KTH Electrical Engineering

Exam in EG2050 System Planning,
26 August 2013, 8:00–13:00, Q22

Allowed aids

In this exam you are allowed to use the following aids:

- Calculator without information relevant to the course.
- One **handwritten**, **single-sided** A4-page with **your own** notes (original, not a copy), which should be handed in together with the answer sheet.

PART I (MANDATORY)

Write all answers on the answer sheet provided. Motivations and calculations do not have to be presented.

Part I can yield 40 points in total. The examinee is guaranteed to pass if the score is at least 33 points. If the result in part I is at least 31 points, then there will be a possibility to complement for passing the exam with the grade E.

Problem 1 (4 p)

Answer the following theoretical questions by choosing *one* alternative, which you find correct.

a) (2 p) In a modern, restructured ("deregulated") electricity market, the system operator is responsible for the short-term balance between generation and consumption. This means that I) The system operator has to ensure that the frequency is kept within given limits, II) If the system operator does not ensure that the system in every moment is supplied as much power as is consumed, then the system operator will have to pay a penalty fee to the balance responsible players, III) If the system operator does not ensure that the system in every trading period (for example an hour) is supplied as much energy as is consumed, then the system operator will have to pay a penalty fee to the balance responsible players.

1. None of the statements is true.
2. Only I is true.
3. Only II is true.
4. Only III is true.
5. I and II are true but not III.

b) (2 p) The consumers in a vertically integrated electricity market has the following choices: I) They can choose which system operator they want, II) They can choose which retailer they want, III) The can choose which player should manage their balance responsibility.

1. None of the statements is true.
2. Only I is true.
3. Only III is true.
4. I and II are true but not III.
5. II and III are true but not I.

Problem 2 (6 p)

The electricity market in Land has perfect competition, perfect information and there are no transmission limitations. The figure below shows the electricity generation in Land during a day. The variable operation costs of the different power sources are given in table 1.

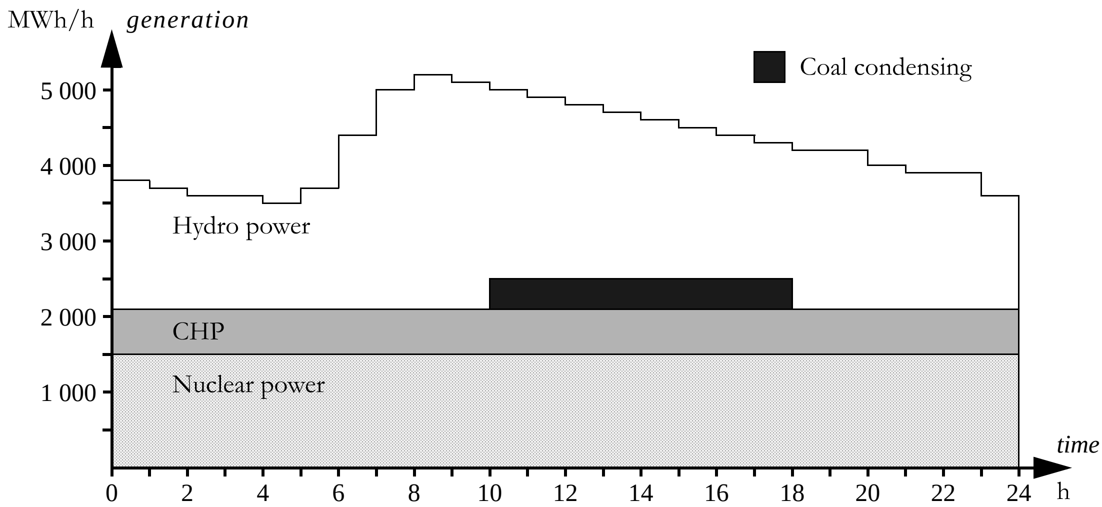


Table 1 Variable costs for the power plants in Land.

Power source	Variable costs [¤/MWh]
Nuclear	100
Combined heat and power (CHP)	250
Coal condensing	350
Hydro power	0

a) (1 p) Which electricity price is there in Land between 6:00 and 7:00?

b) (1 p) Which electricity price is there in Land between 8:00 and 9:00?

c) (1 p) Which electricity price is there in Land between 10:00 and 11:00?

d) (1 p) Which electricity price is there in Land between 19:00 and 20:00?

e) (2 p) Are there any reservoir limitations in the electricity market of Land during this day? Give a brief motivation for your answer!

Problem 3 (6 p)

The power system in Land is divided in two areas (A and B) which are connected by an AC transmission line. This line has a maximal capacity of 1 000 MW and is equipped with a protection system which after a short time delay disconnects the line if the power flow exceeds the maximal capacity of the line.

At 8:45 a fire breaks out in a substation in Stad (which is located in area A). Due to the fire, the entire regional grid of Stad must immediately be disconnected from the national grid in Land, which means that the national grid looses 200 MW generation and 800 MW load. After the disconnection of Stad, the gain in Land is 5 000 MW/Hz in area A and 5 000 MW/Hz in area B. The gain is available in the interval 50 ± 0.5 Hz.

Just before the disconnection of Stad, the frequency of the system was 50.02 Hz and 750 MW was transmitted from area A to area B.

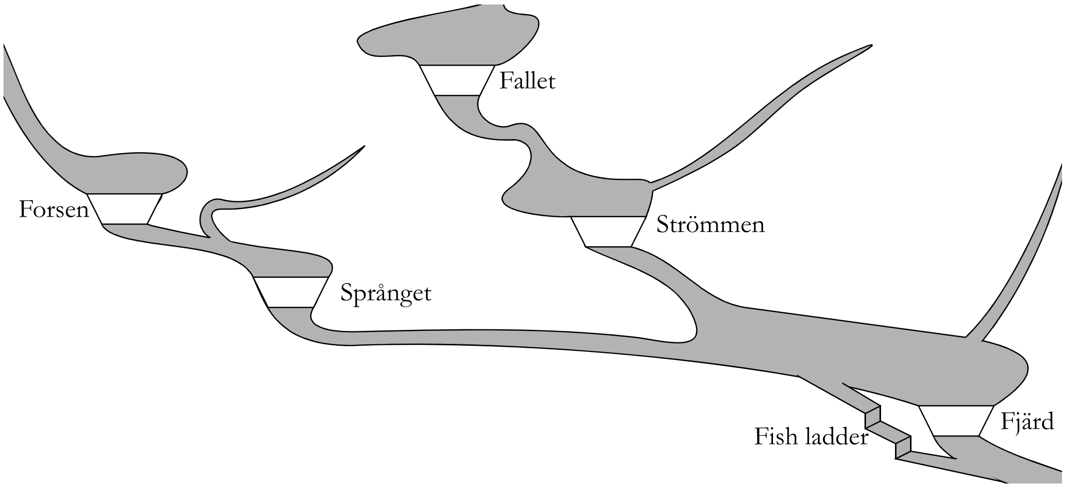
a) (1 p) What happens when the regional grid of Stad is disconnected?

