

# Exam in DD2425 Robotics and Autonomous Systems

Lecturer: Patric Jensfelt

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You are allowed to use a calculator. Please read the entire exam before preparing the answers to make sure that you have a good view of the questions to be answered. You need at least 30 points to pass. The mapping from exam score to 0-10 which is averaged with your score from the project, is such that 30 points on the exam gives you 0 and max points (80) gives you score 10.

**Do not forget to turn the page!**

**Write only on one side of the pages and make sure to put your name and page number on each page and note how many pages are handed in on the front page!**

**GOOD LUCK!**

## Short questions, short answers

1. You want your robot to move from one point to another in a cluttered room without sensor feedback but you have access to the robot's exact location all the time. Would you prefer to navigate based on an occupancy grid or a topological map? (2p)

*An occupancy grid as it models the obstacles that we need to avoid.*

2. Robot A has wheel base (distance between the wheels) 0.1m and robot B has wheel base 0.2m. Assuming the two robots have the same wheel radii which robot will rotate faster if we let the wheels spin with the same speed but different direction? (3p)

*The one with the smallest wheel base, ie A, since  $\omega = r \frac{\omega_R - \omega_L}{\text{wheelbase}}$ .*

3. What happens to the field of view of a camera if you make the image sensor smaller but keep everything else the same? (2p)

*It is decreased (you capture a smaller part of the light cone that comes through the lens).*

4. Assume an object 1m away is visible in both left and right camera in a stereo pair. Does the accuracy in the estimated distance to the object typically improve or degrade with increased base line (ie when the distance between the cameras increases)? (3p)

*Improved accuracy when the baseline increases (the lines that go from a point on the object to the two cameras will have a larger angle between them which makes calculating the position of that point more accurate).*

5. Mention one advantage with having an omnidirectional drive system for a robot. (2p)

*The robot can move in any direction without having to rotate.*

6. Assume a setup with a DC-motor that powers a wheel via a gearbox (with neglectable backlash) that makes the wheel rotate 1 revolution for every 50 revolutions of the motor. We want to equip this system with an encoder and we want to be able to detect as small movements of the wheel. Do we put it on the wheel axis or the motor axis? (2p)

*Better to put it on the motor axis as we will get an increase in resolution by 50.*

## Longer questions, longer answers

7. In object recognition, techniques based on keypoints / interest points are common. Describe roughly how this is done. (5p)

*We acquire training images, i.e. example images of the objects we want to be able to recognise. We extract keypoints in these images using for example SIFT or SURF. When we have a new image we extract keypoints again and then look for a training image which has a similar setup of keypoints (this is enough). To make this fast we need to make use of data structures that makes finding matches quick. It is also worth noting that one has to distinguish between the mechanism we use to detect a keypoint and the way we represent/describe it. For example, you can combine the SIFT descriptor with another detector.*

8. You have a simple phase shift based laser with a modulation wavelength of 10m and a reflecting target at 16m.

- a. What distance will the sensor report? (4p)

*The distance is calculated by looking at the difference in phase between a reference signal and the signal that comes back. The signal is periodic and the difference in phase,  $\theta$ , can be between 0 and  $2\pi$  which corresponds to a distance traveled of 0 to one wavelength. Distances over this will still result in phase differences in the interval  $[0, 2\pi)$  and thus will be interpreted as a distance traveled in  $[0, \lambda)$ . Note that the signal has to travel back and forth which means that the distance from the sensor to the target can only be half a wavelength for an unambiguous distance measurement. A target distance of 16m requires a distance traveled of 32m which will then be measured as the same as a distance of 2m and thus 1m (half the distance traveled) will be reported.*

- b. What wavelength do we need to make sure that the sensor will report distances correctly up to 20m? (4p)

*The wavelength needs to be bigger than twice the distance, so bigger than 40m in this case.*

9. You have implemented a SLAM system using landmarks in the form of points. The robot is in an area where there is only one landmarks which according to the map is expected to be 1.5m away. The robot makes a measurement that says that the landmark is 1m away. Assuming that we have a sensor with really small errors, what will happen (conceptually) with the robot's and the landmark's positions in the map in the following situations?

- a. The uncertainty in the robot position is huge and the uncertainty in the landmark is small. (4p)

*The position of the robot will be adjusted by almost 0.5m so that it is close to a position 1m away from the landmark which will move only very little towards the robot's position (to reduce the distance between them to explain the measurement).*

- b. The uncertainty in the robot position is small and the uncertainty in the landmark is huge. (4p)

*The position of the robot will move very little towards the landmark and the landmark will move almost 0.5m towards the robot, to a position very close to a position 1m away from the robot.*

10. Assume that a differential drive robot with wheel base  $B = 0.2m$  and wheel radii  $r = 0.05m$  is at pose  $(x_0, y_0, \theta_0) = (0, 0, 0)$  at time  $t = 10s$  and that it travels with **constant** wheel speeds (but not necessarily the same for both wheels). What are the left and the right wheel speeds if the robot passes through the position  $(x_1, y_1) = (0.3, 0.15)$  at  $t = 12s$ . (10p)

*Since the wheel speed is constant the robot will move on a circular arc. The second point is to the left so the center of rotation (CoR) must be to the left. The CoR must be along the wheel axis in both positions. This means that it will be at a position  $(x_r, y_r) = (0, R)$  where  $R$  is the radius of the arc. We can solve for  $R$  by looking at the equation for the radius at the second point*

$$R^2 = (x_1 - x_r)^2 + (y_1 - y_r)^2 = 0.3^2 + (R - 0.15)^2 \rightarrow R = 0.3 + 0.15^2/0.3 = 0.375.$$

