Ensemble Learning

Lecture 6, DD2431 Machine Learning

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Outline: Ensemble Learning

We will describe and investigate algorithms to

train weak classifiers/regressors and how to combine them

to construct a classifier/regressor more powerful than any of the individual ones.

They are called Ensemble learning, Commitee machine, etc.

Background/methods:

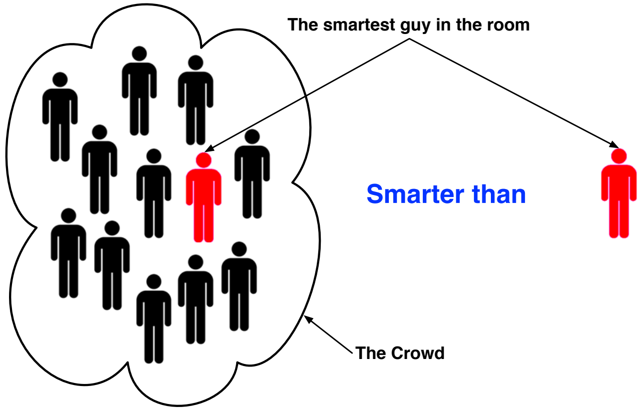
Wisdom of Crowds

Classifier characterization: bias and variance

Bagging: static structure, parallel

Boosting: static structure, serial (Example: face detection)

The Wisdom of Crowds



The **collective knowledge** of a *diverse* and *independent* body of people typically **exceeds** the knowledge of **any single individual** and can be harnessed by voting.

The Wisdom of Crowds - Really?

Crowd wiser than **any individual**

- When ?
- For which questions ?

See **The Wisdom of Crowds** by *James Surowiecki* published in 2004 to see this idea applied to business.

What makes a crowd wise?

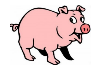
Four elements required to form a wise crow (*J. Surowiecki*):

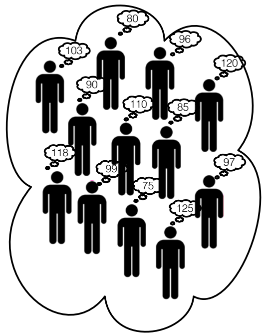
- **Diversity of opinion.** People in crowd should have a range of experiences, education and opinions.
- **Independence.** Prediction by person in crowd is not influenced by other people in the crowd.
- **Decentralization** People have specializations and local knowledge.
- **Aggregation.** There is a mechanism for aggregating all predictions into one single prediction.

Crowd wisdom is best suited for problems that involve optimization, but ill suited for problems that require creativity or innovation. (in **You Are Not a Gadget** by *Jaron Lanier*)

Consider this scenario

Ask each person in the same crowd:

How much does the pig weigh? 



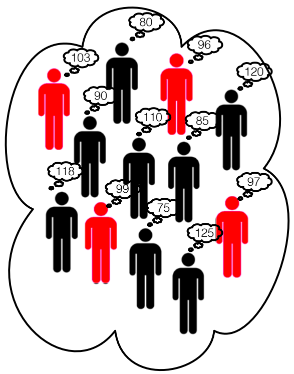
Crowd's prediction:

AVERAGE of all predictions.

$\Leftarrow$ This crowd predicts **99.8333**.

(The pig weighs 99.8333 kg.)

Has crowd made a good estimate?

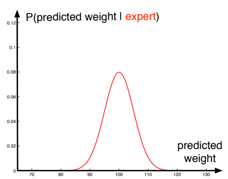


If composition of crowd:

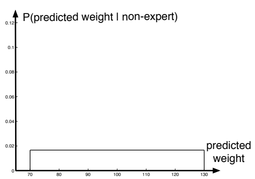
30% **EXPERTS**.
70% **NON-EXPERTS**.

and their level of expertise:

(Say *pig*'s true weight is 100 kg)

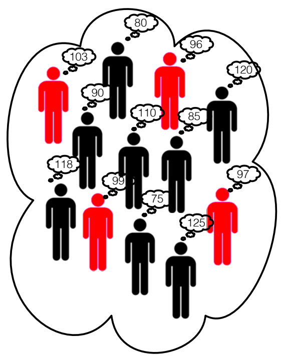


P(pred. weight | expert) :
$\mathcal{N}(100, 5^2)$

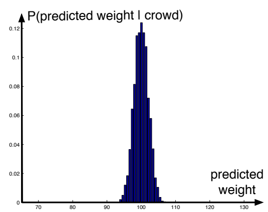


P(pred. weight | non-expert) :
$\mathcal{U}(70, 130)$

Has crowd made a good estimate?



