FOURIER TRANSFORMS AND SAMPLING

① FOURIER SERIES

* DECOMPOSES PERIODIC FUNCTIONS INTO THE SUM A SET OF SIMPLE OSCILLATING FUNCTIONS.

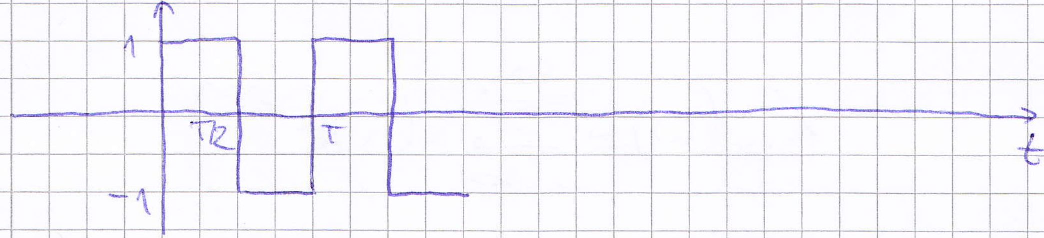
A CONTINUOUS-TIME PERIODIC SIGNAL

$$f(t) = f(t+T), \quad T = \frac{2\pi}{\omega_0}$$

CAN BE WRITTEN AS WEIGHTED SUM

$$f(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}$$

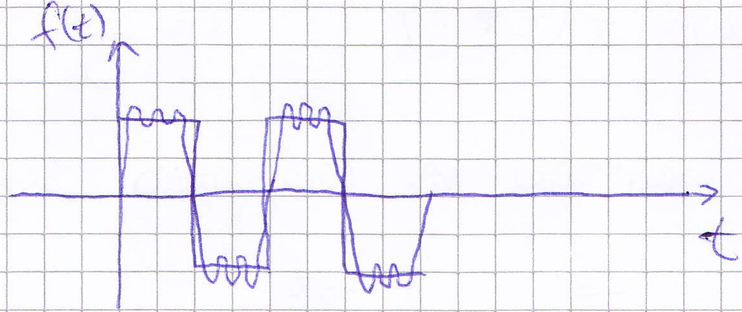
$$c_k = \frac{1}{T}\int_0^T f(t)e^{-jk\omega_0 t}\,dt$$



$$c_k = \begin{cases} -\frac{2j}{k\pi}, & k = 1,3,5,\dots \\ 0, & k = 0,2,4,\dots \end{cases}$$

k=1

k=3



k=5

② DISCRETE-TIME FOURIER TRANSFORM (DTFT)

* TRANSFORMS ONE FUNCTION INTO ANOTHER, CALLED THE FREQUENCY DOMAIN REPRESENTATION

$$X_{2\pi}(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$$

* IT IS A SPECIAL CASE OF Z-TRANSFORM,

$$z = e^{j\omega}$$

③ Sampling

* EVEN AND ODD FUNCTIONS

$f(-n) = f(n)$ - EVEN

$f(-n) = -f(n)$ - ODD

$y(x) = x$: $y(-x) = -x = -y(x)$ - ODD

$y(x) = |x|$: $y(-x) = |-x| = y(x)$ - EVEN

* SAMPLING IS THE REDUCTION OF A CONTINUOUS SIGNAL TO A DISCRETE SIGNAL

CONTINUOUS $\longrightarrow$ DISCRETE

$x(t) \longrightarrow x(nT)$, n - INT

$T = \frac{1}{f_s}$ - SAMPLING PERIOD

$f_s = \frac{1}{T}$ - SAMPLING FREQUENCY

$\nu_0 = \frac{f_0}{f_s} = f_0 T$ - NORMALISED FREQUENCY

$\omega_0 = 2\pi \nu_0$

IF $\nu_0 \leq 0{,}5$ OK!

IF $0{,}5 < \nu_0 \leq 1$ ALIASING

Signal $x(t) = \sin(2\pi f_0 t)$ with frequency $f_0 = 2000\,Hz$ is sampled with $f_s = 8000\,Hz$.

a) Calculate the normalised frequency.

$$\nu_0 = \frac{f_0}{f_s} = \frac{2000}{8000} = \frac{1}{4} < 0{,}5$$

b) Write an expresion for the sampled signal $y(n)$.

$$y(n) = x(nT) = \sin(2\pi f_0 nT) = \sin(2\pi \nu_0 n) =$$

$$= \sin\left(\frac{\pi}{2} n\right) \;//$$

④ Discrete Fourier Transform (DFT)

* converts a finite set of equally spaced samples into a list of coefficients of a finite combination of complex sinusoids.

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-jnk 2\pi/N}$$

$$x(n) = \frac{1}{N} \sum_{n=0}^{N-1} X(k) e^{jnk 2\pi/N}$$

GIVEN THE SIGNAL

$$x(n) \doteq u(n) - u(n-6)$$

DETERMINE AND DRAW $|X(k)|$, WHERE $X(k)$ IS THE FIRST N-POINTS OF THE DFT OF $x(n)$.

a) $N = 6$

b) $N = 12$

$$X(k) = \sum_{n=0}^{N-1} (u(n) - u(n-6)) e^{-jnk2\pi/N}$$

$$u(n) = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

$$\Rightarrow X(k) = \sum_{n=0}^{5} e^{-jkn\pi/3}$$

$$\boxed{\sum_{k=0}^{n} a^k = \frac{a^{n+1} - 1}{a - 1}}$$ - geometric series

$$\Rightarrow X(k) = \frac{e^{-jk2\pi} - 1}{e^{-jk\pi/3} - 1} \;//$$