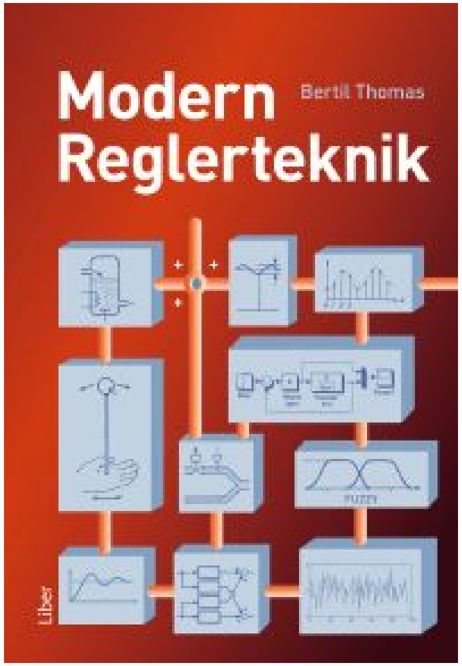
Reglerteknik **4**

Kapitel 8



Köp bok och övningshäfte på kårbokhandeln

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Lektion 4 kap 8

- Blockschemareduktion

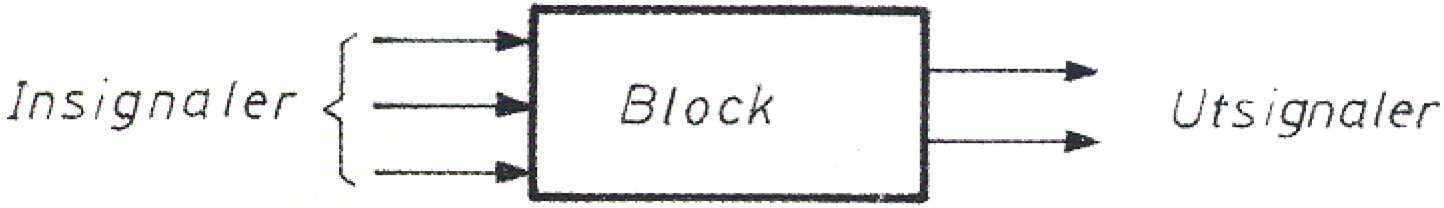
- Förenkla komplicerade blockschemor – jämför med ellärans **ersättningsresistans**.

- Reducera blockschemat till enklaste form – jämför med ellärans **tvåpolssats**.

Blockschema

Blockschemat är ett schematiskt och tydligt sätt att beskriva ett reglersystems funktion. Man använder tre symboler:

- Signaler
- Block
- Signaler



- Summerings/Differens punkter

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Overföringsfunktioner ”baklänges”

Ofta är man också intresserad av överföringsfunktioner mellan andra punkter i blockschemat än den egentliga instorheten och utstorheten.

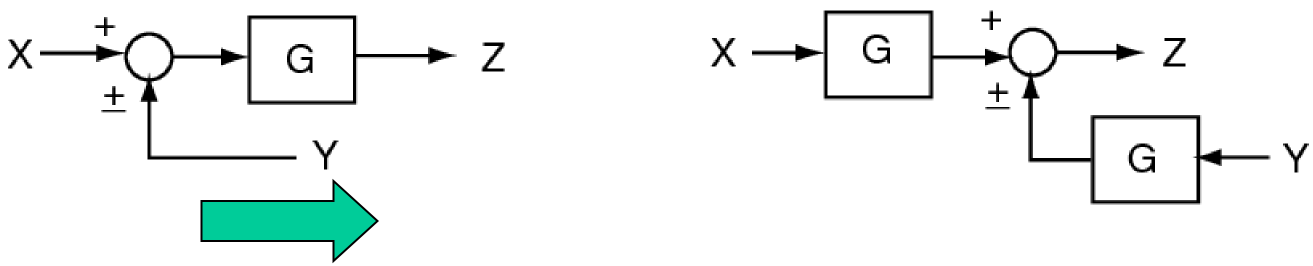
Det kan gälla hur en **störning** fortplantar sig till utgången, eller hur stor en **inre styrsignal** till ett ställdon blir.

Man ritar då om blockschemat så att man får nya blockschemor för dessa andra överföringsfunktioner.

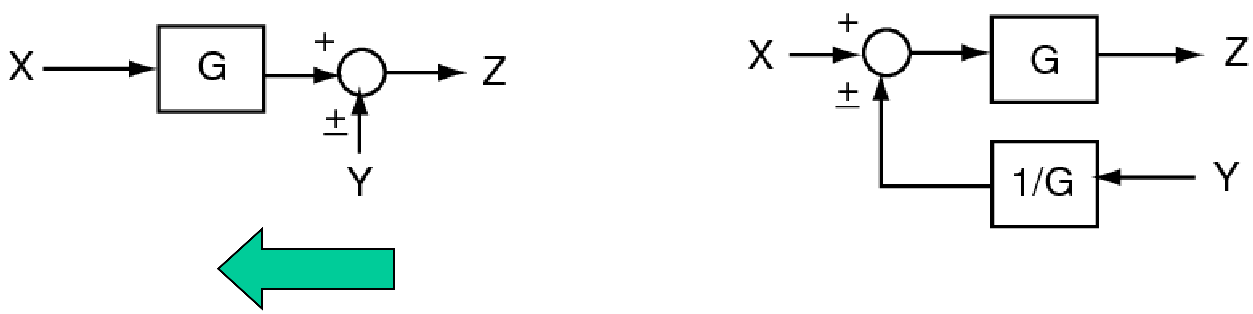
Blockschemareduktionsregler

Cascaded blocks

Moving a summer behind a block



Moving a summer ahead of a block



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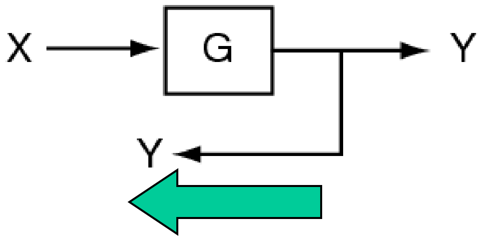
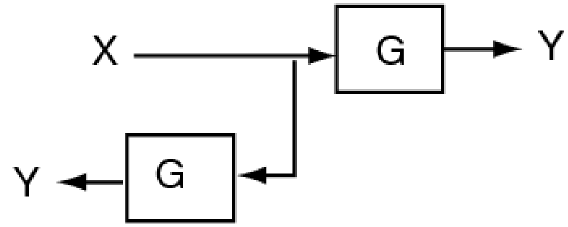
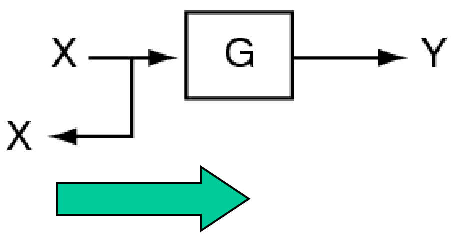
Blockschemareduktionsregler

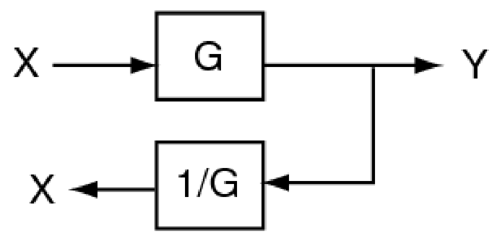
Moving a pickoff ahead of a block



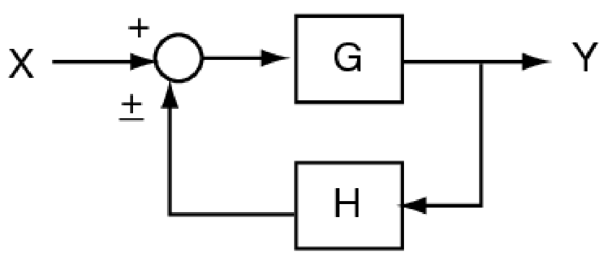


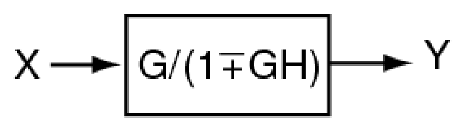
Moving a pickoff behind a block





Eliminating a feedback loop





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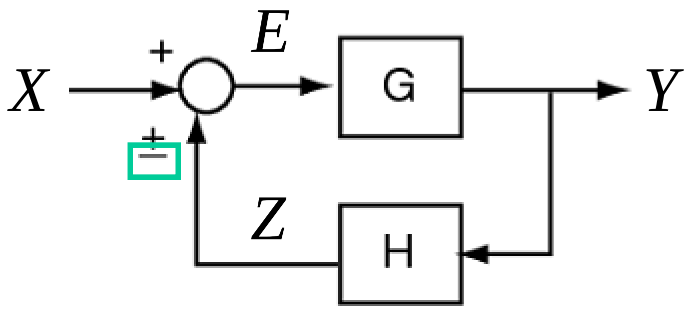
Blockschemareduktionsregler - feedback