If crowd contains ***independent*** **50 people:**



$\Uparrow$

$P(\text{pred. weight}|\text{crowd})$

On average this crowd will make better estimates than the experts.
It is wiser than each of the experts!

But....

Why not just asking asked a bunch of experts??

- Large enough crowd $\Longrightarrow$ high probability a sufficient number of experts will be in crowd (for any question).
- Random selection $\Longrightarrow$ don't make a biased choice in experts.
- For some questions it may be hard to identify a diverse set of experts

Back to machines

The crowd must be careful

In the analysis of the crowd it is implicitly assumed:

- each person is not concerned with the opinions of others,
- The non-experts will predict a completely random wrong answer - these will somewhat cancel each other out.

However, there may be a systematic and consistent bias in the non-experts' predictions.

If the crowd does not contain sufficient experts then *truth by consensus*, rather than fact, leads to **Wikiality**!

(Term coined by *Stephen Colbert* in an episode of the *The Colbert Report* in July 2006.)

Combining classifiers

Will exploit *Wisdom of crowd* ideas for specific tasks by

- combining classifier predictions **and**
- aim to combine independent and diverse classifiers.

But will use labelled training data

- to identify the **expert** classifiers in the pool;
- to identify **complementary** classifiers;
- to indicate how to best combine them.

The bias-variance decomposition

Let us consider

$$\begin{aligned} f(\mathbf{x}) &: \text{ true function} \\ \hat{f}(\mathbf{x}) &: \text{ estimated prediction function (= model)} \\ E[\hat{f}(\mathbf{x})] &: \text{ average of models due to different sample sets} \end{aligned}$$

The mean square error (MSE) for estimating $f(\mathbf{x})$

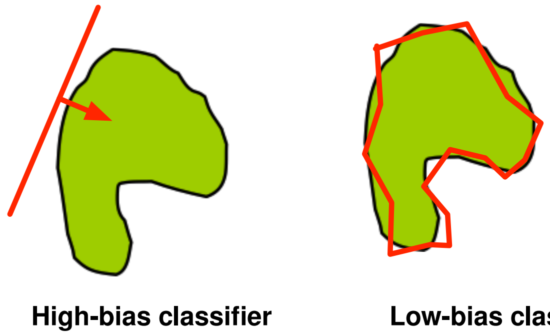
$$\begin{aligned} E[f(\mathbf{x}) - \hat{f}(\mathbf{x})]^2 &= E[(\hat{f}(\mathbf{x}) - E[\hat{f}(\mathbf{x})])^2] + (E[\hat{f}(\mathbf{x})] - f(\mathbf{x}))^2 \\ &= \textbf{Variance} + (\textbf{Bias})^2 \end{aligned}$$

Bias of a classifier is the discrepancy between its averaged estimated and true function

$$E[\hat{f}(\mathbf{x})] - f(\mathbf{x})$$

Characterization of a classifier: Bias

Green region is the true boundary.



Low model complexity (small # of d.o.f.) $\Longrightarrow$ High-bias
High model complexity (large # of d.o.f.) $\Longrightarrow$ Low-bias

Ensemble Prediction: Voting

A **diverse** and **complementary** set of high-bias classifiers, with performance better than chance, combined by **voting**

$$f_V(\mathbf{x}) = \text{sign}\left(\sum_{t=1}^{T} h_t(\mathbf{x})\right)$$

can produce a classifier with a low-bias.

$h_t \in \mathcal{H}$ where $\mathcal{H}$ is a family of possible weak classifiers functions.

Example: Voting of oriented hyper-planes can define convex regions.