1. There is a surplus of energy, which results in a voltage increase in the grid. The control systems of the power plants participating in the primary control responds to the voltage increase by reducing the electricity generation.
2. There is a surplus of energy, which is stored as kinetic energy in all synchronous generators; hence, the frequency of the system increases. The control systems of the power plants participating in the primary control responds to the frequency increase by reducing the electricity generation.
3. There is a deficit of energy, which is compensated using kinetic energy from all synchronous generators; hence, the frequency of the system increases. The control systems of the power plants participating in the primary control responds to the frequency increase by increasing the electricity generation.

b) (2 p) Will the transmission line between area A and B be disconnected due to overloading?

c) (3 p) Which frequency will there be in area A and B respectively, once the primary control has restored the balance between generation and consumption?

Problem 4 (12 p)



AB Vattenkraft owns five hydro power plants located as in the figure above. In order to enable salmon to pass the power plants, the environment court has decided that AB Vattenkraft always must release a flow of 2 m^3/s in the fish ladder at Fjärd. The following symbols have been introduced in a short-term planning problem for these hydro power plants:

Indices for the power plants: Forsen 1, Språnget 2, Fjärd 3, Fallet 4, Strömmen 5.

$M_{i,0}$ = contents of reservoir i at the beginning of the planning period, i = 1, …, 5,
$M_{i,t}$ = contents of reservoir i at the end of hour t, i = 1, …, 5, t = 1, …, 24,
$\overline{M}_i$ = maximal contents of reservoir i, i = 1, …, 5,
$Q_{i,j,t}$ = discharge in power plant i, segment j, during hour t, i = 1, …, 5, j = 1, 2, t = 1, …, 24,
$\overline{Q}_{i,j}$ = maximal discharge in power plant i, segment j, i = 1, …, 5, j = 1, 2,
$S_{i,t}$ = spillage from reservoir i (including the flow through the fish ladder if applicable) during hour t, i = 1, …, 5, t = 1, …, 24,
$\underline{S}_i$ = minimal allowed flow through the fish ladders at reservoir i, i = 1, …, 5,
$\overline{S}_i$ = maximal spillage from reservoir i, i = 1, …, 5,
$V_{i,t}$ = local inflow to reservoir i during hour t, i = 1, …, 5, t = 1, …, 24.

a) (3 p) At installed capacity the hydro power plant Fjärd generates 37 MW and the production equivalent is then 0.25 MWh/HE. The reservoir can hold 6 480 000 m^3. If we start with a full reservoir, how many hours is it then possible to generate the installed capacity at Fjärd before the reservoir is empty? Assume that the power plants up-stream neither discharge or spill any water. The local inflow can be considered negligible, but the Environment Court decision on the fish ladder still applies.

b) (4 p) Formulate the hydrological constraint of Fjärd, hour t. The water delay time between the power plants can be neglected. Use the symbols above.

c) (3 p) Formulate the limits for the optimisation variables in the short-term planning problem of AB Vattenkraft as defined above. To receive full score for this problem, you also have to state the possible index values for each limit!

d) (2 p) Assume that it has been decided that a thermal power plant should not be shut down for a shorter time than four hours, i.e., if the power plant is stopped at 12:00 then it may not be started again before 16:00. Introduce the following symbols:

s_t^+ = start-up variable for hour t (1 if the power plant is starting generation at the beginning of hour t, otherwise 0),

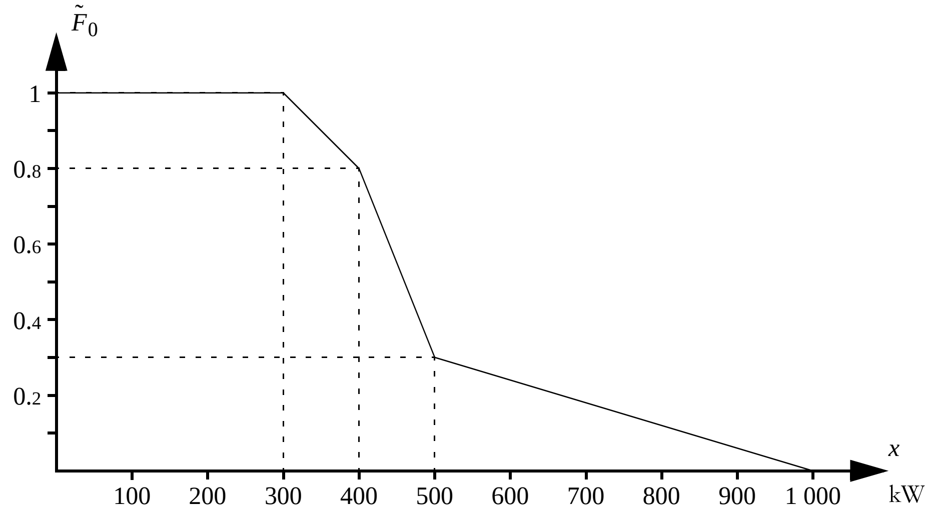
s_t^- = stop variable for hour t (1 if the power plant is stopping generation at the beginning of hour t, otherwise 0).

How should a linear constraint be formulated in order to describe the relation between s_t^-, s_{t+1}^+, s_{t+2}^+ and s_{t+3}^+?

