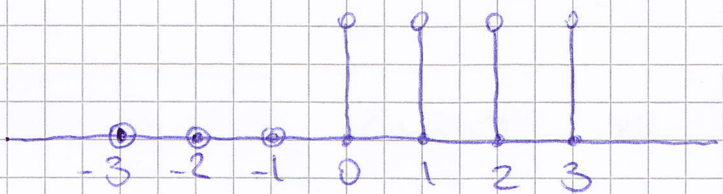
LINEAR TIME-INVARIANT SYSTEMS

* UNIT STEP FUNCTION

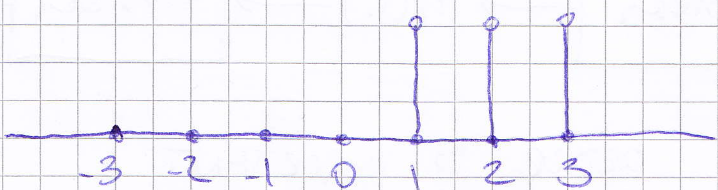
$$u(n) = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases} \quad , n\text{- integer}$$



$u(n)$ - SWITCH

* UNIT IMPULSE FUNCTION

$$\delta(n) = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases} \quad , n\text{- integer}$$



$u(n-1)$

$$\delta(n) = u(n) - u(n-1)$$

* A SYSTEM TRANSFORMS INPUT SIGNAL TO OUTPUT SIGNAL

$$x(n) \rightarrow y(n)$$

$$x(n) \longrightarrow \boxed{\text{SYSTEM}} \longrightarrow y(n)$$

$x(n)$ - INPUT SIGNAL

$y(n)$ - OUTPUT SIGNAL

* INTERCONNECTIONS OF SYSTEMS

- SERIES OF INTERCONNECTIONS

$$x_1(n) \rightarrow y_1(n) \qquad\qquad y_1(n) = x_2(n) \rightarrow y_2(n)$$

$$x_1(n) \longrightarrow \boxed{\text{SYSTEM}} \longrightarrow y_1(n) \longrightarrow \boxed{\text{SYSTEM}} \longrightarrow y_2(n)$$

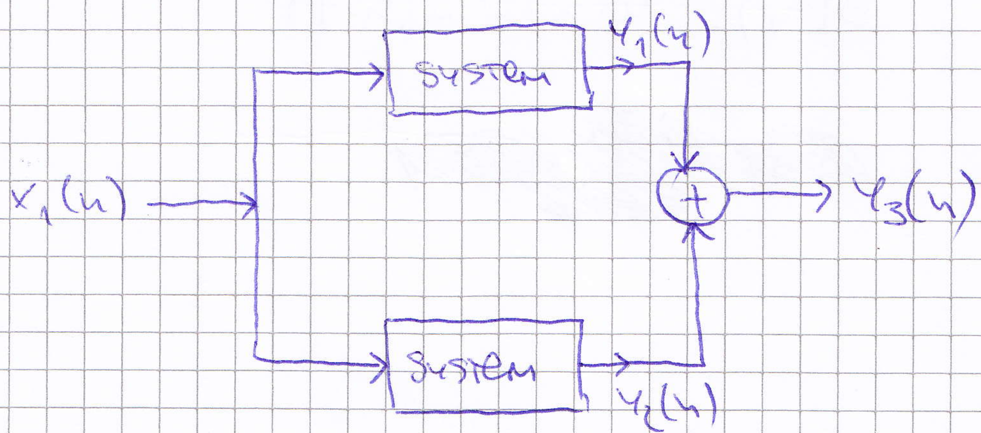
$$\neq$$

$$x_2(n) \rightarrow y_2(n) \qquad\qquad y_2(n) = x_1(n) \rightarrow y_1(n)$$

$$x_2(n) \longrightarrow \boxed{\text{SYSTEM}} \longrightarrow y_2(n) \longrightarrow \boxed{\text{SYSTEM}} \longrightarrow y_1(n)$$

