Multi-channel Noise Reduction

- When more than one microphone is available for sound pickup, the signal enhancement task may be facilitated by exploiting the multivariate deterministic and stochastic properties of the signal.

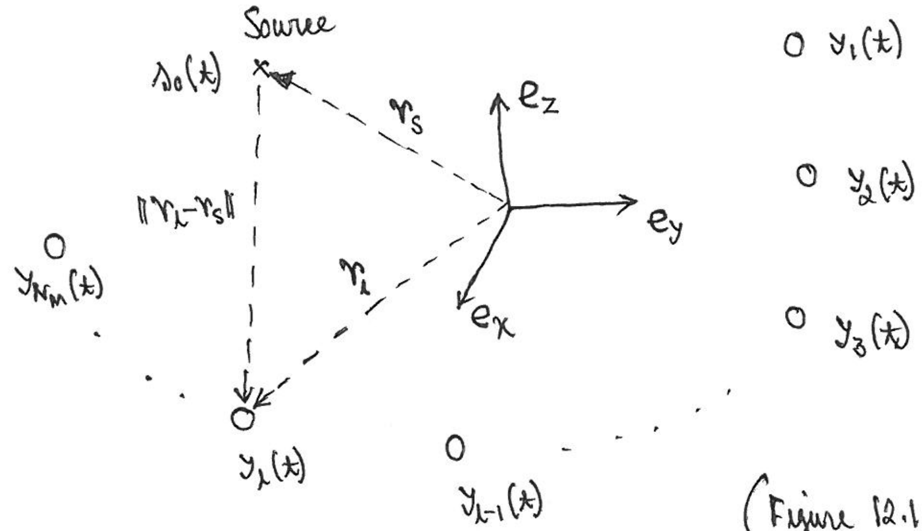
- From a deterministic viewpoint, the signals at the various microphones differ in that they arrive via different acoustic paths at the microphones and thus, also differ in their short term phase and amplitude.

- From a stochastic perspective, multi-channel methods allow the evaluation of the second-order and higher-order statistics of the spatial sound field.

- Sources which are close to array of microphones will generate mostly ~~of~~ coherent microphone signals, while distant and distributed sources lead to uncorrelated signals.

- Thus, short-term amplitude, short-term phase, and the statistics of the signals may be used to differentiate between sources and to perform source separation and enhancement.

Spatial sampling of Sound Fields



(Figure 12.1 of Reference Book)

- Reference coordinate $\{e_x, e_y, e_z\}$
- N_M microphones and single source
- w.r.t reference coordinate system, source position and "l"-th microphone are denoted by vectors r_s and r_l, $l = 1, 2, \cdots N_M$.
- In an ideal environment (no reverberation/noise), the microphone signals $y_l(t)$, $l = 1, 2, \cdots, N_M$, are delayed and attenuated version of the source signal $s_0(t)$,

$$y_l(t) = \frac{1}{\|r_l - r_s\|} s_0(t - \tau_l).$$

Here $\tau_l = \frac{\|r_l - r_s\|}{c}$ [c is sound speed]

- "reference point" — the origin of the reference coordinate
- Signal at reference point

$$s(t) = \frac{1}{\|r_s\|} s_0(t - \tau_0)$$

- The reference point ~~sound~~ could be the geometric ~~ct~~ center of the array.

- We are mainly interested in the delay of the signals relative to the signal which is received at the reference point.

- The relative signal delay $\Delta\tau_l$ is the time delay difference between the received signal at the reference point and at the "l"-th microphone.

$$\Delta\tau_l = \tau_0 - \tau_l = \frac{1}{c}\left(\|r_s\| - \|r_l - r_s\|\right)$$

- What is $y_l(t)$?

$$y_l(t) = \frac{1}{\|r_l - r_s\|}\, s_0(t - \tau_l)$$

$$= \frac{\|r_s\|}{\|r_l - r_s\|} \cdot \frac{1}{\|r_s\|} \cdot s_0\big(t - \tau_0 + \underbrace{\tau_0 - \tau_l}_{\Delta\tau_l}\big)$$

$$= \frac{\|r_s\|}{\|r_l - r_s\|}\, s(t + \Delta\tau_l) \quad \left[\because s(t) = \frac{1}{\|r_s\|} s_0(t - \tau_0)\right]$$

$$\boxed{\text{So, we represent } y_l(t) \text{ as a function of the signal at the reference point.}}$$

The Farfield Model

- The sound source is far away that the microphones.