1. $s_t^- - s_{t+1}^+ - s_{t+2}^+ - s_{t+3}^+ = 0.$
2. $s_t^- - s_{t+1}^+ - s_{t+2}^+ - s_{t+3}^+ \leq 1.$
3. $s_t^- - s_{t+1}^+ - s_{t+2}^+ - s_{t+3}^+ = 1.$
4. $s_t^- + s_{t+1}^+ + s_{t+2}^+ + s_{t+3}^+ \leq 1.$
5. $s_t^- + s_{t+1}^+ + s_{t+2}^+ + s_{t+3}^+ = 1.$

Problem 5 (12 p)

Ekibuga is a town in East Africa. The town is not connected to a national grid, but has a local system of its own. The local grid is supplied by a hydro power plant. The hydro power plant does not have a reservoir, but the water flow is always sufficient to generate the installed capacity (900 kW) and the risk for outages in the power plant is negligible.



a) (2 p) The figure above shows the load duration curve. How large is the expected energy not served per hour in Ekibuga?

b) (2 p) The hydro power plant in Ekibuga does not have sufficient capacity to cover the demand peaks; consequently, there are almost daily reoccurring that load shedding is necessary an hour or so in the evening. To improve the reliability of supply, it is considered to purchase a diesel generator set with a capacity of 150 kW, operation cost 5 ¤/kWh and the availability 90%. Calculate the risk of power deficit in this system.

c) (2 p) The expected energy not served when considering both the hydro power plant and the diesel generator is 0.3 kWh/h. Calculate the expected total operation cost per hour for this system.

d) (2 p) In order to consider the losses and outages in the grid it is necessary to perform a Monte Carlo simulation of Ekibuga. Assume that complementary random numbers are used to improve the accuracy of this simulation. What is the value of the complementary random number, D^*, if the total load of the system is randomised to D = 500 kW?

e) (4 p) Assume that control variates also are used in order to improve the accuracy of the simulation. The detailed model includes losses and outages in the grid. The grid is neglected in the simplified model, which means that the simplified model corresponds to the model used in a probabilistic production cost simulation. The results are shown in table 2. Which estimates of *LOLP* and *ETOC* are obtained from the detailed model?

Table 2 Results from a Monte Carlo simulation of the power system in Ekibuga.

Number of scenarios, n	Results from the detailed model		Results from the simplified model	
	$\sum_{i=1}^{n} lolo_i$	$\sum_{i=1}^{n} toc_i$	$\sum_{i=1}^{n} \tilde{lolo}_i$	$\sum_{i=1}^{n} \tilde{toc}_i$
1 000	14	27 000	5	14 500

PART II (FOR HIGHER GRADES)

All introduced symbols must be defined. Solutions should include sufficient detail that the argument and calculations can be easily followed.

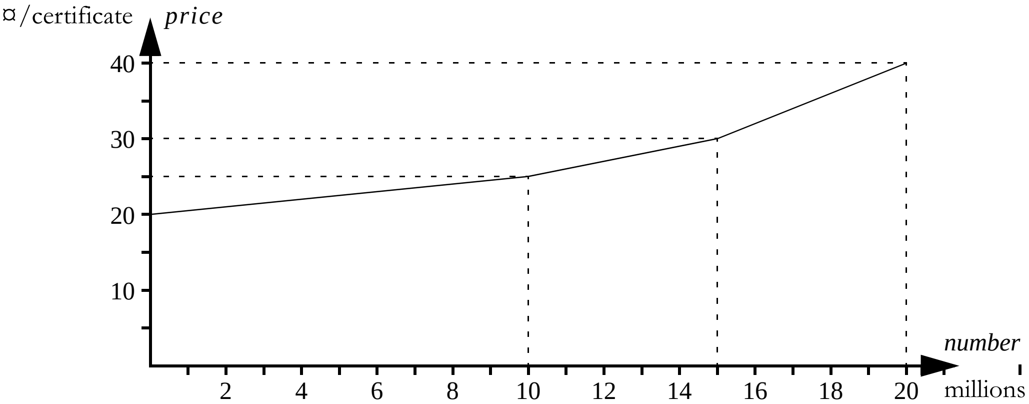
The answer to each problem must begin on a new sheet, but answers to different parts of the same problem (a, b, c, etc.) can be written on the same sheet. The fields *Namn* (Name), *Blad nr* (Sheet number) and *Uppgift nr* (Problem number) must be filled out on every sheet.

Part II gives a total of 60 points, but this part will only be marked if the candidate has obtained at least 33 points in part I. Then the results of parts I and II and the bonus points will be added together to determine the examination grade (A, B, C, D, E).

Problem 6 (10 p)

The Rikish Energy Agency has created a future scenario for the electricity market in 2025. The model of the Energy Agency is based on the assumption that there is perfect competition in the electricity market in Rike, that all players have perfect information and that there are neither capacity, transmission nor reservoir limitations in the system. The Energy Agency also assumes that the electricity consumption in Rike will be 140 TWh/year in 2025 and that the generation capacity will consist of 60 TWh/year hydro power (variable cost 1 ¤/MWh), 50 TWh/year nuclear power (variable cost 10 ¤/MWh) as well as 40 TWh/year fossil fuels (the variable cost is assumed to be linear in the interval 30–50 ¤/MWh; the production is zero if the price is 30 ¤/MWh and the production is maximal at the price level 50 ¤/MWh).

The aim of this future scenario is to study how the electricity market would be affected by investments in a large-scale development of wind power. To support this initiative, it has been proposed that Rike should introduce a system with green certificates. The owner will receive a green certificate for each MWh generated in the new wind power plants. The consumers are then obliged to buy certificates corresponding to 10% of their electricity consumption, which means that a consumer with an annual consumption of 100 MWh will have to buy 10 green certificates per year. The Energy Agency estimates that the supply curve for green certificate would be as in the figure below.



a) (5 p) The opposition of introducing green certificates in Rike argues that wind power is unnecessary (as most of the electricity generation in Rike already is free of carbon dioxide emissions) and that the system will only result in higher costs for the consumers in Rike. Use the data in the Energy Authority scenario above to compute the total cost per MWh for the consumers with and without green certificates respectively. Assume that if there are no green certificates, there will be no wind power in Rike.

b) (5 p) The conclusion in part a is of course depending on the assumptions in the future scenario. Which factor or factors have the most importance for the result? Moreover, give an example of how slightly different assumptions can result in the opposite conclusion compared to part a!