Ensemble Learning & Prediction

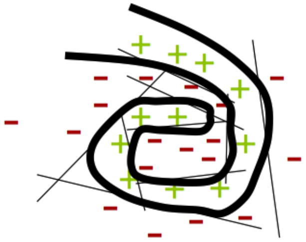
But how can we

- define a set of **diverse** and **complementary** high-bias classifiers, with non-random performance ?
- combine this set of high-biased classifiers to produce a low-bias classifier able to model a complex boundary (superior to voting)?

Ensemble Learning & Prediction (cont.)

How? Exploit labelled training data.

- Train different classifiers using the training data which focus on different subsets of the data.
- Use a weighted sum of these *diversely* trained classifiers.



This approach allows simple high-bias classifiers to be combined to model very complex boundaries.

Ensemble method: **Bagging**

Bootstrap **Agg**regat**ing**

Characterization of a classifier: Variance

Variance of a classifier is the expected divergence of the estimated prediction function from its average value:

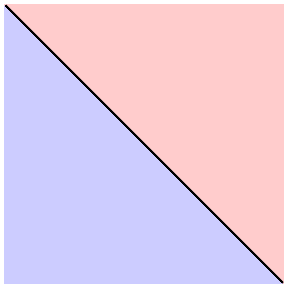
$$E[(\hat{f}(\mathbf{x}) - E[\hat{f}(\mathbf{x})])^2]$$

This measures how dependent the classifier is on the random sampling made in the training set.

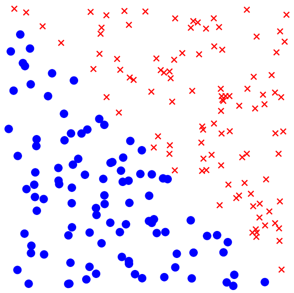
High model complexity (large # of d.o.f.) $\Longrightarrow$ High-variance Low model complexity (small # of d.o.f.) $\Longrightarrow$ Low-variance

Ensemble predictions such as *bagging*, *voting*, *averaging* using diverse high-variance, low-bias classifiers reduce the variance of the ensemble classifier.

Binary classification example



True decision boundary



Training data set $\mathcal{S}_i$

Estimate the true decision boundary with a *decision tree* trained from some labeled training set $\mathcal{S}_i$.

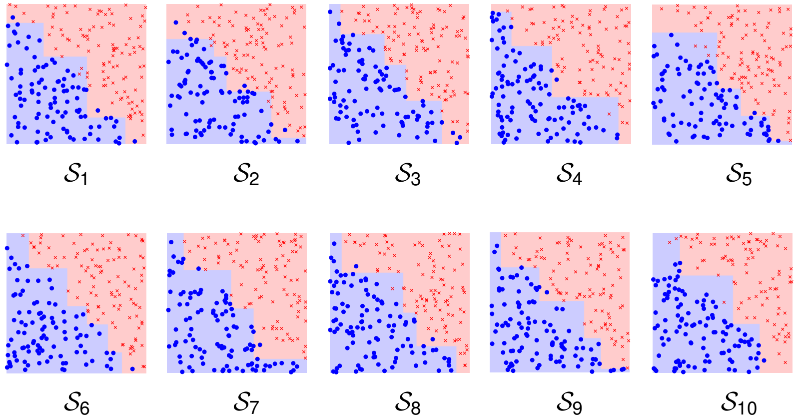
High variance, Low bias classifiers

decision trees

High variance classifiers produce differing decision boundaries which are highly dependent on the training data.

Low bias classifiers produce decision boundaries which on average are good approximations to the true decision boundary.

Estimated decision boundaries found using:



See how the decision boundaries on the previous slide differ from the

expected decision boundary of the decision tree classifier
(with $m = 200$ training points).

Bagging - Bootstrap Aggregating

Input: Training data

$$\mathcal{S} = \{(\mathbf{x}_1, y_1), \ldots, (\mathbf{x}_m, y_m)\}$$

of inputs $\mathbf{x}_j \in \mathbb{R}^d$ and their labels or real values y_j.