Feedback
- negative
+ positive

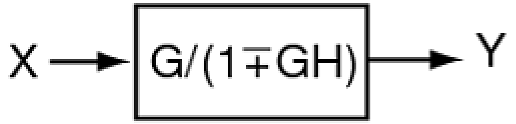


$$Y = E \cdot G \qquad E = X \pm Z \qquad Z = Y \cdot H$$

$$Y = EG = (X \pm Z) \cdot G = (X \pm Y \cdot H) \cdot G$$

$$Y = X \cdot G \pm Y \cdot H \cdot G \quad \Leftrightarrow \quad Y \mp Y \cdot H \cdot G = X \cdot G$$

$$G_{closedloop} = \frac{Y}{X} = \frac{G}{1 \mp GH}$$



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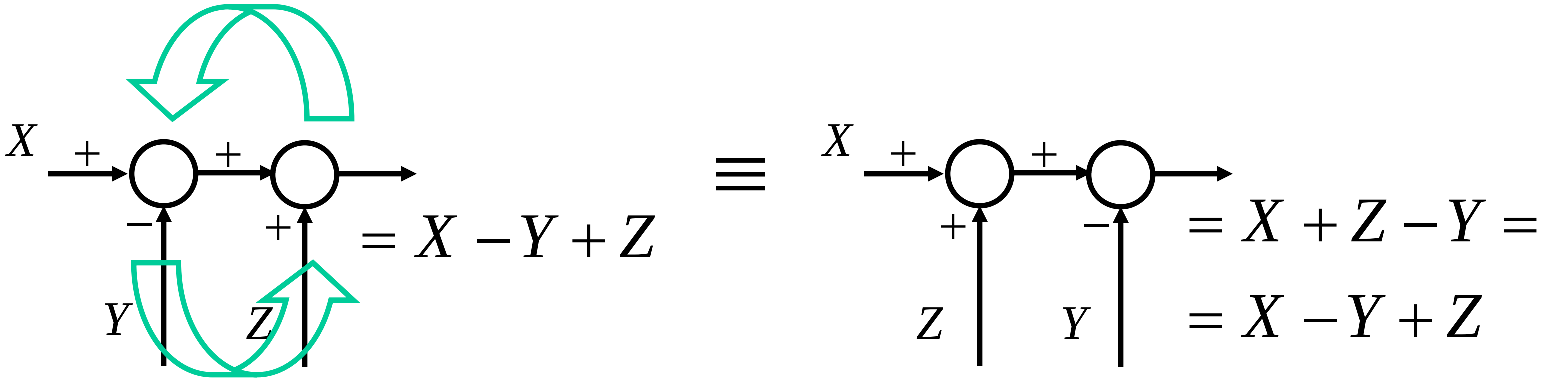
x - y + z = x + z - y



- Man får flytta summations och differenspunkter längs signalvägen.

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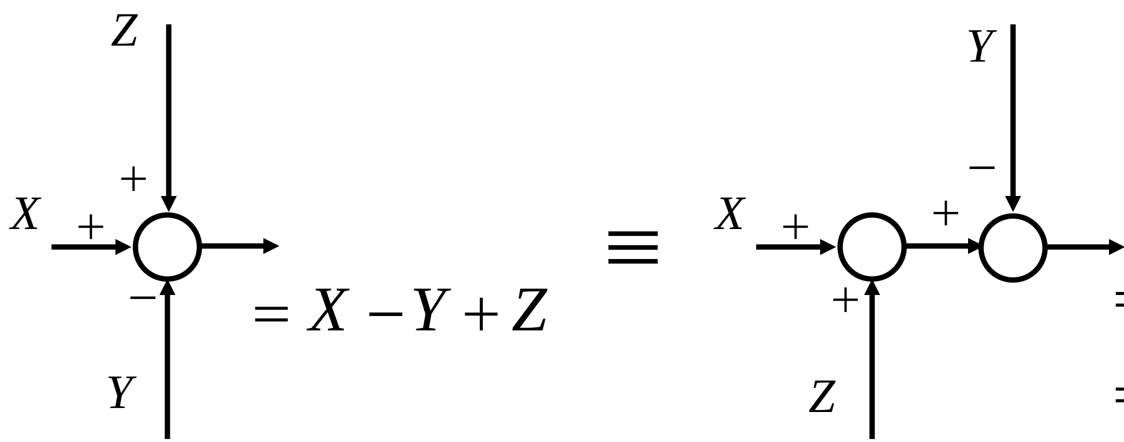
x - y + z = (x + z) - y



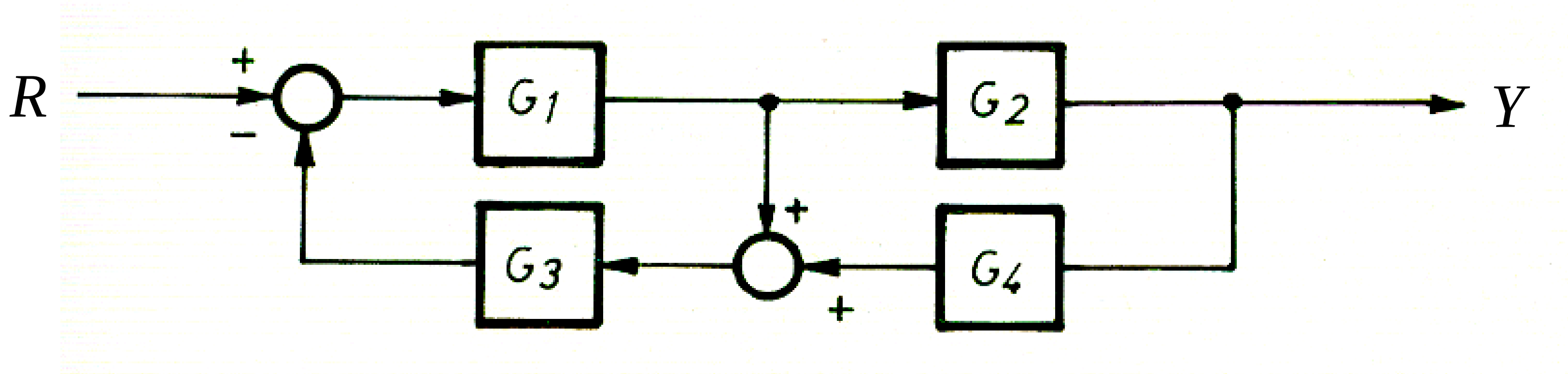
$$= (X + Z) - Y =$$
$$= X - Y + Z$$

- Man kan dela upp, eller slå ihop, summationspunkter.

Ekvationsmetoden

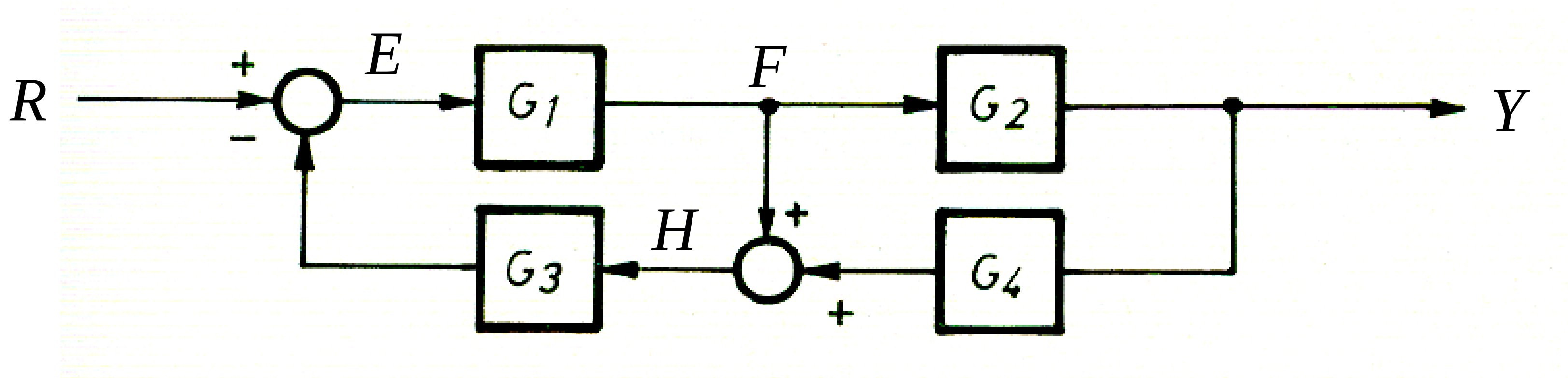
Det matematiska alternativet för blockschemareduktion är att införa mellanvariabler, och ställa upp och lösa ett ekvationssystem för blockschemat.

Ex. Ekvationsmetoden



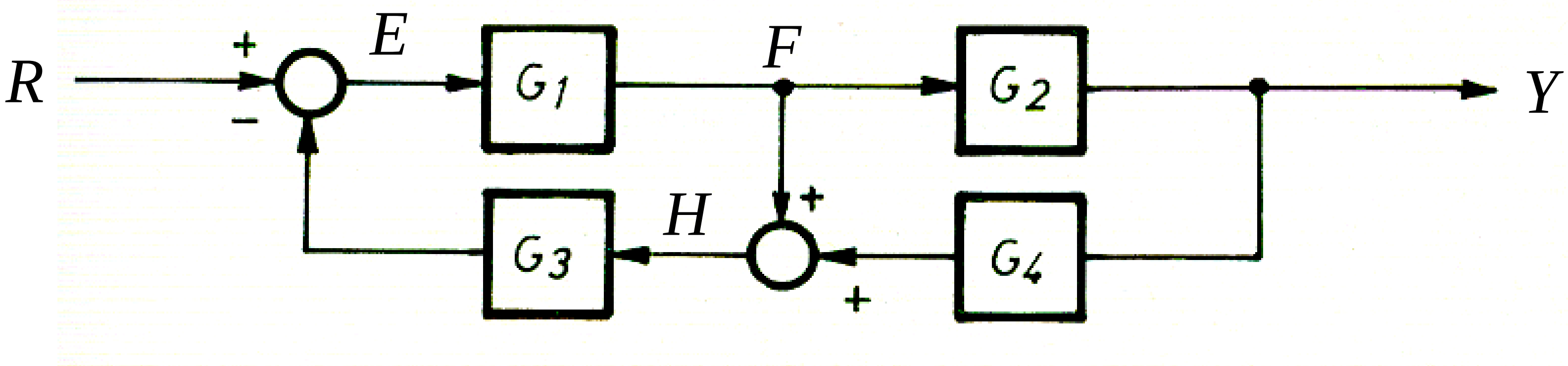
- Inför alla mellanvariabler, *E F H*.