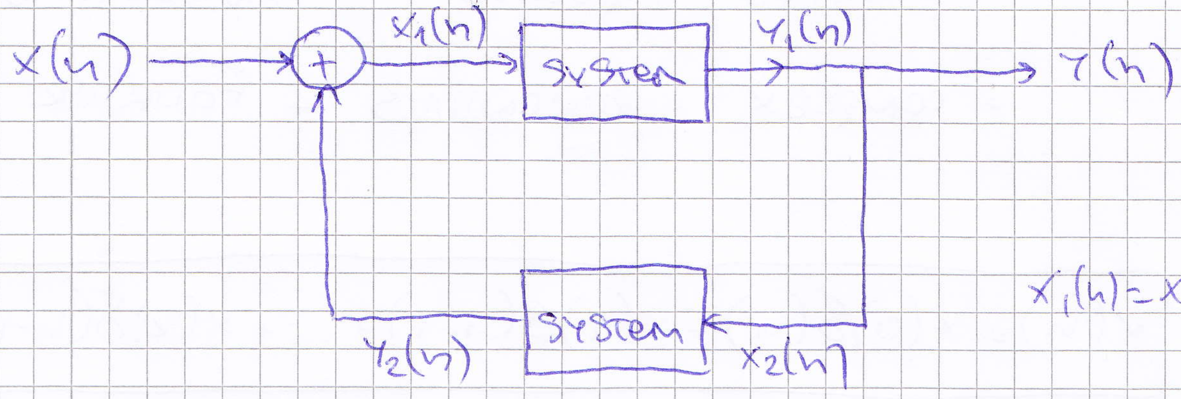
ORDER IS IMPORTANT !

- PARALLEL INTERCONNECTIONS



ORDER IS NOT IMPORTANT (+) !

- FEEDBACK INTERCONNECTIONS



$x_1(n) = x(n) + y_2(n)$

* LINEARITY

$x_1(n) \rightarrow y_1(n)$

$x_2(n) \rightarrow y_2(n)$

$$\boxed{a \cdot x_1(n) + b \cdot x_2(n) \Rightarrow a \cdot y_1(n) + b \cdot y_2(n)}$$

* TIME INVARIANCE

$x(n) \rightarrow y(n)$

$$\boxed{x(n-T) \Rightarrow y(n-T)}$$

* CONVOLUTION - DECOMPOSE THE INPUT SIGNAL INTO LINEAR COMBINATION OF BASIC SIGNALS

- DELAYED IMPULSES - CONVOLUTION
- COMPLEX EXPONENTIALS - FOURIER

$$\boxed{x(n) = x(0)\delta(n) + x(1)\delta(n-1) + \dots + x(k)\delta(n-k)}$$

DECOMPOSITION INTO DELAYED IMPULSES

$$\boxed{x(n) = \sum_{k=-\infty}^{\infty} x(k)\delta(n-k)}$$

LINEAR COMBINATION OF BASIC SIGNALS

* FOR LTI

$$\boxed{y(n) = \sum_{k=-\infty}^{\infty} x(k) . h(n-k)}$$

$$\boxed{y(n) = x(n) * h(n)}$$

IF WE KNOW THE SYSTEM RESPONSE $(h(n))$ FOR $\delta(n)$, WE CAN FIND THE OUTPUT BY CONVOLUTION

① DRAW THE BLOCK DIAGRAM OF A SYSTEM DESCRIBED BY THE FOLLOWING EQUATION.

* GENERAL EQUATION

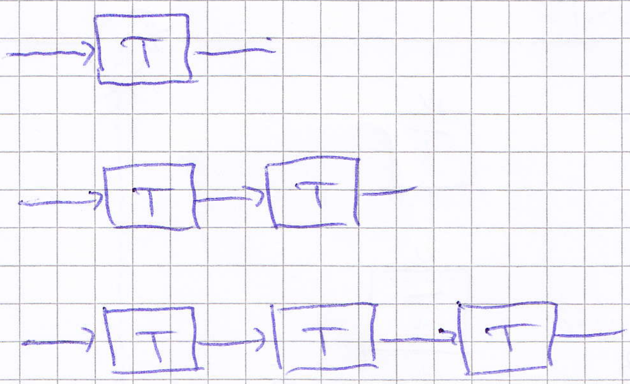
$$\underset{\text{OUT}}{\underset{\downarrow}{\underline{y(n)}}} = \underset{\text{IN}}{\underset{\downarrow}{\underline{x(n)}}} \underset{\text{ADD}}{\underset{\downarrow}{\underline{+}}} \underset{\text{MULT}}{\underset{\downarrow}{\underline{a}}} . \underset{\text{DELAY}}{\underset{\downarrow}{\underline{x(n-k)}}}$$