~~That means $\|r_l\|$~~

- Also assume that the reference point is close to microphones. That is $\|r_l\| \ll \|r_s\|$.

$$\therefore \frac{\|r_s\|}{\|r_l - r_s\|} \approx 1$$

- But we do not assume that $\Delta\tau_l = \frac{1}{c}\left(\|r_s\| - \|r_l - r_s\|\right) = 0$. If we assume that then all $y_l(t)$ are same and of no use to bring diversity.

- So,
$$y_l(t) = \frac{\|r_s\|}{\|r_l - r_s\|} \, s(t + \Delta\tau_l)$$
$$\approx s(t + \Delta\tau_l) \quad \left[\text{as } \frac{\|r_s\|}{\|r_l - r_s\|} \approx 1\right]$$

Beamforming

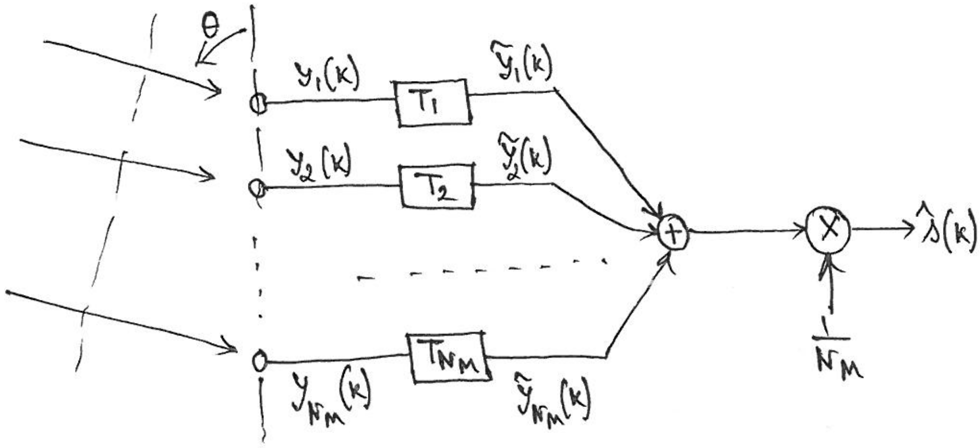
: (a) To combine the microphone signals such that a desired, spatial selectivity is achieved.
(b) In single source case, we need to form a beam of high gain in the source direction.

Delay-and-Sum Beamforming

- Assume insignificant reverberation and low noise.

- Then use "delay" for phase alignment.

- The resulting phase-aligned signals are added to form a single output signal.



- For continuous t, Farfield Model

$$y_l(t) = s_l(t) + n_l(t) = s(t + \Delta\tau_l) + n_l(t),$$

where $s(t)$ is the desired source signal at the reference point and $n_l(t)$ is the noise signal.

- Assuming farfield model, the delayed signal

$$\tilde{y}_l(t) = \tilde{s}_l(t) + \tilde{n}_l(t) = s(t + \Delta\tau_l - T_l) + n_l(t - T_l).$$

- If everything is equalized, then $T_l = T_B + \Delta\tau_l$.

- Now, considering sampling, the output of the delay-and-sum beamformer is

$$\hat{s}(kT) = \frac{1}{N_M}\sum_{l=1}^{N_M} \tilde{y}_l(kT) = \frac{1}{N_M}\sum_{l=1}^{N_M} \tilde{s}_l(kT) + \frac{1}{N_M}\sum_{l=1}^{N_M} \tilde{n}_l(kT)$$

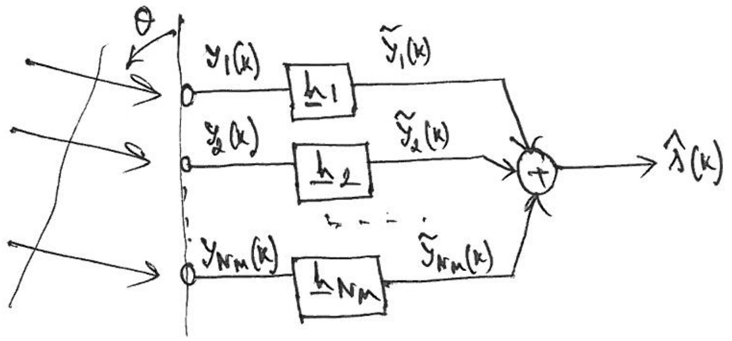
- For perfect equalized case, $T_l = T_B + \Delta\tau_l$

$$\hat{s}(k) = s(k - T_B f_s) + \frac{1}{N_M}\sum_{l=1}^{N_M} \tilde{n}_l(k) .$$

Here f_s = sampling freq. $= \frac{1}{T}$.