Problem 7 (10 p)

Consider a power system divided in four areas. The gain in each area and the current transmission flows are shown in the figure below. Each transmission line is equipped with a protection system which after a short time delay disconnects the line if the power flow exceeds the maximum capacity of the line, which is stated within parentheses at each line.

During normal operation, the frequency in the system should be kept within 50 ± 0.1 Hz. At the occasion shown in the figure below, the frequency in the system is 49.92 Hz and therefore the system operator has decided to activate up-regulation bids from the real-time balancing market. The available up-regulation bids are shown in table 3. Which bids should the system operator activate in order to minimise the costs if the objective is to increase the frequency to at least 49.97 Hz without overloading any transmission line?

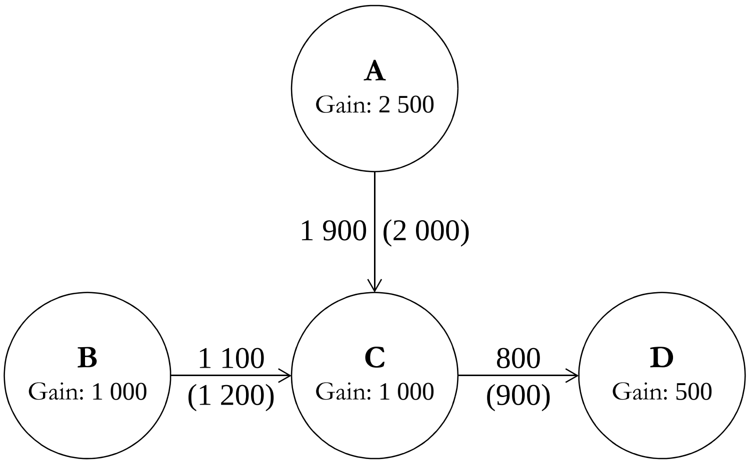
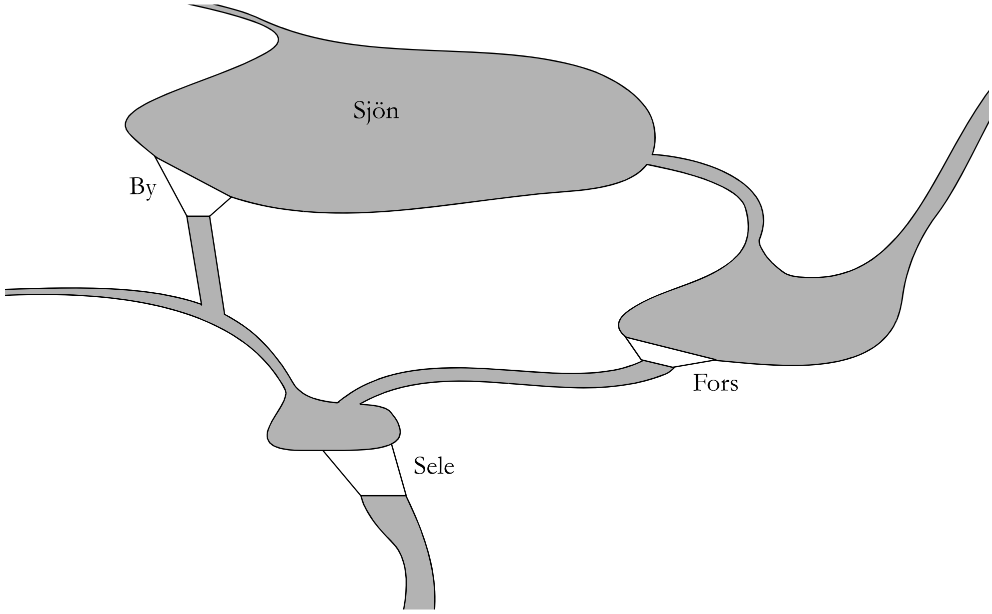


Table 3 Up-regulation bids in the real-time balancing market in Rike.

Bid	Power [MW]	Area	Price [¤/MWh]
1	50	A	400
2	50	A	410
3	100	B	415
4	50	A	425
5	50	C	450
6	50	C	500
7	100	B	520
8	50	D	550

Problem 8 (20 p)



AB Vattenkraft owns three hydro power plants, located as shown in the figure above. Before the hydro power plants were built, the river passed from Sjön via Fors to Sele. Nowadays, water that is discharged at the power plant By is released in a channel where the water eventually ends up in the reservoir of the power plant Sele. Spillage is on the other hand released through the old river bed to Fors. Moreover, a small part of the natural flow from Sjön to Fors still remains and depends on the water level in Sjön. In practice, the natural flow is a nonlinear function of the reservoir contents, as shown in the figure on the next page. The remaining data for the power plants are given in table 4.

The company has a firm power contract of 100 MWh/h with AB Elleverantören. To deliver this quantity, AB Vattenkraft is using their own hydro power plants, but the company has also the possibility to trade at the local power exchange ElKräng. It is assumed that the company can buy and sell unlimited amounts of electricity for the prices stated in table 5. After that, the future electricity price is estimated to 375 ¤/MWh. Stored water is assumed to be used for electricity generation at the best efficiency in each power plant and it is assumed that water stored in Sjön will be used for generation in By.

a) (15 p) Formulate the planning problem of AB Vattenkraft as an LP problem. Use the notation in table 6 for the parameters (it is however permitted to add further symbols if you consider it necessary). The water delay time between the power plants can be neglected.

NOTICE! The following is required to get full score for this problem:

- The symbols for the optimisation variables must be clearly defined.
- The optimisation problem should be formulated so that it is easy to determine what the objective function is, which constraints there are and which limits there are.
- The possible values for all indices should be clearly stated for each equation.

b) (5 p) How must the planning problem from part a be reformulated in order to consider the water delay time between the power plants (see table 7)? Complete new or updated equations do not have to be stated, but it is sufficient to describe the principles how the planning problem must be adjusted. (It can however be recommended to provide a few examples if the description gets complicated!).