*Now we can calculate the orientation of the robot at the second point by looking at the angle from the CoR to the second point,  $\theta_1 = \text{atan}(0.3/(R - 0.15)) = 53^\circ$ . The distance traveled is equal to change in orientation times the radius and therefore the speed  $v = R \frac{\theta_1 - \theta_0}{\Delta t}$  where  $\Delta t$  is the time it takes to move, ie  $\Delta t = 12s - 10s = 2s$ . We also have that  $v = r \frac{\omega_R + \omega_L}{2}$ . Combining the expressions for  $v$  gives us that  $\omega_R + \omega_L = 2R \frac{\theta_1 - \theta_0}{r \Delta t}$ . We can also use the expression for angular velocity as a function of wheel speeds,  $\omega = \frac{\theta_1 - \theta_0}{\Delta t} = r \frac{\omega_R - \omega_L}{B}$  which gives us  $\omega_R - \omega_L = B \frac{\theta_1 - \theta_0}{r \Delta t}$ . Adding the expressions for the sum and difference of the wheel speeds and solving for  $\omega_R$  gives*

$$\omega_R = \frac{1}{2}(2R + B) \frac{\theta_1 - \theta_0}{r \Delta t} = 4.40 \text{rad/s}.$$

*Using the expression for the difference of wheel speeds we can solve for  $\omega_L$  given  $\omega_R$  which gives us*

$$\omega_L = \frac{1}{2}(2R - B) \frac{\theta_1 - \theta_0}{r \Delta t} = 2.55 \text{rad/s}.$$

11. Given a sonar sensor with a very large beam width of  $40^\circ$ , ie the sound spreads  $20^\circ$  around the acoustic axis (we assume an idealized model of a sonar sensor such that the side lobes are ignored in the radiation pattern). Assume that the robot body can be approximated with a block with length  $L = 0.3$ , width  $W = 0.2m$ , height  $H = 0.1m$  and that you tilt the sonar up  $10^\circ$ . Assume that all obstacles extend from the floor to the ceiling. You want to use the sonar to detect objects close to the robot in the front.

- a. How far back should the sonar be placed for the beam to be as wide as the robot at the front the robot? (5p)

*Solve for  $D$  in  $\tan(20^\circ) = (W/2)/D$  where  $D$  is the distance from the front. This gives  $D = (W/2)/\tan(20^\circ) = 0.27m$*

- b. At what height above the robot should the sonar be placed to avoid detecting the robot body? (5p)

*The sensor tilts up  $10^\circ$  ie the beam goes  $10^\circ$  down. Not to hit the robot chassi it has to be at a height  $h$  which we get from the expression  $\tan(10^\circ) = h/D$  which gives  $h = D \tan(10^\circ) = 0.05m$*

- c. At what distance from the front of the robot does the sonar detect something at floor height? (5p)

*The sonar points down  $10^\circ$  and is at height  $h + H$  above the ground. It hits the ground  $\frac{h+H}{\tan(10^\circ)} - D = 0.57m$  in front of the robot.*

12. Describe two advantages of using HSV rather than RGB in some cases. (5p)

*The hue value channel in HSV more or less corresponds to the "color" whereas you need all three channels in RGB to model the color. HSV is also better at handling changing illumination conditions as this does not, ideally, affect the hue channel.*

13. You are building an occupancy grid with an IR-sensor. The size of the square grid cells is 2cm x 2cm. The IR sensor is placed in the center of rotation on a robot that is rotating around its own axis. Assuming that we can sample the IR-sensor at a rate of 5Hz, what is the maximum rotation speed if we want to make sure that the IR-sensor passes through some part of every cell in the grid up to a distance of 40cm. (5p)

*At a distance of 40cm a cell with size 2cm corresponds to an angle of  $2\arctan(1/40)$ .  
The max rotation speed is therefore  $2\arctan(1/40)\text{rad}/(1/5\text{Hz})=0.25\text{rad/s}$ .*

14. You have no idea where the robot is, except that it is in a rectangular room with landmarks in the corners. The landmarks are indistinguishable, ie you can not tell them apart. You have access to some measurements of these landmarks. What can you say about the position of a robot. The sensor is in the center in the two following cases?

- a. The room is 10m by 5m. You measure the range to two of the landmarks as  $\rho_1 = 10\text{m}$  and  $\rho_2 = 10\text{m}$  and the robot has radius 0.2m. (4p)

*Draw circles with radius 10m centered at each landmark. The two intersections inside the room corresponds to the two possible locations. Because of symmetry the position will be 2.5m from each of the long walls. The distance from the short wall will be  $\sqrt{10^2 - 2.5^2} = 9.68\text{m}$  which corresponds to 0.32m from the other wall so the robot with radius 0.2m fits :-)*

- b. The room is 10m by 5m. You measure the bearing to two of the landmarks as  $\alpha_1 = 42^\circ$  and  $\alpha_2 = 227^\circ$  and the robot has radius 0.5m. (6p)

*Given the size of the robot and the fact that the landmarks are in the corners of the room the only possibility is that the sensor is detecting two landmarks diagonally from each other in the room. The difference in bearing is  $175^\circ$  and the largest difference in bearing we can achieve when looking at two landmarks at each end of a wall is when we are at the middle of one of the long walls which then gives  $\pi - 2\arctan(0.5/5)$  which corresponds to  $168.6^\circ$  i.e. too small a difference. The robot must therefore be somewhere along the two curves that connect the landmarks across each other in such a way that every point on the curves has a difference in bearing to the landmarks of  $175^\circ$ .*