- How to equalize ?

<u>Filter-and-Sum beamforming</u>



$$\tilde{y}_l(k) = \sum_{m=0}^{M} h_l(m)\, y_l(k-m) .$$ [M length FIR filter]

$$\underline{y}_l(k) = \left[y_l(k), y_l(k-1), \dots, y_l(k-M)\right]^T$$

$$\underline{h}_l = \left[h_l(0), h_l(1), \dots\dots h_l(M)\right]^T$$

$$\therefore \quad \hat{s}(k) = \sum_{l=1}^{N_M} \underline{h}_l^T y_l(k)$$

- How to find filter coefficients ? Design filters.

- In the farfield model

$$y_l(t) \approx s(t + \Delta\tau_l) .$$

In the frequency domain

$$Y_l(j\omega) = S(j\omega)\, \exp(j\, 2\pi f\, \Delta\tau_l), \quad [\omega = 2\pi f]$$

where we assume that the Fourier Transform of signal $S(j\omega)$ exists.
When the microphone signals are sampled with sampling rate f_S, we may write the signal spectra of the sampled microphone signals $y_l(k)$ as a function of the normalized frequency Ω as

$$Y_l(e^{j\Omega}) = S(e^{j\Omega})\, \exp(j\Omega f_S\, \Delta\tau_l),$$

or, in vector notation,

$$\underline{Y}(e^{j\Omega}) = S(e^{j\Omega})\, \underline{a},$$

with a vector of signal spectra

$$\underline{Y}(e^{j\Omega}) = \left(Y_1(e^{j\Omega}), \ldots, Y_{N_M}(e^{j\Omega})\right)^T$$

and the propagation vector

$$\underline{a} = \left(\exp(j\Omega f_S \Delta\tau_1), \exp(j\Omega f_S \Delta\tau_2), \ldots, \exp(j\Omega f_S \Delta\tau_{N_M})\right)^T.$$

- In the farfield scenario, $\underline{a}^H\, \underline{Y}(e^{j\Omega})$ will yield the sum of perfectly phase aligned microphone signals.

- The output signal of the filter-and-sum beamformer

$$\hat{s}(k) = \sum_{l=1}^{N_M} \underline{h}_l^T \underline{y}_l(k) .$$

For the signal $s(k)$ of a single (desired) source and the farfield scenario, we obtain in the frequency domain

$$\hat{S}(e^{j\Omega}) = \sum_{l=1}^{N_M} H_l(e^{j\Omega}) \, Y_l(e^{j\Omega})$$

$$= S(e^{j\Omega}) \sum_{l=1}^{N_M} H_l(e^{j\Omega}) \exp(j\Omega f_s \Delta\tau_l)$$

$H_l(e^{j\Omega})$ denotes the frequency response of the 'l'-th FIR filter. Thus, for a fixed source position and fixed microphone positions r_l, the array response

$$\sum_{l=1}^{N_M} H_l(e^{j\Omega}) \exp(j\Omega f_s \Delta\tau_l) = \underline{H}^T(e^{j\Omega}) \, \underline{a} ,$$

where $\underline{H}(e^{j\Omega}) = (H_1(e^{j\Omega}), H_2(e^{j\Omega}), \ldots, H_{N_M}(e^{j\Omega}))^T$.

- When a delay-and-sum beamformer is steered towards the source in the farfield, we obtain a special case of the filter-and-sum beamformer with the filter coefficient vector

$$\underline{H}(e^{j\Omega}) = \frac{\exp(-j\Omega f_s T_B)}{N_M} \, a^* = \frac{1}{N_M} \underline{e}$$

where $\underline{e} = (\exp(-j\Omega f_s T_1), \exp(-j\Omega f_s T_2), \ldots, \exp(-j\Omega f_s T_{N_M}))^T$

is the steering vector.