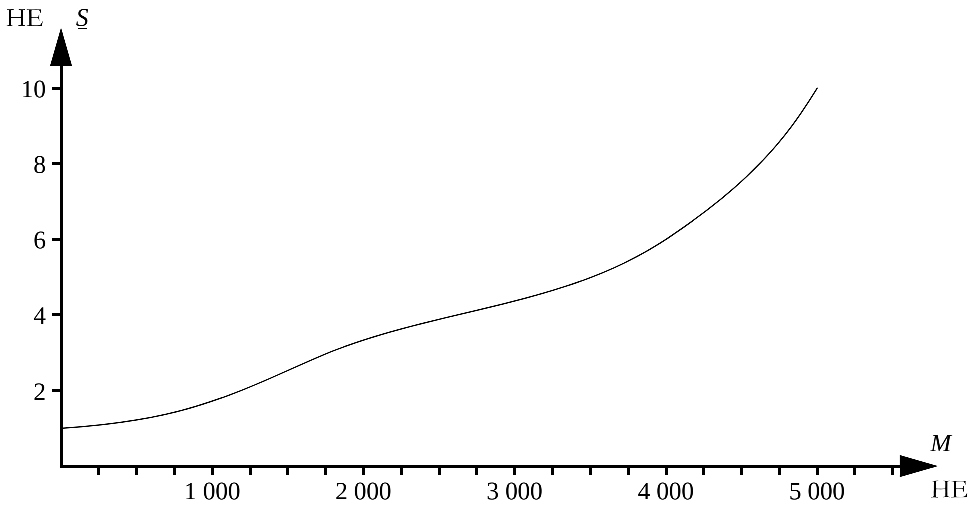


Table 4 Data or the hydro power plants of AB Vattenkraft.

Power plant	Start contents of reservoir [HE]	Maximal contents of reservoir [HE]	Marginal production equivalents [MWh/HE]		Maximal discharge [HE]		Local inflow [HE]
			Segment 1	Segment 2	Segment 1	Segment 2	
By/Sjön	2 000	5 000	0,72	0,64	125	40	160
Fors	800	1 700	0,34	0,31	40	15	30
Sele	200	600	0,52	0,48	140	50	15

Table 5 Expected prices at ElKräng.

Hour	0-1	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10	10-11	11-12
Price at ElKräng [SEK/MWh]	285	265	265	265	265	285	340	355	375	385	405	420
Hour	12-13	13-14	14-15	15-16	16-17	17-18	18-19	19-20	20-21	21-22	22-23	23-24
Price at ElKräng [SEK/MWh]	435	380	375	370	365	360	370	355	350	350	365	320

Table 6 Notation for the planning problem of AB Vattenkraft.

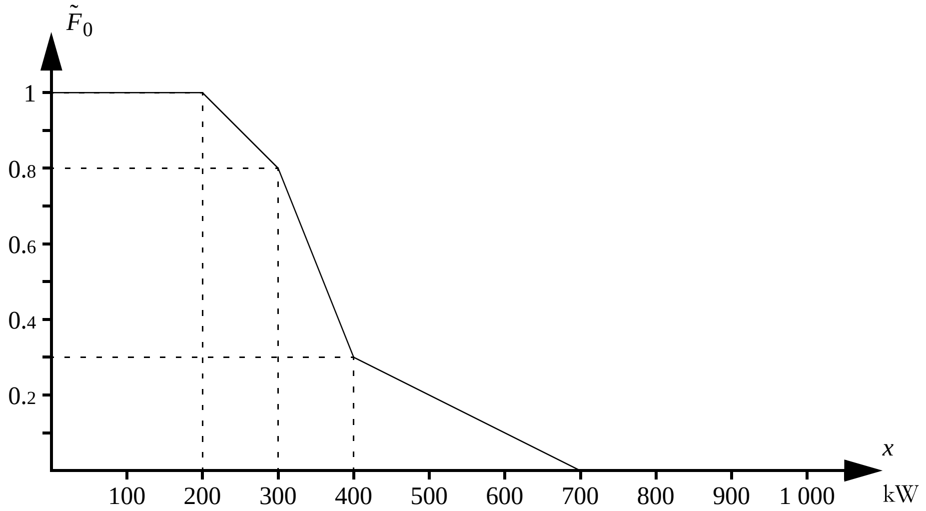
Symbol	Explanation	Value
$\underline{S}(M)$	Natural outflow from Sjön as function of reservoir contents	See figure
$M_{i,0}$	Start contents of reservoir *i*	See table 4
$\overline{M}_i$	Maximal contents of reservoir *i*	See table 4
$\mu_{i,j}$	Marginal production equivalent in power plant *i*, segment *j*	See table 4
$\overline{Q}_{i,j}$	Maximal discharge in power plant *i*, segment *j*	See table 4
V_i	Local inflow to reservoir *i*	See table 4
λ_t	Expected price at ElKräng hour *t*	See table 5
D	Contracted load	100
λ_f	Expected future electricity price	375

Table 7 Water delay time between the reservoirs and power plants of AB Vattenkraft.

From	To	Delay time [h]
By	Sele	2
Sjön	Fors	1
Fors	Sele	3

Problem 9 (20 p)

Mji is a town in East Africa. Despite many promises from the electricity company in Nchi, Mji is still not connected to the national grid. A number of entrepreneurs and citizens in Mji are therefore considering to start a cooperative, Mji Electricity Consumers Cooperative Limited (MECCO), which will build and operate a local power system in Mji, The idea is that members will be allowed to connect to the local grid and that all members should pay a uniform tariff per consumed kWh. This tariff, which will be the only income of MECCO, should be set to cover all costs (fixed and variable) in an annual basis. Moreover, the tariffs should generate a surplus around 5%, which can be used for future investments in the power system. However, before they are ready start a cooperative, a feasibility study is requested to verify that it is possible to set tariffs that are economically sustainable for the cooperative at the same time as they are reasonable for the consumers. Based on experience from other parts of Nchi, it has been decided to use the load duration curve shown in the figure below for the feasibility study.