Ex. Ekvationsmetoden



- Ställ upp ekvationssystemet.

Ex. Ekvationsmetoden



$$E = R - G_3 H \quad (1)$$

$$F = EG_1 \quad (2)$$

$$Y = FG_2 \quad (3)$$

$$H = F + G_4 Y \quad (4)$$

$$(2) \rightarrow (3)$$

$$(2) \rightarrow (4)$$

$$E = R - G_3 H \quad (1)$$

$$Y = EG_1 G_2 \quad (3')$$

$$H = EG_1 + G_4 Y \quad (4')$$

Ex. Ekvationsmetoden

$$E = R - G_3 H \qquad (1)$$

$$Y = EG_1G_2 \qquad (3')$$

$$H = EG_1 + G_4 Y \qquad (4')$$

$$(4') \rightarrow (1)$$

$$E = R - G_3(EG_1 + G_4 Y_3) \qquad (1')$$

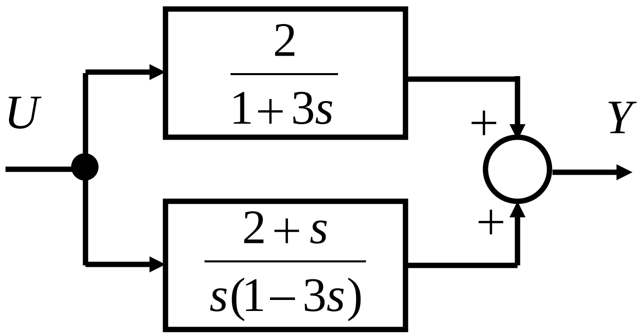
$$Y = EG_1G_2 \qquad (3')$$

$$Y = EG_1G_2 \Rightarrow \frac{Y}{G_1G_2} = E \qquad (3') \rightarrow (1')$$

$$\frac{Y}{G_1G_2} = R - G_3\left(\frac{Y}{G_2} + G_4 Y\right) \quad \Rightarrow \quad Y = RG_1G_2 - Y(G_1G_3 + G_1G_2G_3G_4)$$

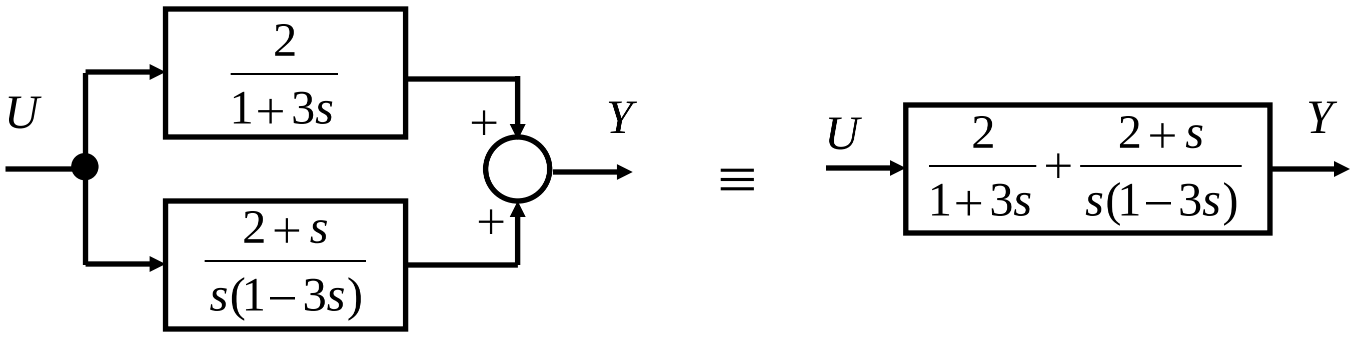
$$\Rightarrow \quad G\left(\tfrac{Y}{R}\right) = \frac{G_1G_2}{1 + G_1G_3 + G_1G_2G_3G_4}$$

8.1 Blockschemareduktion



$$G = \frac{Y}{U} = ?$$

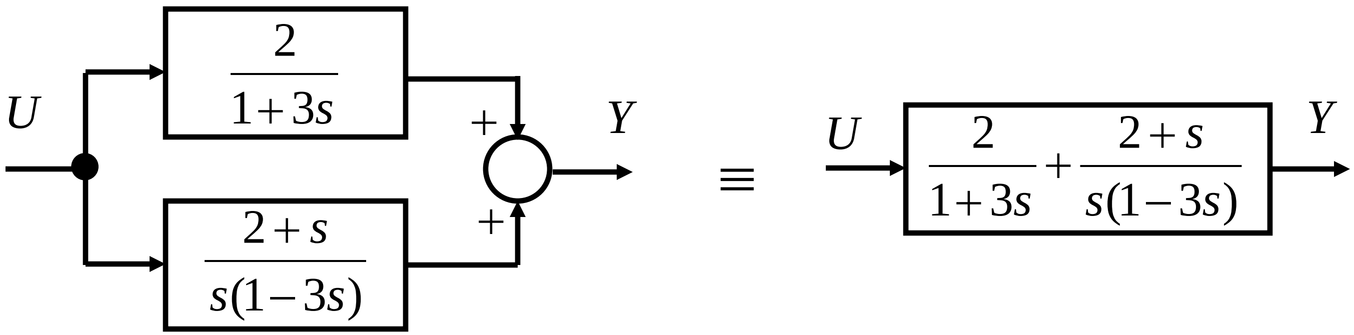
8.1 lösn. Blockschemareduktion



$$\frac{2}{1+3s}+\frac{2+s}{s(1-3s)} \quad \Rightarrow \quad \frac{2s(1-3s)+(2+s)(1+3s)}{s(1+3s)(1-3s)}$$

$$\Rightarrow \quad G=\frac{Y}{U}=\frac{9s-3s^2+2}{s(1-9s^2)}$$

8.1 med Matlab

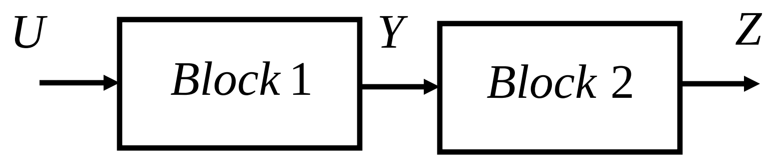



```
G1=tf([2],[3,1])
Transfer function:
   2
-------
3 s + 1
```

```
G2=tf([1,2],[-3,1,0])
Transfer function:
 -s – 2
---------
3 s^2 - s
```

```
G=G1+G2
Transfer function:
 3 s^2 - 9 s – 2
---------------
   9 s^3 - s
```

8.2 Blockschemareduktion

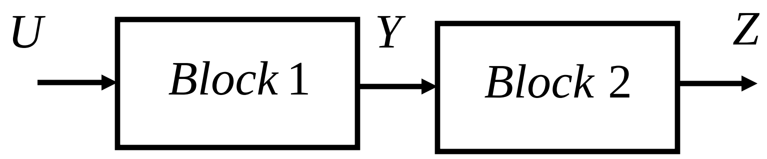


$$G = \frac{Z}{U} = ?$$

$$Block1: \quad \ddot{y} + 2\dot{y} + 3y = 5\ddot{u} + u$$

$$Block2: \quad 3\dot{z} + 2z = 6y$$

8.2 lösn. Blockschemareduktion



$$G = \frac{Z}{U} = ?$$

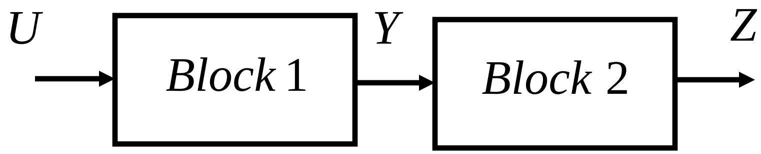
$$\ddot{y} + 2\dot{y} + 3y = 5\ddot{u} + u$$

$$\{L:\} \quad Y(s^2 + 2s + 3) = U(5s^2 + 1) \quad \Rightarrow \quad G_1\left(\tfrac{Y}{U}\right) = \frac{5s^2 + 1}{s^2 + 2s + 3}$$

$$3\dot{z} + 2z = 6y$$

$$\{L:\} \quad Z(3s + 2) = Y(6) \quad \Rightarrow \quad G_2\left(\tfrac{Z}{Y}\right) = \frac{6}{3s + 2}$$

