for example the case when we have the maximal net load decrease in northern Rike, $\Delta N_N = -35$ MW, at the same time as we have the maximal net load decrease in southern Rike,$\Delta N_S = -245$ MW. The primary control must in total change the generation by $\Delta G = -280$ MW. As 80% of the gain in the normal operation reserve is in norther Rike, the generation change in that part will be $\Delta G_N = 0.8\Delta G = -224$ MW, whereas the change in souther Rike is $\Delta G_S = 0.2\Delta G = -56$ MW. The power plants in northern Rike are decreasing their generation by a larger amount than the local net load decrease; therefore, the flow from north to south must change by $\Delta P_{N\to S} = \Delta G_N - \Delta N_N = -189$ MW (the same conclusion can also be obtained by studying the difference in net load change in southern Rike and how the generation is changind there).

Similarly, we can study the other combinations of net load changes in the system:

ΔN_N	ΔN_S	$\Delta G = \Delta N_N + \Delta N_S$	$\Delta G_N = 0{,}8\Delta G$	$\Delta G_S = 0{,}2\Delta G$	$\Delta P_{N\to S} = \Delta G_N - \Delta N_N = \Delta N_S - \Delta G_S$
–35	–245	–280	–224	–56	–189
–35	+230	+195	+156	+39	+191
+30	–245	–215	–172	–43	–202
+30	+230	+260	+208	+52	+178

From this calculations we can conclude that the normal operation reserve may need to use 191 MW of the transmission capacity from north to south (for the case when there is a maximal net load decrease in the northern part and a maximal net load increase in the southern part), and 202 MW of the transmission capacity from north to south (for the case when there is a maximal net load increase in the northern part and a maximal net load decrease in the southern part).

Problem 8

The problem we want to solve is

maximise *value of sold electricity – generation costs – start-up costs,*
subject to *load balance of the accumulator tank,*
limitations in heat generation.

The challenge in this problem is how to model the operation cost as a function of the heat generation. The figure in the problem text shows that the cost is a piecewise linear function with two segments, and that there is a forbidden interval as well. We can also see that the marginal generation cost is lower in the second segment than in the first, which means that the model will prefer to use the second segment rather than the first. This is however not corresponding to the actual usage of the power plant and we must therefore use a binary variable, which prevents segment 2 from being used until segment 1 is fully utilised. To accomplish this, we introduce the following optimisation variables:

$G_{g,j,t}$ = heat generation in block g, segment j, during hour t,
$u_{g,j,t}$ = unit commitment of block g, segment j, during hour t.

The power plant must be committed in order to use the first segment, and the generation is then equal to at least $\underline{G}_g$. We then add the generation in the two segments, which means that the heat generation for one specific hour can be expressed as

$$\underline{G}_g u_{g,1,t} + G_{g,1,t} + G_{g,2,t}.$$

From this expression it is straightforward to compute the electricity generation and the generation cost for a specific hour. Hence, we can now formulate the complete planning problem.

Indices for the power plants

Block I 1, block II 2, block II 3.

Parameters

In addition to the parameters defined in table 7, we introduce the following symbols:

$\overline{G}_{g,j}$ = maximal heat generation in block g, segment j = {computed using data from table 4 in the problem text} =

$$\begin{cases} 65-25 = 40 & g = 1, j = 1, \\ 75-65 = 10 & g = 1, j = 2, \\ 90-40 = 50 & g = 2, j = 1, \\ 105-90 = 15 & g = 2, j = 2, \\ 50-21 = 29 & g = 3, j = 1, \\ 60-50 = 10 & g = 3, j = 2, \end{cases}$$

$\beta_{g,j}$ = marginal generation cost in block g, segment j = {computed using data from table 4 in the problem text} =

$$= \begin{cases} (33\,000-15\,000)/40 = 450 & g = 1, j = 1, \\ (37\,000-33\,000)/10 = 400 & g = 1, j = 2, \\ (42\,500-21\,000)/50 = 430 & g = 2, j = 1, \\ (48\,350-42\,500)/15 = 390 & g = 2, j = 2, \\ (27\,000-13\,515)/29 = 465 & g = 3, j = 1, \\ (31\,100-27\,000)/10 = 410 & g = 3, j = 2, \end{cases}$$

$\underline{\beta}_g$ = generation cost at minimal heat generation when block g is committed =

$$= \{\text{see table 4 in the problem text}\} = \begin{cases} 15\,000 & g = 1, \\ 21\,000 & g = 2, \\ 13\,515 & g = 3, \end{cases}$$

C_g^+ = start-up cost of block g = {see table 5 in the problem text} =

$$= \begin{cases} 60\,000 & g = 1, \\ 85\,000 & g = 2, \\ 50\,000 & g = 3, \end{cases}$$

$u_{g,0}$ = unit commitment of block g at the beginning of the planning period =

$$= \{\text{stated in the problem text}\} = \begin{cases} 1 & g = 1, \\ 1 & g = 2, \\ 0 & g = 3. \end{cases}$$

Optimisation variables

$G_{g,j,t}$ = heat generation in block g, segment j, during hour t, $g = 1, 2, 3, j = 1, 2, t = 1, \ldots, 24$,

H_t = electricity consumption in the electric boiler during hour t, $t = 1, \ldots, 24$,

M_t = contents of the accumulator tank at the end of hour t, $t = 1, \ldots, 24$,

$s_{g,t}^+$ = start-up of block g at the beginning of hour t, $g = 1, 2, 3, t = 1, \ldots, 24$,

$u_{g,j,t}$ = unit commitment of block g, segment j, during hour t, $g = 1, 2, 3, j = 1, 2, t = 1, \ldots, 24$.