MECCO would be able to take over two diesel generator sets located in a school in Mji (which the school itself lacks funding to operate). These diesel generator sets have a capacity of 100 kW each, the availability is assumed to be around 80% and the operation cost 10 ¤/kWh. MECCO would also be able to build their own line connecting the local grid to the national system. Such a line would have a capacity of 2 000 kW. The line in itself is expected to be very reliable (i.e., it is possible to neglect the risk of outages in the line); however, the electricity demand in the national grid is large compared to the generation capacity and rotating load curtailment is common practice. MECCO estimates that Mji will be disconnected in average 438 hours per year. MECCO would be able to buy electricity from the national electricity company at the price 5 ¤/kWh. The consumption of MECCO would however be measured at the connection point to the national grid, which means that the losses on the line would have to be paid by the cooperative. These losses can be computed as

$$L = \beta_L \cdot P^2,$$

where

L = losses on the line [kW],
β_L = loss coefficient [kW^{-1}] = 0.0001,
P = injected power on the line [kW].

a) (8 p) Suggest a method to compute the expected operation cost of the power system in Mji. Describe which assumptions you do and how you do to obtain a result that is as accurate as possible considering the limited data given above and the limited time you have at disposal.

b) (12 p) Which tariff will MECCO need to charge according to the conditions above? Assume that the fixed costs of the cooperative (for example capital costs, salaries for staff and maintenance costs) are 23 M¤/year. The losses in the distribution grid in Mji can be considered negligible.

Table 8 Random numbers from a $U(0, 1)$ distribution.

0.81	0.10	0.16	0.14	0.66
0.91	0.28	0.97	0.42	0.04

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Answer sheet for part I

Name: ....

Personal number: ....

Problem 1

a) Alternative .... is correct.

b) Alternative .... is correct.

Problem 2

a) .... ¤/MWh **b)** .... ¤/MWh

c) .... ¤/MWh **d)** .... ¤/MWh

e) ....

....

Problem 3

a) Alternative .... is correct. **b)** ....

c) Area A: .... Hz Area B: .... Hz

Problem 4

a) .... hours.

b) ....

c) ....

....

....

d) Alternative .... is correct.

Problem 5

a) .... kWh/h **b)** .... %

c) .... ¤/h **d)** .... kW

e) *LOLP* .... % *ETOC* .... ¤/h

Problem 1

a) 2, **b)** 1.

Problem 2

a) The electricity price is in this our set by the CHP generation ⇒ 250 ¤/MWh.

b) The electricity price is in this our set by the CHP generation ⇒ 250 ¤/MWh.

c) The electricity price is in this our set by the coal condensing generation ⇒ 350 ¤/MWh.

d) The electricity price is in this our set by the CHP generation ⇒ 250 ¤/MWh.

e) Yes. (There are no capacity limitations in they hydro power, as there is no extra coal condensing used during the peak demand period between 8 and 9. If there were no reservoir limitations the same thermal power plants would be used during the entire day. However, in this case the stored water at 10 o'clock plus the inflow between 10 and 18 is not sufficient to avoid starting the coal condensing.)

Problem 3

a) 2.

b) After the disconnection of Stad, the primary control has to reduce the generation by 600 MW. Area B has half the gain in the system and will consequently contribute to half of the generation decrease. Since the load and the other generation in area B remains the same, the generation decrease will have to be compensated by increased import from area A. The transmission on the line will therefore increase to 1 500 MW, which is more than the capacity of the line. Hence, the line will be disconnected.

c) When the line has been disconnected, area A has lost 200 MW generation, 800 MW load and 750 MW export. In total, the primary control in area A must reduce the generation by 1 350 MW, which results in a frequency increase $\Delta f = \Delta G/R$ = 1 350/5 000 = 0.27 Hz, i.e., the new frequency is 50.02 + 0.27 = 50.29 Hz.

Area B has lost 750 MW import, which must be replaced by the primary control in area B. This results in a frequency decrease $\Delta f = \Delta G/R$ = 750/5 000 = 0.15 Hz, i.e., the new frequency is 50.02 – 0.15 = 49.87 Hz.

Problem 4

a) The discharge at installed capacity is given by the relation $Q = H/\gamma(Q)$ = 37/0.25 = 148 HE. We also need 2 HE for the flow through the fish ladder. Consequently, 150 HE are released from the reservoir, which means that as the reservoir can store 6 480 000/3 600 = 1 800 HE, a full reservoir will be enough for 12 hours maximal generation.

b) $M_{3,t} = M_{3,t-1} + V_{3,t} + Q_{2,1,t} + Q_{2,2,t} + S_{2,t} + Q_{5,1,t} + Q_{5,2,t} + S_{5,t} - Q_{3,1,t} - Q_{3,2,t} - S_{3,t}$.

c) The optimisation variables involved in the problem are reservoir contents, discharge and spillage, which yields the following limits:

$$0 \le M_{i,t} \le \overline{M}_i, \qquad i = 1, \ldots, 5, t = 1, \ldots, 24,$$

$$0 \le Q_{i,j,t} \le \overline{Q}_{i,j} \qquad i = 1, \ldots, 5, j = 1, 2, t = 1, \ldots, 24,$$

$$\underline{S}_i \le S_{i,t} \le \bar{S}_i, \qquad i = 1, \ldots, 5, t = 1, \ldots, 24.$$

d) 4.

Problem 5

a) Since the hydro power plant is assumed to have 100% availability we get that $\tilde{F}_1(x) = \tilde{F}_0(x)$. Hence, the unserved energy during an hour is given by

$$EENS_1 = 1 \cdot \int_{900}^{\infty} \tilde{F}_0(x)dx = 0{,}06 \cdot 100/2 = 3 \text{ kWh/h}.$$

b) The risk of power deficit is given by $\tilde{F}_2(1\,050) = 0.9\tilde{F}_1(1\,050) + 0.1\tilde{F}_1(900) = 0 + 0.1 \cdot 0.06$ = 0.6%.

c) The expected generation in the diesel generator set is $EG_2 = EENS_1 - EENS_2 = 3 - 0.3$ = 2.7 kWh/h. Thus, the expected total operation cost is $ETOC = 5EG_2$ = 13.5 ¤/h.