8.2 lösn. Blockschemareduktion



$$G = \frac{Z}{U} = ?$$

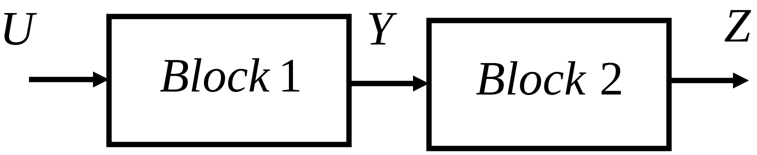
$$G_1\left(\tfrac{Y}{U}\right) = \frac{5s^2+1}{s^2+2s+3} \qquad G_2\left(\tfrac{Z}{Y}\right) = \frac{6}{3s+2}$$

$$G\left(\tfrac{Z}{U} = \tfrac{Y}{U}\cdot\tfrac{Z}{Y}\right) = G_1 \cdot G_2 = \frac{6(5s^2+1)}{(3s+2)(s^2+2s+3)} = \frac{30s+6}{3s^3+8s^2+13s+6}$$

8.2 med Matlab

$$G = \frac{Z}{U} = ?$$

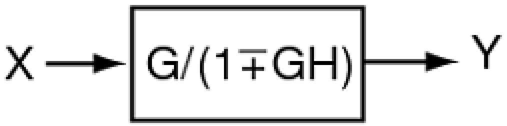
$$G_1\left(\frac{Y}{U}\right) = \frac{5s^2 + 1}{s^2 + 2s + 3} \qquad G_2\left(\frac{Z}{Y}\right) = \frac{6}{3s + 2}$$

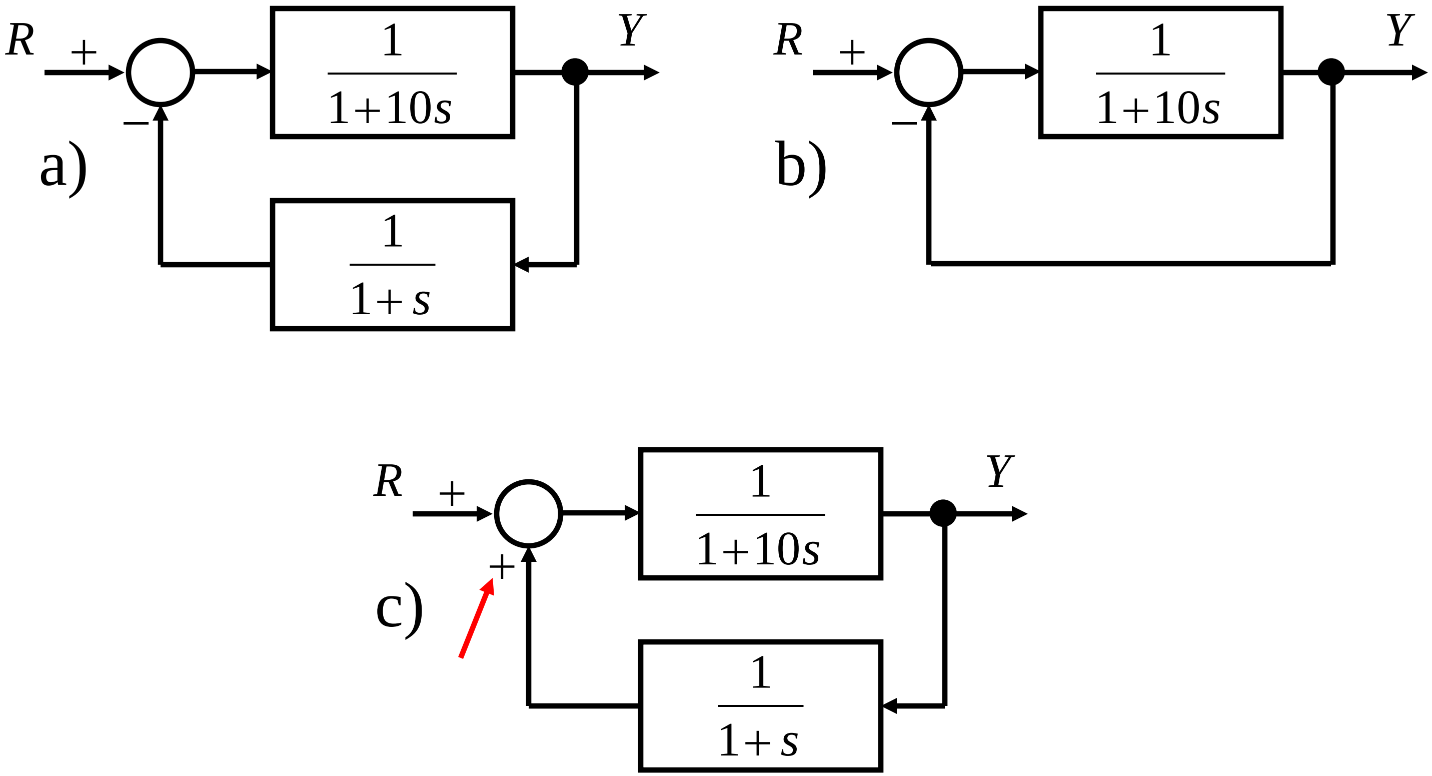



```
G1=tf([5,0,1],[1,2,3])
Transfer function:
  5 s^2 + 1
-------------
s^2 + 2 s + 3

G2=tf([6],[3,2])
Transfer function:
   6
-------
3 s + 2
```

```
G=G1*G2
Transfer function:
       30 s^2 + 6
------------------------
3 s^3 + 8 s^2 + 13 s + 6
```

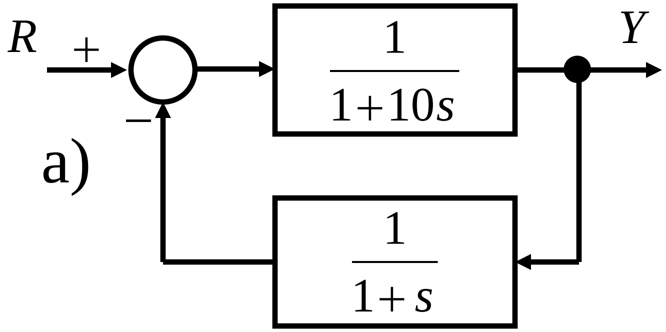
8.3 Feedback





8.3 a lösn. Feedback

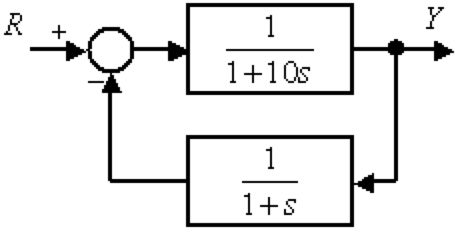
$$G_{closedloop}\left(\tfrac{Y}{X}\right) = \frac{G}{1 \mp GH}$$



$$G_1 = \frac{1}{10s+1} \quad G_2 = \frac{1}{s+1}$$

$$G_{closedloop}\left(\tfrac{Y}{R}\right) = \frac{G_1}{1+G_1 \cdot G_2} = \frac{\frac{1}{10s+1}}{1+\frac{1}{10s+1}\cdot\frac{1}{s+1}}$$

8.3 a lösn. Feedback

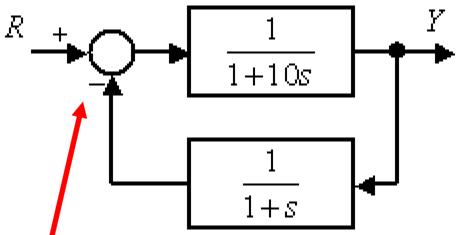


$$\frac{G_1}{1+G_1\cdot G_2}=\frac{\dfrac{1}{10s+1}}{1+\dfrac{1}{10s+1}\cdot\dfrac{1}{s+1}}=\frac{s+1}{(10s+1)(s+1)+1}=$$

$$=\frac{s+1}{10s^2+11s+2}$$

8.3 a med Matlab

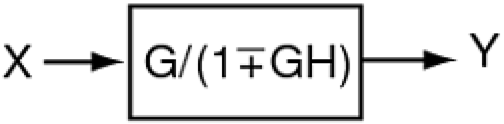
$$G_1 = \frac{1}{10s+1} \quad G_2 = \frac{1}{s+1}$$



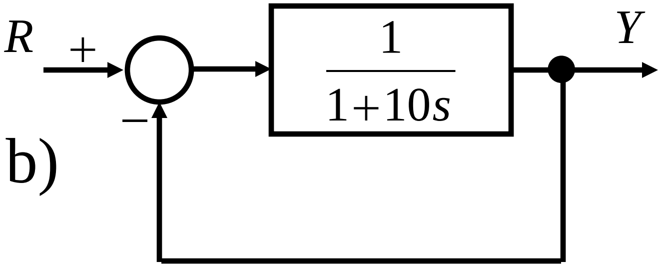

```
G1=tf([1],[10,1])
Transfer function:
    1
--------
10 s + 1

G2=tf([1],[1,1])
Transfer function:
   1
-------
1 s + 1
```

```
G=feedback(G1,G2,-1)
Transfer function:
     s + 1
-----------------
10 s^2 + 11 s + 2
```

