ÖVNING 10: Implementering

Uppgifter: 11.2, 11.1, 11.3

TEORI:

- Vi har i denna kurs tittat på tidskontinuerliga regulatorer som:
- PID: $u(t) = K\left(e(t) + \frac{1}{T_I}\int_{t_0}^{t} e(\tau)\,d\tau + T_D\frac{de(t)}{dt}\right)$
- Lead/Lag: $U(s) = K\left(\frac{\tau_D s + 1}{\beta\tau_D s + 1}\right)\left(\frac{\tau_I s + 1}{\tau_I s + \gamma}\right)E(s)$

Hur ska vi implementera dessa i datorer?

- Datorer arbetar i diskret tid, så vi måste approximera våra kontinuerliga system och regulatorer med diskreta motsvarigheter.

Approximationer:

- Euler bakåt: $\dot{x}(t) \approx \Delta_e x(t) = \frac{1}{T}\left(x(t) - x(t-T)\right)$
- Tustins formel: $\dot{x}(t) \approx \Delta_t x(t)$ där $\Delta_t x(t)$ ges av:

$$\frac{1}{2}\left(\Delta_t x(t) + \Delta_t x(t-T)\right) = \frac{1}{T}\left(x(t) - x(t-T)\right)$$

Notation:

- Deriveringsoperatorn p: $\dot{x}(t) = p\,x(t)$

$$\ddot{x}(t) = p^2 x(t)$$

- Förskjutningsoperatorn q_T: $x(t+T) = q_T\,x(t)$

$$x(t-T) = q_T^{-1}x(t)$$

Approximationer:

- Euler bakåt: $px(t) \approx \frac{1}{T}(1-q^{-1})x(t)$
$$\Rightarrow \underline{p \approx \frac{1}{T}(1-q^{-1})}$$

- Tustins formel: $px(t) \approx \frac{2}{T} \frac{1-q^{-1}}{1+q^{-1}} x(t)$
$$\Rightarrow \underline{p} \approx \frac{2}{T} \underline{\frac{1-q^{-1}}{1+q^{-1}}}$$

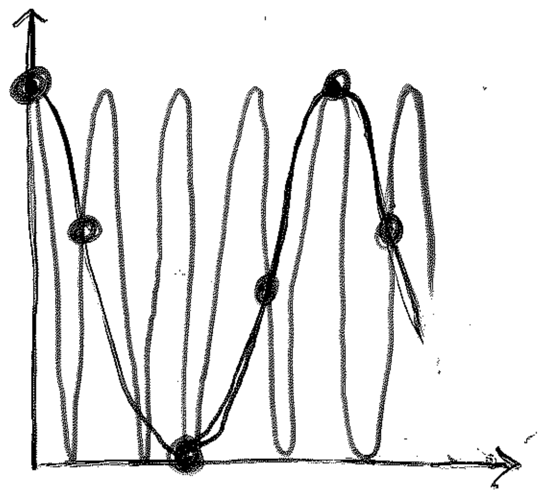
Sampling:

T: samplingsintervall; tiden mellan punkter som används.

ω_s: samplingsfrekvens $\quad \omega_s = \frac{2\pi}{T}$

ω_N: nyquist frekvens $\quad \omega_N = \frac{\omega_s}{2} = \frac{\pi}{T}$

OBS! Aliaseffekten! (s.218)
$\omega > \omega_N$ kan inte skiljas från en långsammare frekvens.



11 Implementation

11.1 If you "translate" the compensator

$$U(s) = KN(\frac{s+b}{s+bN})E(s)$$

with Tustin's formula you get a controller of the form

$$u(t) = \beta_1 u(t-T) + \alpha_1 e(t) + \alpha_2 e(t-T)$$

What are the values of α_1, α_2, and β_2, if $T = 0.1$, $N = 10$, $b = 0.1$, and $K = 2$?

11.2 Consider the system

$$\dot{y}(t) = u(t)$$

Suppose it is controlled with a computer, so that the control signal is constant over the sampling interval, that is,

$$u(t) = u_k, \quad kT \leq t < (k+1)T$$

a) Introduce the notation $y_k = y(kT)$ and derive a relation between y_{k+1}, y_k, and u_k.

b) Suppose we use the proportional feedback

$$u_k = -Ky_k$$

and that $y(0) = y_0$. What are the values of K, for which the closed loop system is stable?

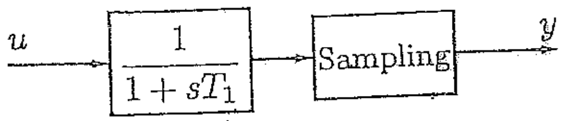


Figure 11.3a.

11.3 Consider the system in Figure 11.3a, which illustrates sampling with prefiltering. Suppose we are sampling with the sampling period T and that $u = u_0 + u_1$, where u_0 is an "interesting" low frequency signal in the frequency interval $0 < \omega < \pi/T$ and that u_1 is a sinusoidal control signal

$$u_1(t) = \sin \omega_2 t, \quad \frac{\pi}{T} < \omega_2 < \frac{2\pi}{T}$$

Since the sampling causes aliasing, the output will be

$$y(t) = y_0 + y_1$$

where y_0 is interesting and y_1 is a disturbance signal

$$y_1(kT) = A \sin(\omega_1 kT + \varphi), \quad \omega_1 < \pi/T$$

a) What are A, ω_1 and φ?

b) It is clear from a) that the choice of T affects the amplitude of the disturbance signal y_1. What is the smallest amplitude you can get if you do not want to damp any frequencies in u_0 more than $\sqrt{2}$ times?