Transfer functions in Control Structure

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1 Definitions

The transfer functions provided are given for the notation and structure given in Figure 1.

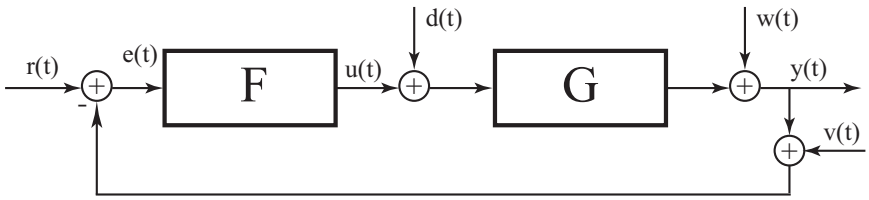

Figure 1: Block Diagram

1.1 Laplace transforms

$$
\begin{aligned}
\mathfrak{L}\big(y(t)\big) &= Y(s) \\
\mathfrak{L}\big(u(t)\big) &= U(s) \\
\mathfrak{L}\big(r(t)\big) &= R(s) \\
\mathfrak{L}\big(e(t)\big) &= E(s) \\
\mathfrak{L}\big(d(t)\big) &= D(s) \\
\mathfrak{L}\big(w(t)\big) &= W(s) \\
\mathfrak{L}\big(v(t)\big) &= V(s)
\end{aligned}
$$

If upper case letters are used it is assumed that they are the Laplace-transform of the continues version as above.

1.2 Disturbances

The signals $d(t)$, $w(t)$ and $v(t)$ in Figure 1 are disturbances entering the system. The disturbance $d(t)$ is called *load disturbance*, the disturbance $w(t)$ is called *output disturbance* and $v(t)$ is called *measurement disturbance.*

2 Signal expressions

2.1 Expression for Y

$$Y = W + G(D + F(R - (Y + V)))$$

$$Y = W + GD + GFR - GFY - GFV$$

$$Y + GFY = W + GD + GFR - GFV$$

$$Y(1 + GF) = W + GD + GFR - GFV$$

$$Y = \underbrace{\frac{1}{1+GF}}_{S(s)} W + \frac{G}{1+GF} D + \underbrace{\frac{GF}{1+GF}}_{G_c(s)} R - \underbrace{\frac{GF}{1+GF}}_{T(s)} V$$

2.2 Expression for U

$$U = F(R - (V + W + G(D + U)))$$

$$U = FR - FV - FW - GFD - GFU$$

$$U + GFU = FR - FV - FW - GFD$$

$$U(1 + GF) = FR - FV - FW - GFD$$

$$U = \frac{F}{1+GF} R - \frac{F}{1+GF} V - \frac{F}{1+GF} W - \frac{GF}{1+GF} D$$

2.3 Expression for E

$$E = R - (V + W + G(D + FE))$$

$$E = R - V - W - GD - GFE$$

$$E + GFE = R - V - W - GD$$

$$E(1 + GF) = R - V - W - GD$$

$$E = \frac{1}{1+GF}R - \frac{1}{1+GF}V - \frac{1}{1+GF}W - \frac{G}{1+GF}D$$