Iterate: for $b = 1, \ldots, B$

1. Sample training examples, *with replacement*, m times from $\mathcal{S}$ to create $\mathcal{S}_b$.
2. Use this bootstrap sample $\mathcal{S}_b$ to estimate the regression or classification function f_b.

Output: The bagging estimate for

Classification:

$$f_{\text{bag}}(\mathbf{x}) = \arg \max_{1 \le k \le K} \sum_{b=1}^{B} \text{Ind}\left(f_b(\mathbf{x}) = k\right)$$

Note: $\text{Ind}\,(x) = 1$ if $x = \text{TRUE}$ otherwise $\text{Ind}\,(x) = 0$

Regression:

$$f_{\text{bag}}(\mathbf{x}) = \frac{1}{B} \sum_{b=1}^{B} f_b(\mathbf{x})$$

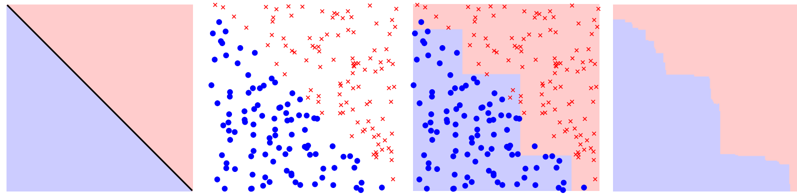
Bagging

is a procedure to reduce the variance of our classifier when labelled training data is limited.

Bias of **bagged classifier** may be marginally less than the base classifiers.

Note: it only produces good results for **high variance**, **low bias** classifiers.

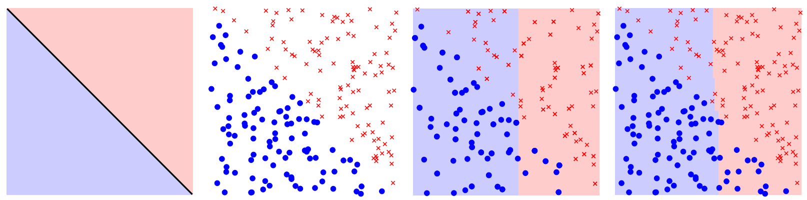
Apply bagging to the original example



Ground truth | $\mathcal{S}_1$ | **Classifier from** $\mathcal{S}_1$ | **Bagged classifier** $B = 100$.

Apply bagging to the original example (cont.)

If we bag a **high bias, low variance** classifier - *oriented horizontal and vertical lines* - we don't get any benefit.



Ground truth | $\mathcal{S}_1$ | **Classifier from** $\mathcal{S}_1$ | **Bagged classifier** $B = 100$.

Ensemble method: **Boosting**

Started from a question:

Can a set of weak learners create a single strong classifier where a weak learner performs only slightly better than a chance? (Kearns, 1988)

Ensemble Method: Boosting

Input: Training data $\mathcal{S} = \{(\mathbf{x}_1, y_1), \ldots, (\mathbf{x}_m, y_m)\}$ of inputs $\mathbf{x}_i$ and their labels $y_i \in \{-1, 1\}$ or real values.

$\mathcal{H}$: a family of possible weak classifiers/regression functions.

Output: A strong classifier/regression function

$$f_T(\mathbf{x}) = \text{sign}\left(\sum_{t=1}^{T} \alpha_t h_t(\mathbf{x})\right) \quad \text{or} \quad f_T(\mathbf{x}) = \sum_{t=1}^{T} \alpha_t h_t(\mathbf{x})$$

weighted sum of weak classifiers

$h_t \in \mathcal{H} \quad t = 1, ..., T$
α_t: confidence/reliability

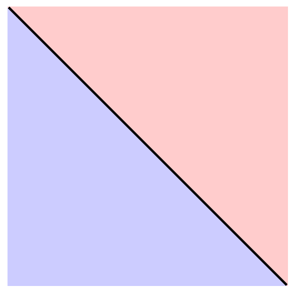
Ensemble Method: Boosting

How ?? (Just consider case of classification.)