- DELAY

$x(n-1)$

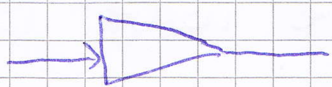
$x(n-2)$

$x(n-3)$



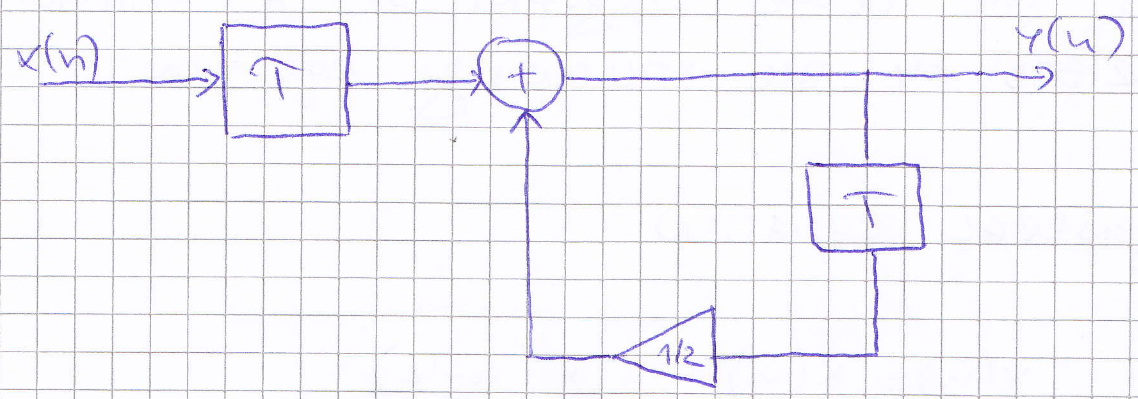
- ADDITION

- MULTIPLICATION

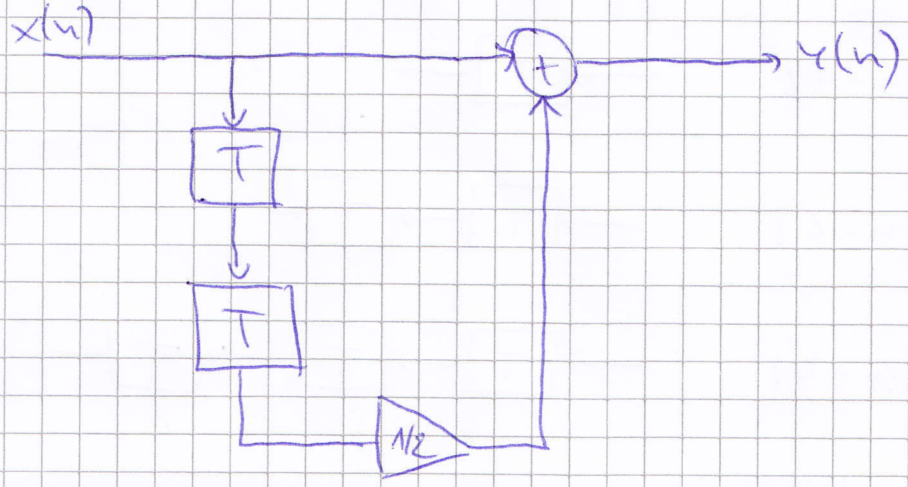


a) $y(n) - \frac{1}{2} y(n-1) = x(n-1)$

$\Rightarrow y(n) = x(n-1) + \frac{1}{2} y(n-1)$



b) $y(n) = x(n) + \frac{1}{2} x(n-2)$



② A SYSTEM HAS THE FOLLOWING IMPULSE RESPONSE $h(n) = [3, 2, 1]$. COMPUTE THE OUTPUT OF THE SYSTEM IF THE INPUT IS,

a) $x(n) = [3, 2, 1]$

$$y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k) \cdot h(n-k)$$

n	0	1	2	3	4	5	6
$x(n)$	3	2	1				
$h(n)$	3	2	1				
$x(0).h(n-0)$	9	6	3				
$x(1).h(n-1)$	0	6	4	2			
$x(2).h(n-2)$	0	0	3	2	1		
	9	12	10	4	1		

$\Rightarrow y(n) = [9, 12, 10, 4, 1]$

b) $x(n) = [1, 2, 3]$

$$y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k).h(n-k)$$

n	0	1	2	3	4	5	6
$x(n)$	1	2	3				
$h(n)$	3	2	1				
$x(0).h(n-0)$	3	2	1				
$x(1).h(n-1)$	0	6	4	2			
$x(2).h(n-2)$	0	0	9	6	3		
	3	8	14	8	3		