8.3 b lösn. Feedback



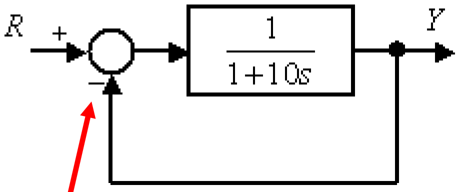
b)



$$G_1 = \frac{1}{10s+1} \quad G_2 = 1$$

$$G_{closedloop}\left(\tfrac{Y}{R}\right) = \frac{G_1}{1+G_1 \cdot G_2} = \frac{\frac{1}{10s+1}}{1+\frac{1}{10s+1}\cdot 1} = \frac{1}{10s+2}$$

8.3 b med Matlab



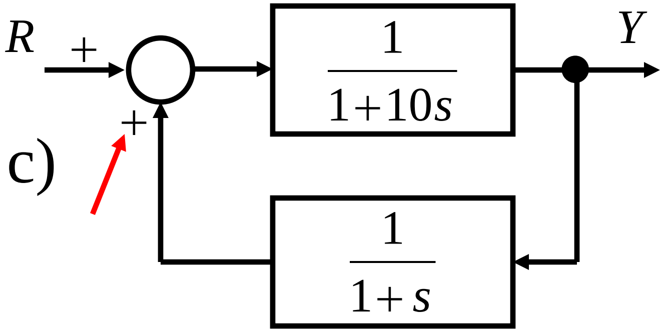
$$G_1 = \frac{1}{10s+1} \quad G_2 = 1$$

```
G1=tf([1],[10,1])
Transfer function:
    1
--------
10 s + 1

G2=tf(1)

Transfer function:
1
Static gain.
```

```
G=feedback(G1,G2,-1)
Transfer function:
   1
--------
10 s + 2
```

8.3 c lösn. Feedback

c)

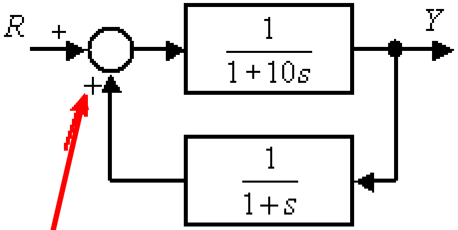


$$G_1 = \frac{1}{10s+1} \quad G_2 = \frac{1}{s+1}$$

$$\frac{G_1}{1 - G_1 \cdot G_2} = \frac{\frac{1}{10s+1}}{1 - \frac{1}{10s+1} \cdot \frac{1}{s+1}} = \frac{s+1}{(10s+1)(s+1)-1} =$$

$$= \frac{s+1}{10s^2+11s}$$

8.3 c med Matlab

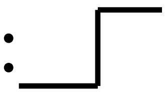


$$G_1 = \frac{1}{10s+1} \quad G_2 = \frac{1}{s+1}$$

```
G1=tf([1],[10,1])
Transfer function:
    1
--------
10 s + 1

G2=tf([1],[1,1])
Transfer function:
   1
-------
1 s + 1
```

```
G=feedback(G1,G2,+1)
Transfer function:
    s + 1
-------------
10 s^2 + 11 s
```

(Begynnelse) och Slutvärde

Kommer Du ihåg? $L:$  $\Rightarrow \boxed{\dfrac{1}{s}}$

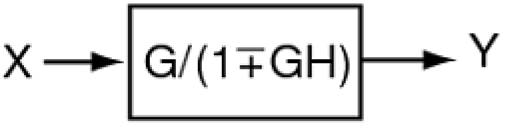
$$\lim_{t \to 0+} f(t) = \lim_{s \to \infty} s \cdot F(s)$$

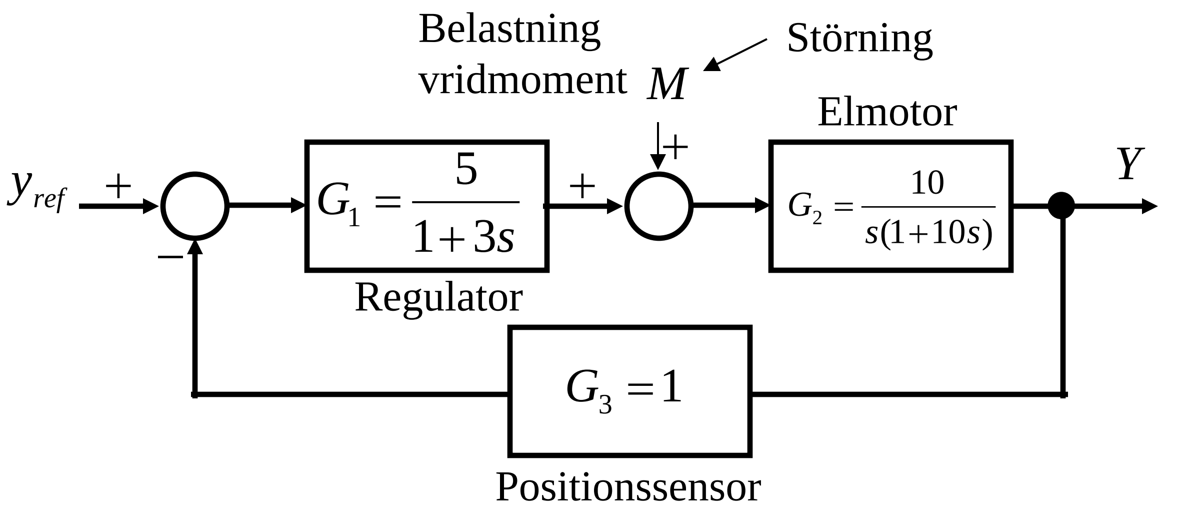
$$\boxed{\lim_{t \to \infty} f(t) = \lim_{s \to 0} \boxed{s} \cdot F(s) \qquad \bullet \text{ slutvärde}}$$

Vad som händer efter lång tid avgörs av laplacetransformens lågfrekvensegenskaper. Slutvärdet efter ett steg om man låter s → 0.

$$\boxed{\frac{1}{s} \times s = 1}$$

8.4 Positionsreglering





a) $G_{y_{ref} \to Y} = ?$

För reglerstorheten.

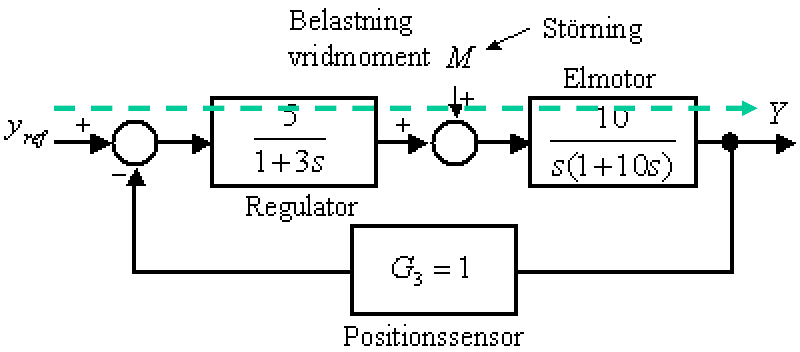
b) $G_{M \to Y} = ?$

För störstorheten.

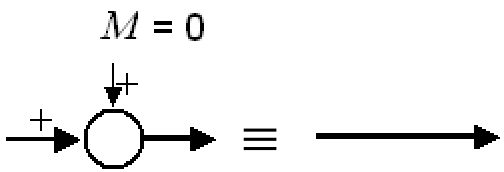
Den storhet som *inte* ingår i överföringsfunktionen kan sättas = 0. För linjära system gäller ju superposition.

8.4 a lösn. Positionsreglering

$$G_1 = \frac{5}{3s+1} \quad G_2 = \frac{10}{s(s+1)} \quad G_3 = 1$$



Antag $M = 0$



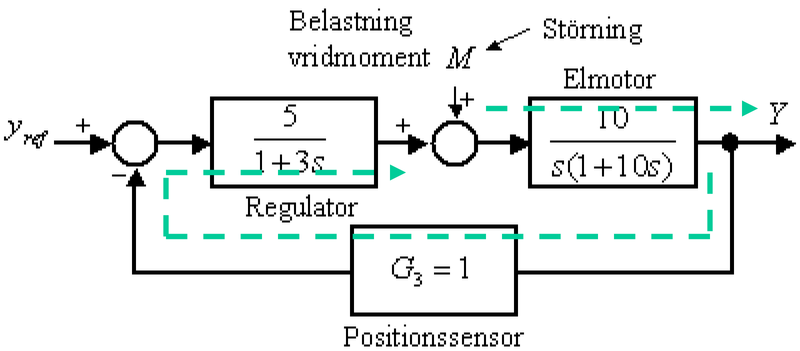
$$G\left(\frac{Y}{Y_{REF}}\right) = \frac{G_1 \cdot G_2}{1 + G_1 \cdot G_2 \cdot G_3} = \frac{\frac{5}{3s+1} \cdot \frac{10}{s(s+1)}}{1 + \frac{5}{3s+1} \cdot \frac{10}{s(s+1)} \cdot 1} = \frac{50}{s(3s+1)(s+1)+50} =$$

$$= \frac{50}{\cancel{30s^3} + \cancel{13s^2} + \cancel{s} + 50}$$

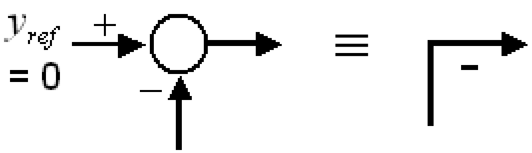
($s = 0 \rightarrow G = 1$, en *stegändring* hos styrstorheten slår igenom med 100% på utstorhetens *slutvärde*. $s \cdot 1/s = 1$)

