Understanding the influence of the P-, I-, and D-part in PID controllers

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The aim here is to give a simple understanding of how the different parts of the PID-controller works and how they interact. The explanation is based on the P-, PI-, and PD-controller and how they can be approximated to give insight on how the PID-controller works. The PID-controller discussed is shown in Figure 1 showing how the controller may be represented by the sum of a PI-controller, a PD-controller minus a P-controller.

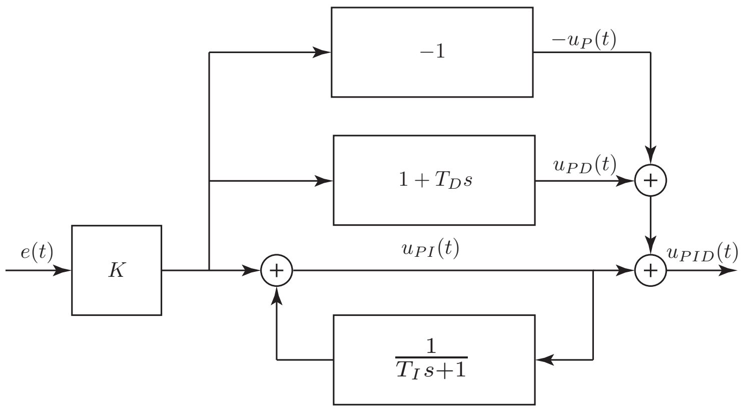

Figure 1: A PID controller

1 The PI-controller

The PI-control signal u_{PI} in Figure 1 is given by

$$u_{PI} = Ke + \frac{1}{T_I s + 1} u_{PI} \quad \Rightarrow \quad u_{PI} \quad = Ke + \frac{K}{T_I s} e$$

We note that when $T_I s$ is small the following hold

$$\frac{1}{1 + T_I s} = \sum_{n=0}^{\infty} (-T_I s)^n \approx 1 - T_I s.$$

From this we may approximate the controller to be

$$u_{PI} = Ke + \frac{1}{T_I s + 1} u_{PI} \approx Ke + (1 - T_I s) u_{PI} \Rightarrow$$
$$u_{PI}(t) \approx Ke(t) + u_{PI}(t) - T_I \frac{d}{dt} u_{PI}(t)$$

We note that this is the first term in the Taylor expansion so that for small T_I we have

$$u_{PI}(t) \approx Ke(t) + u_{PI}(t - T_I).$$

2 The PD-controller

The PD-control signal u_{PD} in Figure 1 is given by

$$u_{PD} = Ke + T_D s Ke \quad \Rightarrow \quad u_{PD}(t) = Ke(t) + KT_D \frac{d}{dt} e(t)$$

We note that this is the first term in the Taylor expansion so that for small T_D we have

$$u_{PD}(t) \approx Ke(t + T_D)$$

3 The PID-controller

The PID-control signal in Figure 1 is given by $u_{PID} = u_{PI} + u_{PD} - u_P$ or

$$u_{PID} = Ke + \frac{K}{T_I s} e + Ke + KT_D s e - Ke$$
$$u_{PID}(t) = Ke(t) + \frac{K}{T_I} \int_0^t e(s) ds + KT_D \frac{d}{dt} e(t)$$

or in simplified terms

$$\begin{aligned} u_{PID}(t) &\approx Ke(t) + u_{PI}(t - T_I) + Ke(t + T_D) - Ke(t) \\ &= u_{PI}(t - T_I) + Ke(t + T_D). \end{aligned}$$

We may further express the following

$$\begin{aligned} u_{PI}(t - T_I) &= u_{PID}(t - T_I) - u_{PD}(t - T_I) + u_P(t - T_I) \\ &\approx u_{PID}(t - T_I) - Ke(t + T_D - T_I) + Ke(t - T_I). \end{aligned}$$

So that

$$u_{PID}(t) \approx Ke(t - T_I) + Ke(t + T_D) - Ke(t + T_D - T_I) + u_{PID}(t - T_I).$$

4 Summary

P-control:	$u(t) = Ke(t)$		
PI-control:	$u(t) \approx Ke(t)$	$+u(t - T_I)$	
PD-control:	$u(t) \approx Ke(t + T_D)$		
PID-control:	$u(t) \approx Ke(t + T_D)$	$+u(t - T_I)$	$+\underbrace{Ke(t - T_I) - Ke(t + T_D - T_I)}_{\neq Ke(t)}$