d) The inverse transform method states that $D = F_D^{-1}(U)$, where U is a $U(0, 1)$-distributed random number. Since it is the duration curve that is given in the problem, we may as well use the transform $D = \tilde{F}_D^{-1}(U)$. The original random number must then have been $U = \tilde{F}_0(500) = 0.3$. Hence, $U^* = 1 - U = 0.7$, which results in $D^* = \tilde{F}_0^{-1}(U^*) = 420$ kW

e) $$m_{LOLO} = m_{(LOLO - \tilde{LOLO})} + \mu_{\tilde{LOLO}} = \frac{1}{n}\left(\sum_{i=1}^{n} lolo_i - \sum_{i=1}^{n} \tilde{lolo}_i\right) + 0.006 =$$

$$= \frac{1}{1\,000}(14 - 5) + 0.006 = 1.5\%.$$

$$m_{ENS} = m_{(TOC - \tilde{TOC})} + \mu_{\tilde{TOC}} = \frac{1}{n}\left(\sum_{i=1}^{n} toc_i - \sum_{i=1}^{n} \tilde{toc}_i\right) + 13.5 =$$

$$= \frac{1}{1\,000}(27\,000 - 14\,500) + 13.5 = 26 \text{ ¤/h}.$$

Problem 6

a) Without the green certificates all hydro power and nuclear power will be used. In addition to that, 30/40 of the fossil fuelled generation will be needed, which means that 3/4 of the price interval for fossil fuels will be used. The electricity price for the consumers will then be 45 ¤/MWh.

If green certificates are introduced, 14 TWh wind power (10% of the total consumption) will be supplied to the system. Consequently, only 16 out of 40 TWh fossil fuels will be needed and the electricity price will then be 38 ¤/MWh. To compute the total cost per MWh for the consumers, we also need to include that they have to buy a tenth of a certificate for each MWh. According to the figure, the price of green certificates will be 29 ¤/certificate if the demand corresponds to

14 TWh certificates. The total price is then 40.9 ¤/MWh. Introducing green certificates is thus profitable for the consumers in Rike.

b) The two factors that decide if the green certificates are a profit or a loss for the consumers are how much the electricity price is decreasing for each TWh wind power supplied to the system, as well as the cost of the certificates. If the fossil fuels would be in the price interval 40–50 ¤/MWh instead and the certificate prices are 25% higher, then the corresponding computations as in part a gives the electricity price 47.5 ¤/MWh without certificates and a total cost of 44 + 3.625 = 47.625 ¤/MWh.

Problem 7

It is necessary to activate $\Delta G = R{\cdot}\Delta f = 5\,000{\cdot}0{,}05 = 250$ MW up-regulation in order to increase the frequency by 0.05 Hz. The least expensive alternative would be to activate bids 1–4. The question is if this is possible considering the limitations in the transmission system.

In area A the generation is increased by 150 MW as a consequence of the activated up-regulation, but at the same time the generation in the power plants participating in the primary control is decreased by 125 MW (because the total up-regulation is 250 MW and area A has half the gain in the system). The new flow from A to C is then 1 925 MW, which can be managed by the transmission line.

In area B the generation is increased by 100 MW as a consequence of the activated up-regulation, but at the same time the generation in the primary control is decreased by 50 MW (because area B has 20% of the gain in the system). The new flow from B to C is then 1 150 MW, which can be managed by the transmission line.

No up-regulation bids are activated in area C, whereas the generation in the primary control is decreased by 50 MW (because area C has 20% of the gain in the system). At the same time, the import is increased by 75 MW. The new flow from C to D is then 825 MW, which can be managed by the transmission line.

No up-regulation bids are activated in area D, whereas the generation in the primary control is decreased by 25 MW (because area D has 10% of the gain in the system), but this can—as concluded above—be compensated by increased import from area C.

Thus, the conclusion is that the system operator can activate the first four bids without overloading any transmission line.

Problem 8

a) The challenge in this problem is to linearise the relation between spillage and reservoir contents in Sjön. In this solution, we choose to use two linear segments, where the breakpoint is placed at the content 4 000 HE (because the spillage is significantly increasing at that level). There is no need to introduce integer variables, as the spillage from Sjön is unfavourable (more electricity per HE is obtained in By compared to Fors); hence, the solution will use the first use the first segment, where the marginal spillage is lower.

The problem we want to solve is then

maximise	*value of sold electricity + value of stored water – cost of purchased electricity*,
subject to	*hydrological balance for the reservoirs*,
	relation between spillage and contents in Sjön ,
	load balance,
	limitations in reservoirs, discharge and spillage.

Indices for the power plants

By/Sjön 1, Fors 2, Sele 3.

Parameters

Most of the parameters are defined in table 6 in the problem text, but a few more parameters are necessary to manage the nonlinear relation between the natural outflow and the contents of the reservoir Sjön, $\underline{S}(M)$. Therefore, we introduce the following parameters:

$$\sigma_j = \text{marginal spillage in Sjön, segment } j = \{\text{read from figure}\} = $$
$$= \begin{cases} (6-1)/4\,000 = 0.00125 & j = 1, \\ (10-6)/1\,000 = 0.004 & j = 2. \end{cases}$$

$$\overline{M}_{1,j} = \text{maximal contents in Sjön, segment } j = \begin{cases} 4\,000 & j = 1, \\ 1\,000 & j = 2. \end{cases}$$

Optimisation variables

$Q_{i,j,t}$ = discharge in power plant i, segment j, during hour t, $i = 1, 2, 3, j = 1, 2, t = 1, \ldots, 24$,

$S_{i,t}$=spillage from reservoir i during hour t, $i = 1, 2, 3, t = 1, \ldots, 24$,

$M_{1,j,t}$=contents of Sjön, segment j, at the end of hour t, $j = 1, 2, t = 1, \ldots, 24$,

$M_{i,t}$ = contents of reservoir i at the end of hour t, $i = 2, 3, t = 1, \ldots, 24$,

p_t = purchase from ElKräng during hour t, $t = 1, \ldots, 24$,

r_t = sales to ElKräng during hour t, $t = 1, \ldots, 24$.