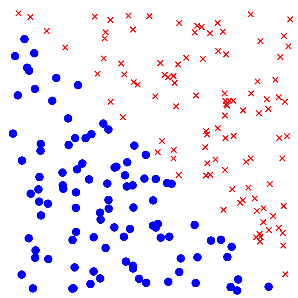
- Performance of classifiers $h_1, \ldots, h_t$ helps define h_{t+1}.
- Maintain **weight** $w_i^{(t)}$ for each training example in $\mathcal{S}$.
- Large $w_i^{(t)} \implies \mathbf{x}_i$ has greater influence on choice of h_t.
- Iteration t: $w_i^{(t)}$ increased if $\mathbf{x}_i$ wrongly classified by h_t.
- Iteration t: $w_i^{(t)}$ decreased if $\mathbf{x}_i$ correctly classified by h_t.

Remember: Each $h_t \in \mathcal{H}$

Binary classification example



True decision boundary

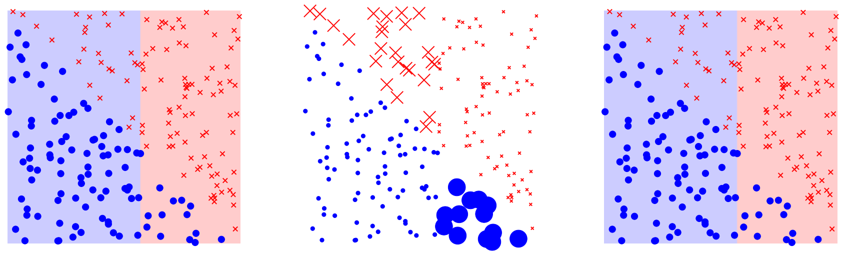


Training data

$\mathcal{H}$ is the set of all possible oriented vertical and horizontal lines.

Example

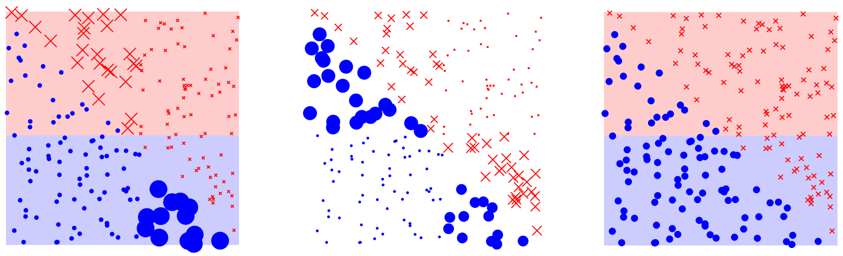
Round 1



Chosen weak classifier — Re-weight training points — Current strong classifier

$\epsilon_1 = 0.19$, $\alpha_1 = 1.45$ — $w_i^{(2)}$'s — $f_2(\mathbf{x})$

Round 2



Chosen weak classifier — Re-weight training points — Current strong classifier

Adaboost Algorithm (Freund & Schapire, 1997)

Given:

- Labeled training data

$$\mathcal{S} = \{(\mathbf{x}_1, y_1), \ldots, (\mathbf{x}_m, y_m)\}$$

of inputs $\mathbf{x}_j \in \mathbb{R}^d$ and their labels $y_j \in \{-1, 1\}$.

- A set/class $\mathcal{H}$ of T possible weak classifiers.

Initialize:

- Introduce a weight, $w_j^{(1)}$, for each training example.
- Set $w_j^{(1)} = \frac{1}{m}$ for each j.

Adaboost Algorithm (cont.)

Iterate: for $t = 1, \ldots, T$

1. Train classifier $h_t \in \mathcal{H}$ using $\mathcal{S}$ and $w_1^{(t)}, \ldots, w_m^{(t)}$ which minimizes the training error:

$$\epsilon_t = \sum_{j=1}^{m} w_j^{(t)} \,\text{Ind}\,(y_j \neq h_t(\mathbf{x}_j))$$

Note: $\text{Ind}\,(x) = 1$ if $x =$ TRUE otherwise $\text{Ind}\,(x) = 0$

2. Compute the reliability coefficient:

$$\alpha_t = \log\left(\frac{1 - \epsilon_t}{\epsilon_t}\right)$$

ϵ_t must be less than 0.5. Break out of loop if $\epsilon_t \approx .5$

3. Update weights using:

$$w_j^{(t+1)} = w_j^{(t)} exp(\alpha_t \,\text{Ind}\,(y_j \neq h_t(\mathbf{x}_j)))$$

4. Normalize the weights so that they sum to 1.

Chosen weak classifier — Re-weight training points — Current strong classifier

Properties of the Boosting algorithm

Training Error: Training error $\rightarrow 0$ exponentially.