$\Rightarrow y(n) = [3, 8, 14, 8, 3]$

(3) A SYSTEM IS DESCRIBED BY THE FOLLOWING EQUATION

$$y(n) = \frac{1}{2}x(n) + \frac{1}{2}x(n-2)$$

THE SYSTEM IS FED WITH INPUT SIGNAL,

$$x(n) = \cos(w_0 n)$$

DETERMINE THE OUTPUT SIGNAL WHEN,

a) $w_0 = 0$

b) $w_0 = \pi/4$

c) $w_0 = \pi/2$

d) $w_0 = 3\pi/4$

e) $w_0 = \pi$

$$\Rightarrow y(n) = \frac{1}{2}\cos(w_0 n) + \frac{1}{2}\cos(w_0(n-2))$$

$$\boxed{\cos(\phi) = \frac{e^{j\phi} + e^{-j\phi}}{2}}$$

$$\Rightarrow y(n) = \frac{e^{jw_0 n} + e^{-jw_0 n}}{4} + \frac{e^{jw_0(n-2)} + e^{-jw_0(n-2)}}{4}$$

$$\Rightarrow y(n) = \frac{1}{4}\left(e^{jw_0 n} + e^{-jw_0 n} + e^{jw_0 n} \cdot e^{-2jw_0} + e^{-jw_0 n} \cdot e^{2jw_0}\right)$$

$$\Rightarrow y(n) = \frac{1}{4}\left(e^{jw_0 n}\left(1 + e^{-2jw_0}\right) + e^{-jw_0 n}\left(1 + e^{2jw_0}\right)\right)$$

$$\boxed{\begin{aligned} e^{jw_0 - jw_0} &= e^{-jw_0 + jw_0} = e^{0} = 1 \\ e^{-2jw_0} &= e^{-jw_0} \cdot e^{-jw_0} \\ e^{2jw_0} &= e^{jw_0} \cdot e^{jw_0} \end{aligned}}$$

$$\Rightarrow y(n) = \frac{1}{4}\left(e^{jw_0 n}\left(e^{-jw_0 + jw_0} + e^{-2jw_0}\right) + e^{-jw_0 n}\left(e^{jw_0 - jw_0} + e^{2jw_0}\right)\right)$$

$$\boxed{e^{-jw_0 + jw_0} = e^{-jw_0} \cdot e^{jw_0}}$$

$$\Rightarrow y(n) = \frac{1}{4}\left(e^{jw_0 n}\left(e^{-jw_0} \cdot e^{jw_0} + e^{-jw_0} \cdot e^{-jw_0}\right) + e^{-jw_0 n}\left(e^{jw_0} \cdot e^{-jw_0} + e^{jw_0} \cdot e^{jw_0}\right)\right)$$

$$\Rightarrow y(n) = \frac{1}{4}\left(e^{j\omega_0 n} \cdot e^{-j\omega_0}\left(e^{j\omega_0} + e^{-j\omega_0}\right) + e^{-j\omega_0 n} \cdot e^{j\omega_0}\left(e^{j\omega_0} + e^{-j\omega_0}\right)\right)$$

$$\Rightarrow y(n) = \frac{1}{4}\left(\underbrace{\left(e^{j\omega_0} + e^{-j\omega_0}\right)}_{= 2\cos(\omega_0)}\underbrace{\left(e^{j\omega_0(n-1)} + e^{-j\omega_0(n-1)}\right)}_{2\cos(\omega_0(n-1))}\right)$$

$$\Rightarrow y(n) = \cos(\omega_0) \cdot \cos(\omega_0(n-1))$$

a) $y(n) = \cos 0 \cdot \cos 0 = 1$

b) $y(n) = \cos\frac{\pi}{4} \cdot \cos\left(\frac{\pi}{4}(n-1)\right) = \frac{\sqrt{2}}{2} \cdot \cos\left(\frac{\pi}{4}(n-1)\right)$

c) $y(n) = \cos\frac{\pi}{2} \cdot \cos\left(\frac{\pi}{2}(n-1)\right) = 0$

d) $y(n) = \cos\frac{3\pi}{4} \cdot \cos\left(\frac{3\pi}{4}(n-1)\right) = \frac{\sqrt{2}}{2} \cdot \cos\left(\frac{3\pi}{4}(n-1)\right)$

e) $y(n) = \cos\pi \cdot \cos(\pi(n-1)) = \cos(\pi)^2 \cdot \cos(\pi n) = \cos(\pi n)$