8.4 b lösn. Positionsreglering

$$G_1 = \frac{5}{3s+1} \quad G_2 = \frac{10}{s(s+1)} \quad G_3 = 1$$



Antag $y_{\text{ref}} = 0$



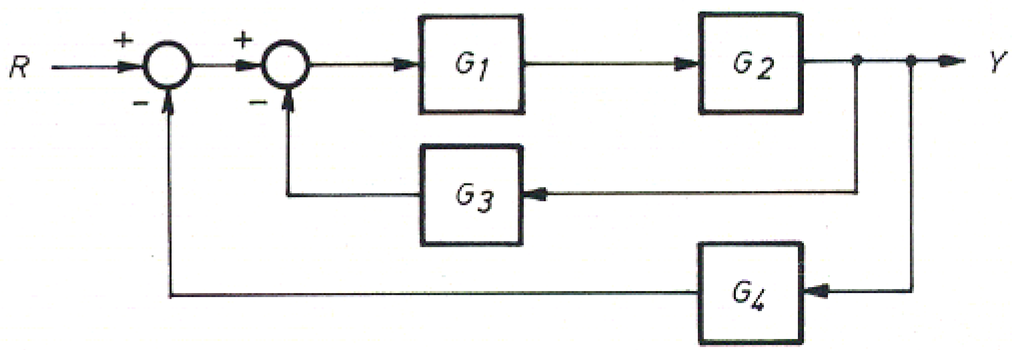
$$G\left(\tfrac{Y}{M}\right) = \frac{G_2}{1+G_3\cdot G_1\cdot G_2} = \frac{\frac{10}{s(s+1)}}{1+1\cdot\frac{5}{3s+1}\cdot\frac{10}{s(s+1)}} = \frac{10(3s+1)}{s(s+1)(3s+1)+5\cdot 10} =$$

$$= \frac{\cancel{30s}+10}{\cancel{3s^3}+\cancel{4s^2}+\cancel{s}+50}$$

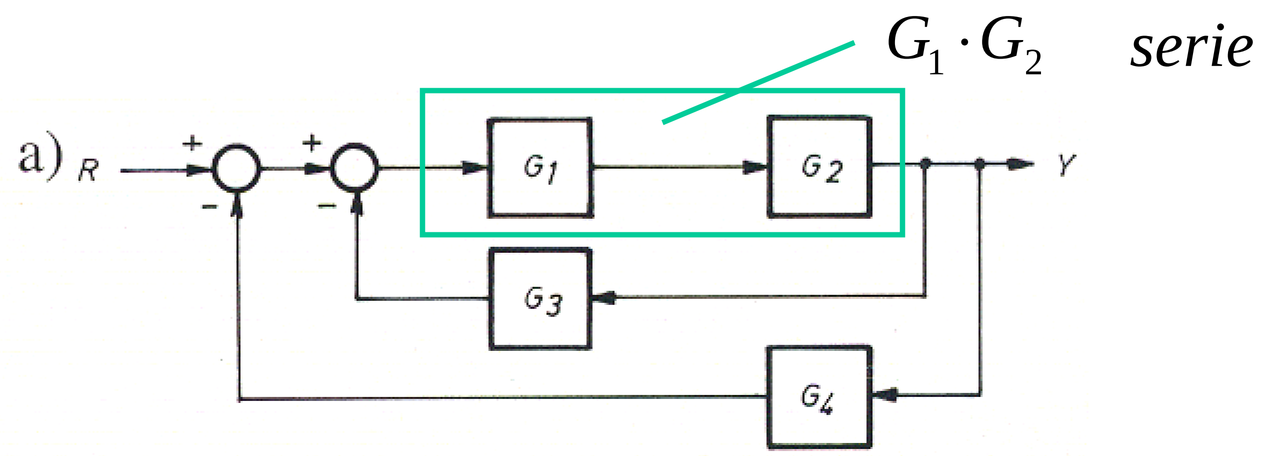
($s = 0 \rightarrow G = 1/5$, en *stegändring* hos störningen påverkar *slutvärdet* med 20%. $s\cdot 1/s = 1$)

8.5 a Blockschemareduktion

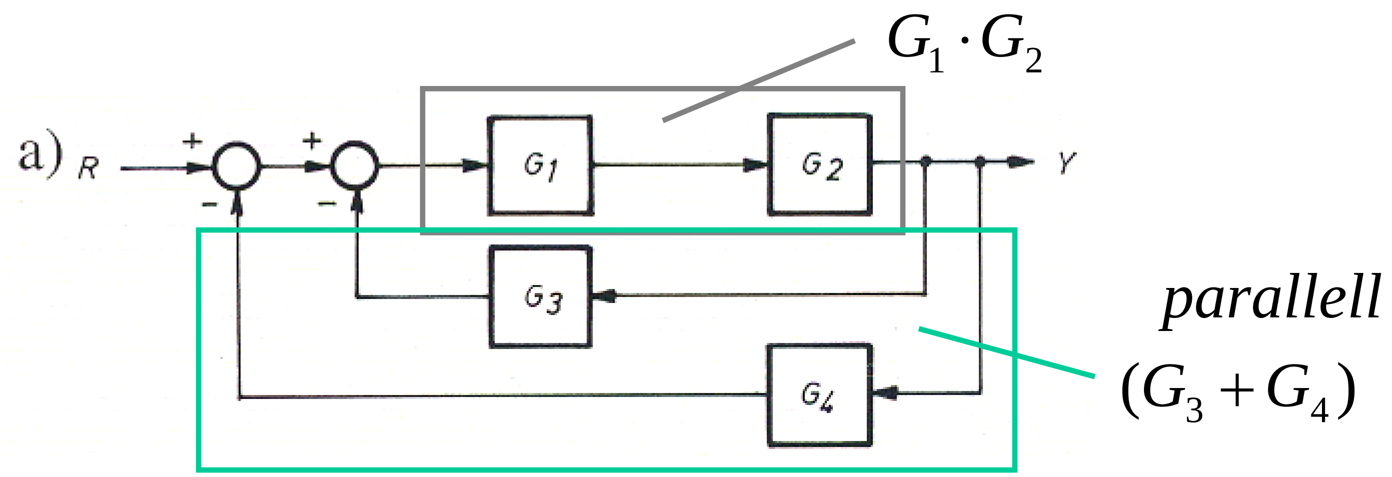
a)