Objective function

$$\text{maximise} \quad \sum_{t=1}^{24} \sum_{g=1}^{3} \left(\lambda_t c_g (\underline{G}_g u_{g,t} + G_{g,1,t} + G_{g,2,t}) - \underline{\beta}_g u_{g,t} - \sum_{j=1}^{2} \beta_{g,j} G_{g,j,t} - C_g^+ s_{g,t}^+ \right) - \sum_{t=1}^{24} \lambda_t H_t.$$

Constraints

Load balance of the accumulator tank:

$$M_{1,t} = M_{1,t-1} + \sum_{g=1}^{3} (\underline{G}_g u_{g,t} + G_{g,1,t} + G_{g,2,t}) + \eta_H H_t - D_t, \qquad t = 1, \ldots, 24.$$

Samband mellan Relation between unit commitment and start-up:

$$u_{g,1,t} - u_{g,1,t-1} \le s_{g,t}^+, \qquad g = 1, 2, 3, t = 1, \ldots, 24.$$

Maximal generation in the segments:

$$G_{g,j,t} \le \overline{G}_{g,j} u_{g,j,t}, \qquad g = 1, 2, 3, j = 1, 2, t = 1, \ldots, 24.$$

Minimal generation in segment 1:

$$G_{g,j,t} \ge u_{g,t} \overline{G}_{g,j}, \qquad g = 1, 2, 3, t = 1, \ldots, 24.$$

(This constraint means that segment 1 must be fully utilised if segment 2 is to be used.)

Variable limits

$0 \le G_{g,j,t} \le \overline{G}_{i,j}$, $\quad g = 1, 2, 3, j = 1, 2, t = 1, \ldots, 24$,

$0 \le H_t \le \overline{H}$, $\quad t = 1, \ldots, 24$,

$0 \le M_t \le \overline{M}$, $\quad t = 1, \ldots, 23$,

$\underline{M}_{24} \le M_{24} \le \overline{M}$,

$s_{g,t}^+ \in \{0, 1\}$, $\quad g = 1, 2, 3, t = 1, \ldots, 24$,

$u_{g,j,t} \in \{0, 1\}$, $\quad g = 1, 2, 3, j = 1, 2, t = 1, \ldots, 24$.

Problem 9

a) According to the problem text, the power company wants to study if the wind power can be used more efficiently if the local grids are interconnected into a common grid. An advantage of a common grid is that if there is a surplus of wind power generation in one island, the surplus can be exported to the other islands, which lowers the operation costs of the system. We should then simulate how the investment affects *ETOC*. Another advantage by interconnecting the system is that the reliability is improved (lower *LOLP*). This does not seem to be of vital importance in this case, but it might as well be included in the simulation results.

The system can of course be simulated using Monte Carlo methods, but as the transmission losses are negligible and the capacity of the lines is not a limiting factor, it will be possible to simulate also the common grid using probabilistic production cost simulation. In this case, we choose to use probabilistic production in order to avoid the inevitable uncertainty when estimating system indices in a Monte Carlo simulation. The study will then be carried out as follows:

1. Use probabilistic production cost simulation to compute *ETOC* and *LOLP* for each local power system.

2. Compute the expected operation cost for the entire system (before the investment in new interconnections) by adding *ETOC* of the local systems.

3. If we assume that power deficit in the local systems are independent events (this is a slight simplification as the loading of the local systems are likely to follow similar patterns) then we can compute the risk of power deficit in the entire system by studying the probability that there is not power deficit in any of the local systems (cf example 6.3 in the compendium).

4. Use probabilistic production cost simulation to compute *ETOC* and *LOLP* for the interconnected system.

5. Compare *ETOC* and *LOLP* before and after the investment respectively. Are the cost savings sufficient to make the investment profitable? If not, is the total impact (lower operation cost and better reliability) sufficient to motivate the investment?

b) In this case the objective of the study is to investigate how improved transmission capacity from north to south affects the operation costs, the reliability (that the reliability is important is indicated by the argument to choose an HVDC link for increased controllability) an the electricity prices (as they have been subject to complaints). To study these issues, we apparently need to consider the limited transmission capacity, the losses and the improved controllability. This requires a more advanced model than the one used in probabilistic production cost simulation and we are therefore forced to used Monte Carlo simulation. As we are studying rather small differences and rare events, we should also use the three variance reduction techniques complementary random numbers, control variates and stratified sampling. The study will then be carried out as follows:

1. Use probabilistic production cost simulation to simulate the system before and after the investment in increased transmission capacity. These simulations provide expectation values of the control variates.

2. Formulate an advanced electricity market model which considers limited transmission capacity, losses and improved controllability. Define a suitable strata tree for this model.

3. Carry out Monte Carlo simulations of the system before and after the investment in increased transmission capacity.

4. Compare *ETOC* and *LOLP* before and after the investment respectively. Are the cost savings sufficient to make the investment profitable? If not, is the total impact (lower operation cost, better reliability and more evenly distributed electricity prices) sufficient to motivate the investment?