Objective function

$$\text{maximise} \quad \sum_{t=1}^{24} \lambda_t(r_t - p_t) + \lambda_f((\mu_{1,1} + \mu_{3,1})M_{1,24} + (\mu_{2,1} + \mu_{3,1})M_{2,24} + \mu_{3,1}M_{3,24}).$$

Constraints

Hydrological balance for By/Sjön:

$$M_{1,1,t} + M_{1,2,t} = M_{1,1,t-1} + M_{1,2,t-1} - Q_{1,1,t} - Q_{1,2,t} - S_{1,t} + V_{1,t}, \qquad t = 1, \ldots, 24.$$

Hydrological balance for Fors:

$$M_{2,t} = M_{2,t-1} - Q_{2,1,t} - Q_{2,2,t} - S_{2,t} + S_{1,t} + V_{2,t}, \qquad t = 1, \ldots, 24.$$

Hydrological balance for Sele:

$$M_{3,t} = M_{3,t-1} - Q_{3,1,t} - Q_{3,2,t} - S_{3,t} + Q_{1,1,t} + Q_{1,2,t} + Q_{2,1,t} + Q_{2,2,t} + S_{2,t} + V_{2,t}, \qquad t = 1, \ldots, 24.$$

Relation between spillage and reservoir contents at Sjön:

$$S_{1,t} \geq \sum_{j=1}^{2} \sigma_j M_{1,j,t}, \qquad t = 1, \ldots, 24.$$

Load balance:

$$\sum_{i=1}^{3} \sum_{j=1}^{2} \mu_{i,j} Q_{i,j,t} + p_t = D + r_t, \qquad t = 1, \ldots, 24.$$

Variable limits

$0 \le Q_{i,j,t} \le \overline{Q}_{i,j}$, $\quad i = 1, 2, 3, j = 1, 2, t = 1, \ldots, 24$,

$0 \le S_{i,t}$ $\quad i = 2, 3, t = 1, \ldots, 24$,

$0 \le M_{1,j,t} \le \overline{M}_{1,j}$, $\quad j = 1, 2, t = 1, \ldots, 24$,

$0 \le M_{i,t} \le \overline{M}_i$, $\quad i = 2, 3, t = 1, \ldots, 24$,

$0 \le p_t$, $\quad t = 1, \ldots, 24$,

$0 \le r_t$, $\quad t = 1, \ldots, 24$.

b) The objective function must be updated to include in the stored water the water that is on its way from a power plant to the downstream reservoir at the end of the planning period. It is also necessary to update the hydrological constraints so that the inflow to a reservoir for a specific hour that depending on water released from an upstream reservoir, is based on the discharge and spillage from an earlier hour. For example, in the hydrological constraint of Fors, $S_{1,t-1}$ should be used for water spilled from Sjön, as it takes one hour for water to travel from Sjön to Fors. This means that we will also need new parameters for discharge and spillage a couple of hours before the start of the planning problem.

Problem 9

a) We have to cases to study: During 5% of the time (438 hours out of 8 760) MECCO will be disconnected from the national grid, and during the remaining time, the entire load in Mji can be supplied by the national grid (which is preferable as the electricity from the grid is less expensive than the electricity generated in the diesel generator sets. In order to compute the total operation costs, we may therefore introduce two strata: in the first the line is disconnected (stratum weight 0.05) and in the second the line is available (stratum weight 0.95).

In the first stratum, we do not need to consider any losses, as the two diesel generator sets are located in Mji and we may neglect distribution losses. The expected operation cost of this stratum can thus be computed analytically using probabilistic production cost simulation.

In the second stratum, we should consider the transmission losses which are not insignificant ((64 kW at the maximum load 800 kW). As each level of electricity demand in Mji (the probability distribution of which is known) we get a specific transmission loss, we could calculate a new duration curve for the sum of the load and the losses and then compute the expected transmission from the national grid analytically. However, this is quite a complex procedure without a computer; hence, in this case it is probably preferable to use Monte Carlo simulation to compute the expected operation cost of the second stratum. In this simulation it would of course also be straightforward to apply complementary random numbers.

b) We start by computing the expected operation cost according to the methods described above.

The first stratum is simulated using probabilistic production cost simulation. As the diesel generator sets only have two possible states each (available or unavailable) we can the formula

$$EG_g = T \cdot p_g \int_{\hat{G}_{g-1}^{tot}}^{\hat{G}_g^{tot}} \tilde{F}_{g-1}(x)dx$$

$$\Rightarrow \quad EG_1 = 0.8 \int_0^{100} \tilde{F}_0(x)dx = 80 \text{ kWh/h},$$

$$EG_2 = 0.8 \int_{100}^{200} \tilde{F}_1(x)dx = \{\tilde{F}_1(x) = \tilde{F}_0(x) \text{ for } x \le 200\} = 80 \text{ kWh/h}.$$

Hence we get $ETOC_1 = 10 \cdot (80 + 80) = 1\,600$ ¤/h.

Using the random variables provided in the problem text and their complementary random numbers, we can generate twenty scenarios for the second stratum:

Random number, u_i	Load, d_i [kW]	Transmission losses, l_i [kW]	Operation cost, toc_i [¤/h]	Complementary random number, u_i^*	Load, d_i^* [kW]	Transmission losses, l_i^* [kW]	Operation cost, toc_i^* [¤/h]
0.81	295	8.7	1 519	0.19	510	26.0	2 680
0.10	600	36.0	3 180	0.90	250	6.2	1 281
0.16	540	29.2	2 846	0.84	280	7.8	1 439
0.14	560	31.4	2 957	0.86	270	7.3	1 386
0.66	328	10.8	1 694	0.34	392	15.4	2 037
0.91	245	6.0	1 255	0.09	610	37.2	3 236
0.28	420	17.6	2 188	0.72	316	10.0	1 630
0.97	215	4.6	1 098	0.03	670	44.9	3 574
0.42	376	14.1	1 951	0.58	344	11.8	1 779
0.04	660	43.6	3 518	0.96	220	4.8	1 124

The mean of these twenty observations is 2 119 ¤/h, which is our estimate of $ETOC_2$.

Now we get $ETOC = 0.05 ETOC_1 + 0.95 ETOC_2 = 2\,093$ ¤/h or 18.56 M¤/year. Including the fixed costs we get a total cost of 41,56 M¤/year. In addition to this, there should be a 5% margin, which means that the income must be 43.64 M¤/year.

In order to compute the tariff, we now need to know how much electric energy MECCO will deliver in a year. This can be computed using some of the results from the simulation of the operation cost. In the first stratum, the expected generation of the two diesel generator sets 160 kWh/h and this equals the delivery to the consumers as the distribution losses are neglected. In the second stratum, we use the mean of the twenty observed load values, which results in the expected load 405 kWh/h.Combining these results yields an average delivery around 393 kWh/h or 3 441 MWh/year. Hence, we can conclude that the tariff need to be around 12.68 ¤/kWh.