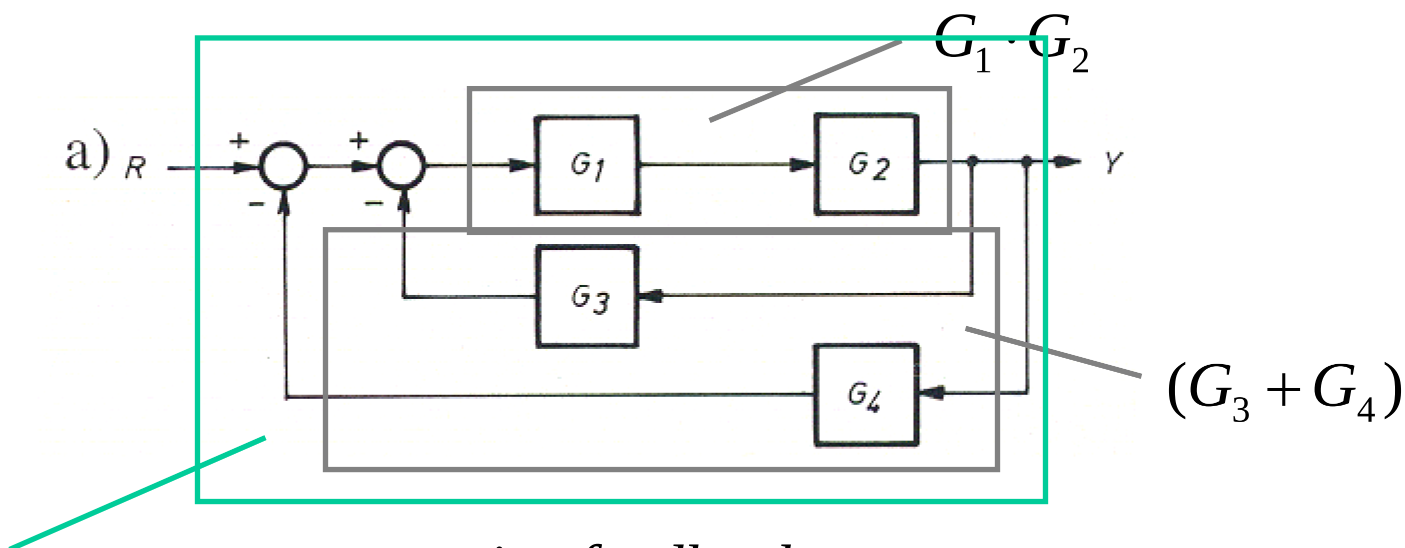
8.5 a lösn. Blockschemareduktion



8.5 a lösn. Blockschemareduktion



8.5 a lösn. Blockschemareduktion



negative feedback

$$\frac{G_1G_2}{1+G_1G_2(G_3+G_4)}$$

$$G\left(\tfrac{Y}{R}\right)=\frac{G_1G_2}{1+G_1G_2G_3+G_1G_2G_4}$$

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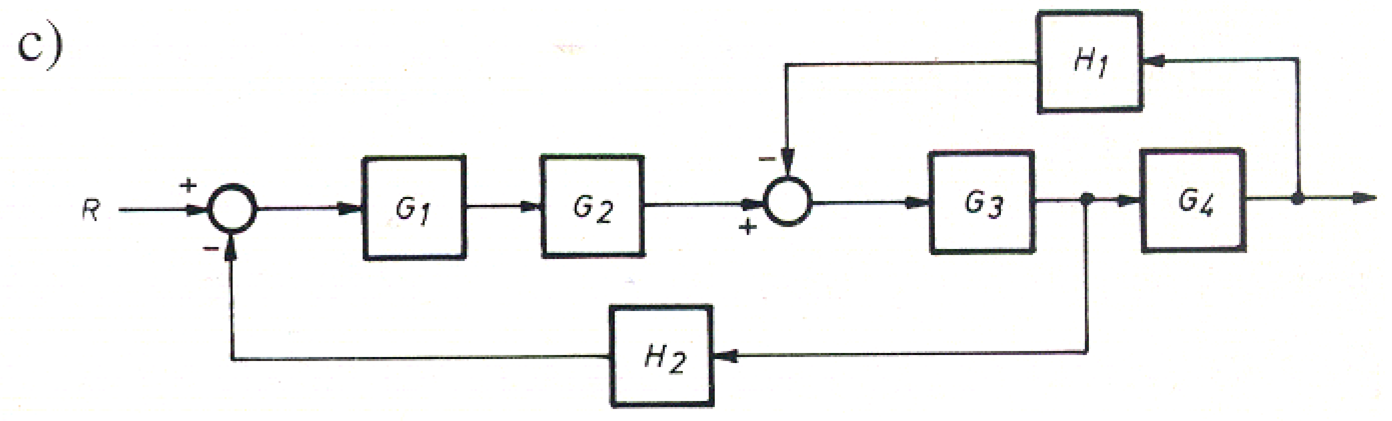
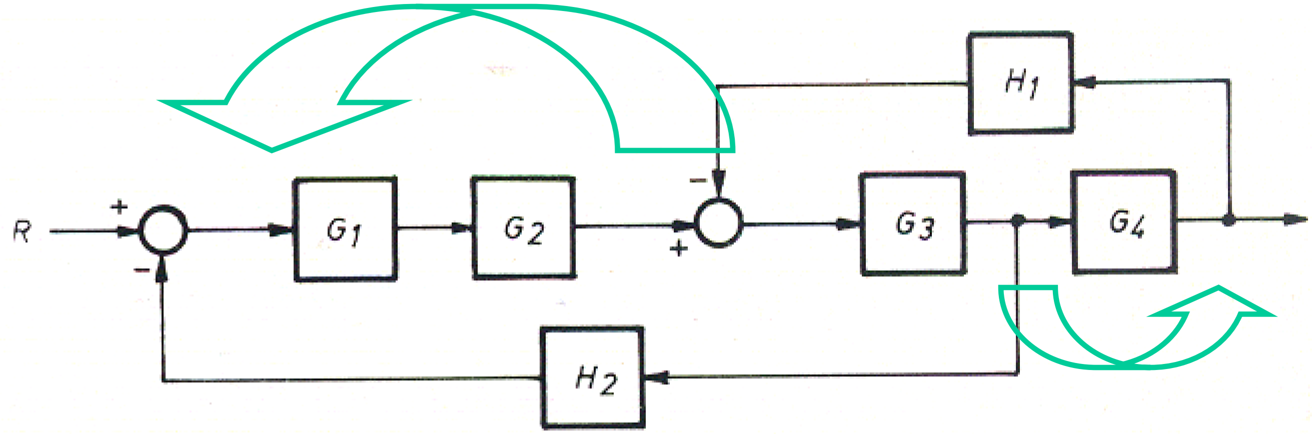
8.5 c Blockschemareduktion

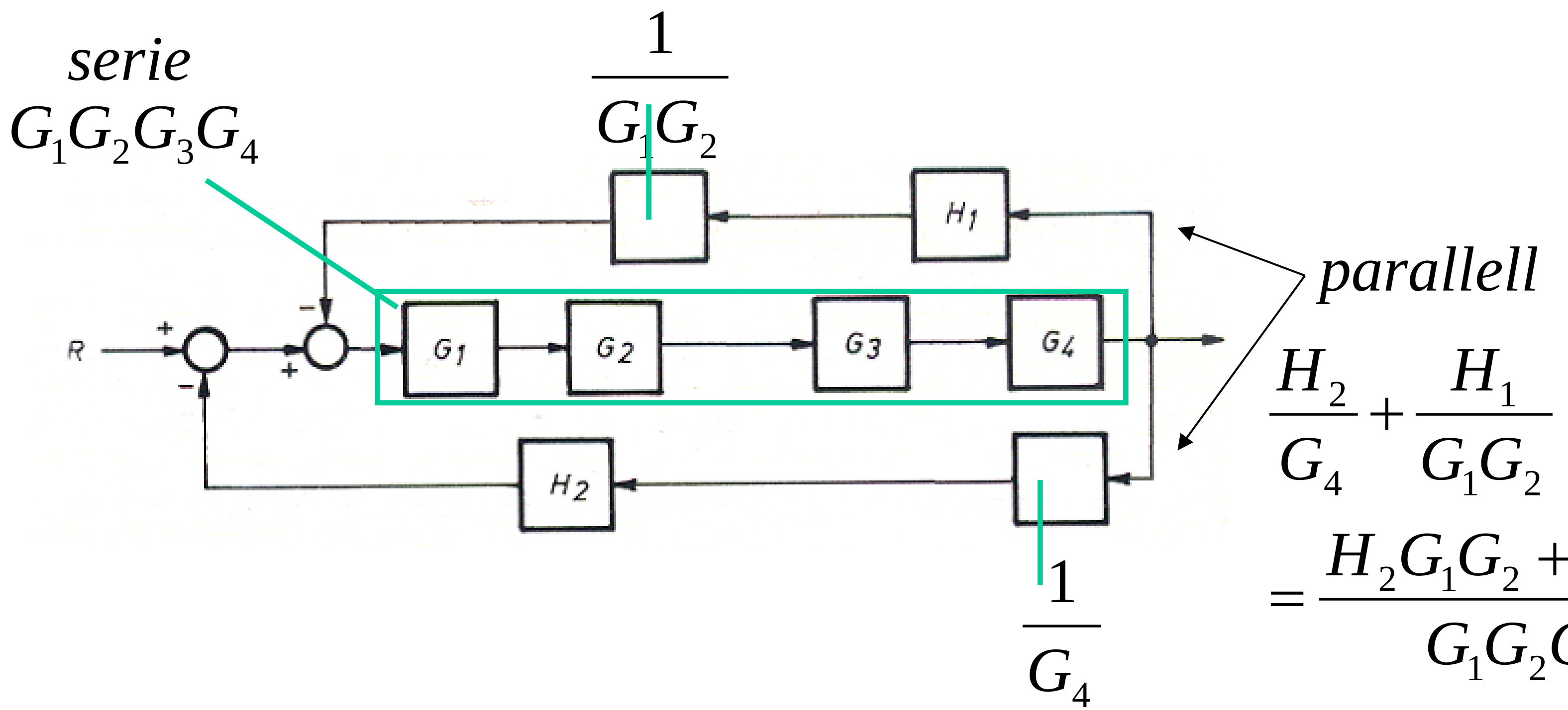


8.5 c Blockschemareduktion

c)