Good Generalization Properties: Would expect over-fitting but even when training error vanishes the test error asymptotes

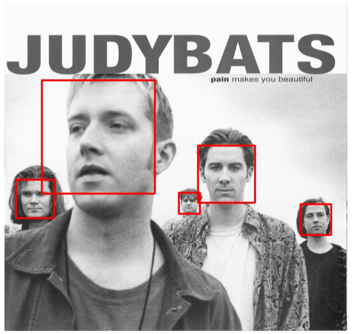
Why? Boosting tries to increase the margin of the training examples even when the training error is zero:

$$f_T(\mathbf{x}) = \text{sign}\left(\sum_{t=1}^{T} \alpha_t h_t(\mathbf{x})\right) = \text{sign}\,(\phi_T(\mathbf{x}))$$

Margin of a correctly classified example is: $y_i\,\phi_T(\mathbf{x}_i)$

The larger the margin $\Longrightarrow$ further example is from the decision boundary $\Longrightarrow$ better generalization ability.

Example: Viola & Jones Face Detection



- Most state-of-the-art face detection on mobile phones, digital cameras etc. are based on this algorithm.
- Example of a classifier constructed using the Boosting algorithm.

Viola & Jones: Training data

Positive training examples: Image patches corresponding to faces - $(\mathbf{x}_i, 1)$.

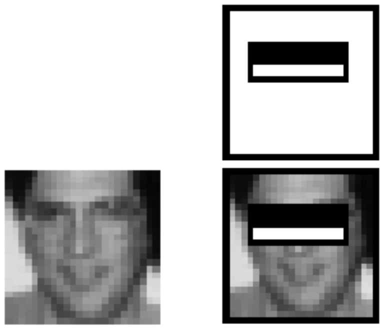
Negative training examples: Random image patches from images not containing faces - $(\mathbf{x}_j, -1)$.

Note: All patches are re-scaled to have same size.

⇑

Positive training examples

Viola & Jones: Weak classifier



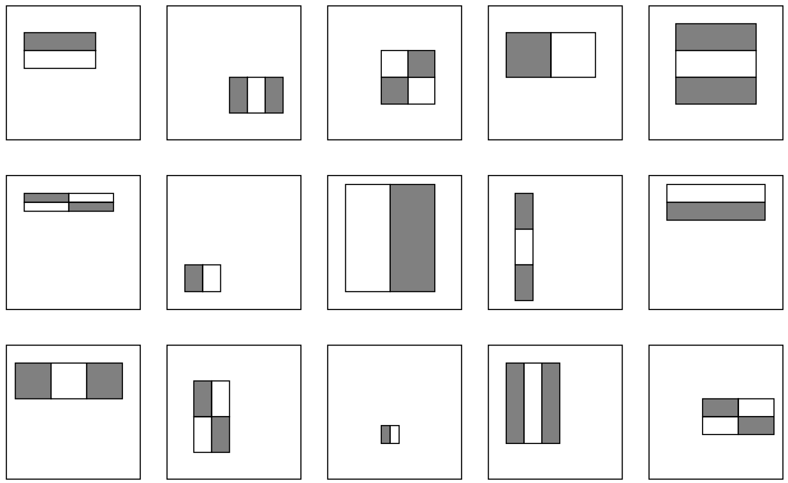
FACE or **NON-FACE**

Input: x **Apply filter:** $f^j(\mathbf{x})$ **Output:** $h(\mathbf{x}) = (f^j(\mathbf{x}) > \theta)$

Filters used compute differences between sums of pixels in adjacent rectangles. (These can be computed very quickly using **Integral Images**.)

Viola & Jones: Filters Considered

Huge **library** of possible Haar-like filters, $f^1, \ldots, f^n$ with $n \approx 16,000,000$.



Viola & Jones: AdaBoost training

Recap: define weak classifier as

$$h_t(\mathbf{x}) = \begin{cases} 1 & \text{if } f^{j_t}(\mathbf{x}) > \theta_t \\ -1 & \text{otherwise} \end{cases}$$

Use AdaBoost to efficiently choose the **best weak classifiers** and to **combine** them.

Remember: a weak classifier corresponds to a filter type and a threshold.

Viola & Jones: AdaBoost training (cont.)

For $t = 1, \ldots, T$

- for each filter type j
  1. Apply filter, f^j, to each example.
  2. Sort examples by their filter responses.
  3. Select best threshold for this classifier: θ_{tj}.
  4. Keep record of error of this classifier: ϵ_{tj}.
- Select the filter-threshold combination (weak classifier j^*) with minimum error. Then set $j_t = j^*$, $\epsilon_t = \epsilon_{tj^*}$ and $\theta_t = \theta_{tj^*}$.
- Re-weight examples according to the AdaBoost formualae.

Note: (There are many tricks to make this implementation more efficient.)

Viola & Jones: Sliding window

Remember: Better classification rates if use a classifier, f_T, with large T.

Given a new image, I, detect the faces in the image by:

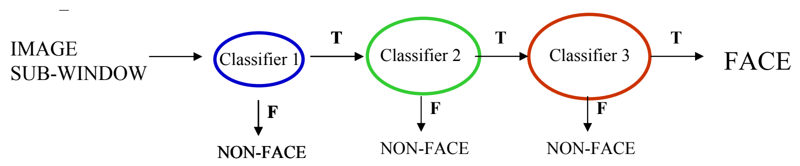
- for each plausible face size s
  - for each possible patch centre c
    1. Extract sub-patch of size s at c from I.
    2. Re-scale patch to size of training patches.
    3. Apply detector to patch.
    4. Keep record of s and c if the detector returns positive.

This is a **lot** of patches to be examined. If T is very large processing an image will be very slow!

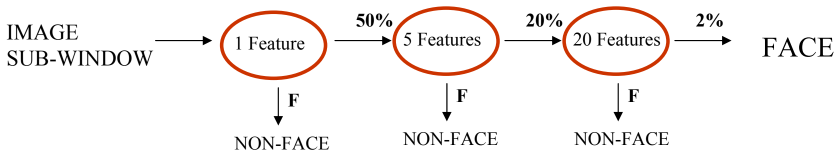
Viola & Jones: Cascade of classifiers

But:
only a tiny proportion of the patches will be faces **and** many of them will not look anything like a face.

Exploit this fact: Introduce a cascade of increasingly strong classifiers



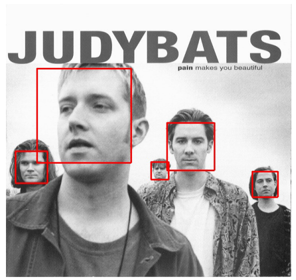
Viola & Jones: Cascade of classifiers



- A 1 feature classifier achieves 100% detection rate and about 50% false positive rate.
- A 5 feature classifier achieves 100% detection rate and 40% false positive rate (20% cumulative) - using data from previous stage.
- A 20 feature classifier achieves 100% detection rate with 10% false positive rate (2% cumulative).

Summary

Viola & Jones: Typical Results



P. Viola, M. J. Jones, **Robust real-time face detection**. *International Journal of Computer Vision* 57(2): 137-154, 2004.

Summary: Ensemble Prediction

Can combine many weak classifiers/regressors into a stronger classifier; voting, averaging, bagging

- if weak classifiers/regressors are better than random.
- if there is sufficient de-correlation (independence) amongst the weak classifiers/regressors.

Can combine many (high-bias) weak classifiers/regressors into a **strong** classifier; boosting

- if weak classifiers/regressors are **chosen** and **combined** using knowledge of how well they and others performed on the task on training data.
- The selection and combination encourages the weak classifiers to be complementary, diverse and de-correlated.