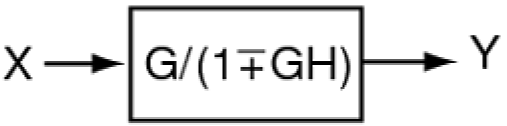
8.5 c Blockschemareduktion



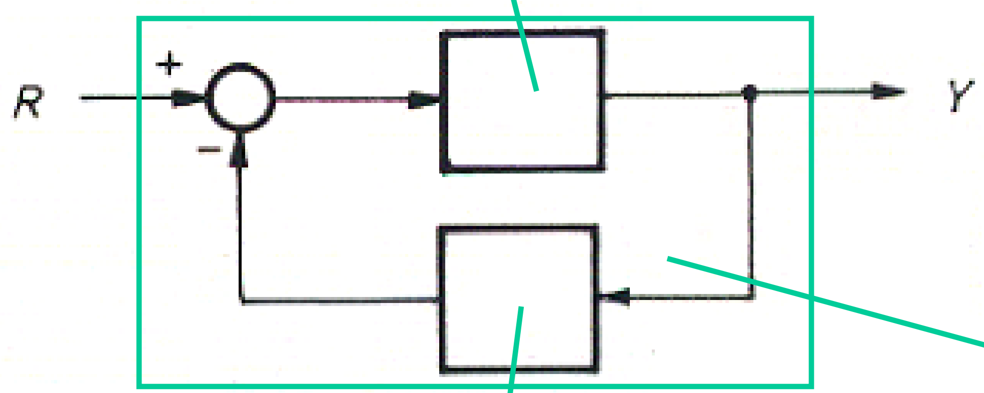
$$\frac{H_2}{G_4}+\frac{H_1}{G_1G_2}=$$

$$=\frac{H_2G_1G_2+H_1G_4}{G_1G_2G_4}$$

8.5 c Blockschemareduktion



$G_1G_2G_3G_4$



$$\frac{H_2G_1G_2 + H_1G_4}{G_1G_2G_4}$$

negative feedback

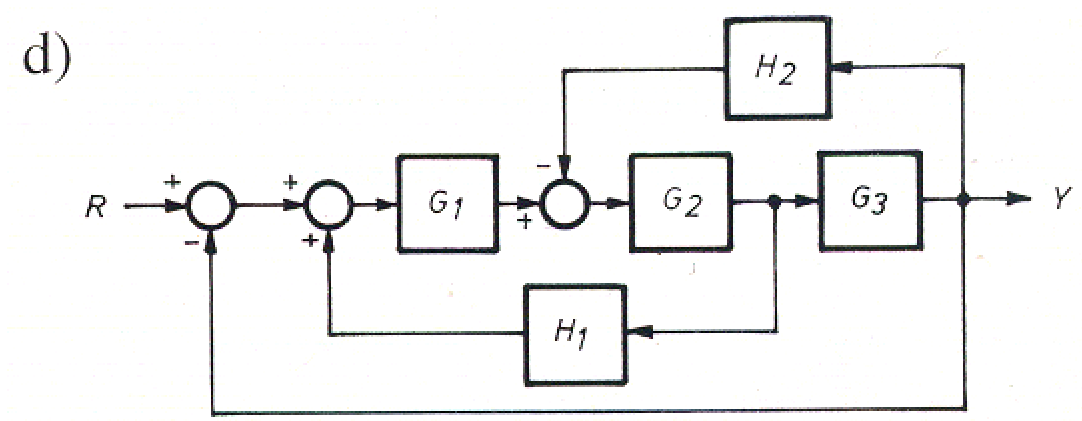
$$\frac{G_1G_2G_3G_4}{1 + G_1G_2G_3G_4 \cdot \dfrac{H_2G_1G_2 + H_1G_4}{G_1G_2G_4}}$$

8.5 c Blockschemareduktion

$$\frac{G_1G_2G_3G_4}{1+\cancel{G_1}\cancel{G_2}G_3\cancel{G_4}\cdot\dfrac{H_2G_1G_2+H_1G_4}{\cancel{G_1}\cancel{G_2}\cancel{G_4}}}=\frac{G_1G_2G_3G_4}{1+G_3(H_2G_1G_2+H_1G_4)}=$$

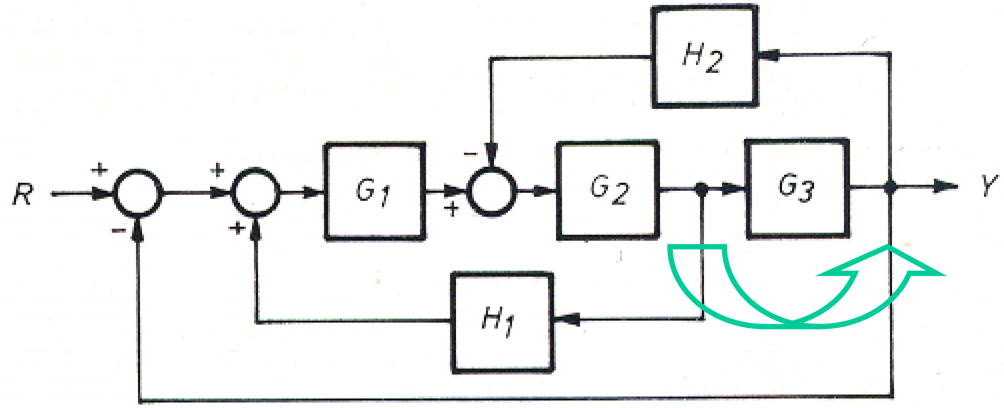
$$=\frac{G_1G_2G_3G_4}{1+H_2G_1G_2G_3+H_1G_3G_4}$$

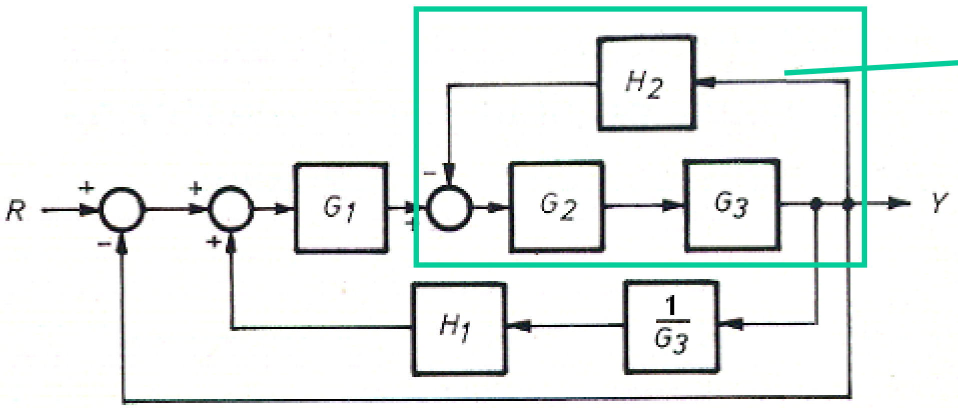
8.5 d Blockschemareduktion



8.5 d Blockschemareduktion

d)





negative feedback

$$\frac{G_2G_3}{1+G_2G_3H_2}$$

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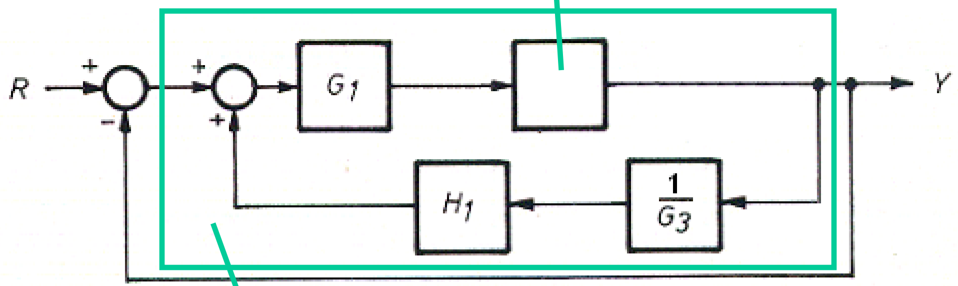
8.5 d Blockschemareduktion

$$\frac{G_2 G_3}{1+G_2 G_3 H_2}$$



positive feedback

$$\frac{\dfrac{G_1 G_2 G_3}{1+G_2 G_3 H_2}}{1-\dfrac{H_1}{G_3}\cdot\dfrac{G_1 G_2 G_3}{1+G_2 G_3 H_2}}$$

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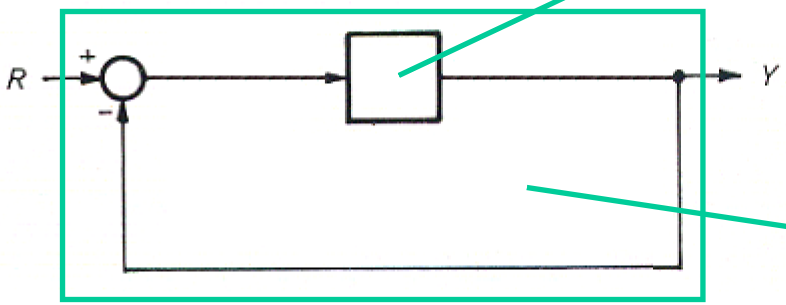
8.5 d Blockschemareduktion

$$\frac{\dfrac{G_1G_2G_3}{1+G_2G_3H_2}}{1-\dfrac{H_1}{G_3}\cdot\dfrac{G_1G_2G_3}{1+G_2G_3H_2}}\cdot\frac{1+G_2G_3H_2}{1+G_2G_3H_2}=\frac{G_1G_2G_3}{1+G_2G_3H_2-H_1G_1G_2}$$



negative feedback

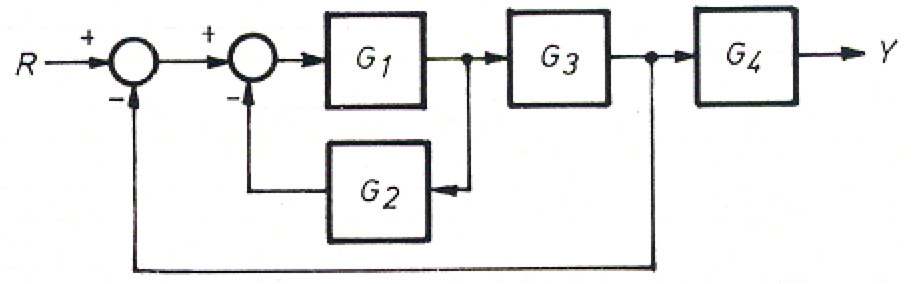
$$\frac{\dfrac{G_1G_2G_3}{1+G_2G_3H_2-H_1G_1G_2}}{1+\dfrac{G_1G_2G_3}{1+G_2G_3H_2-H_1G_1G_2}}$$

8.5 d Blockschemareduktion

$$\frac{\dfrac{G_1G_2G_3}{1+G_2G_3H_2-H_1G_1G_2}}{1+\dfrac{G_1G_2G_3}{1+G_2G_3H_2-H_1G_1G_2}}=\frac{G_1G_2G_3}{1+G_2G_3H_2-H_1G_1G_2+G_1G_2G_3}$$

8.5 e Blockschemareduktion

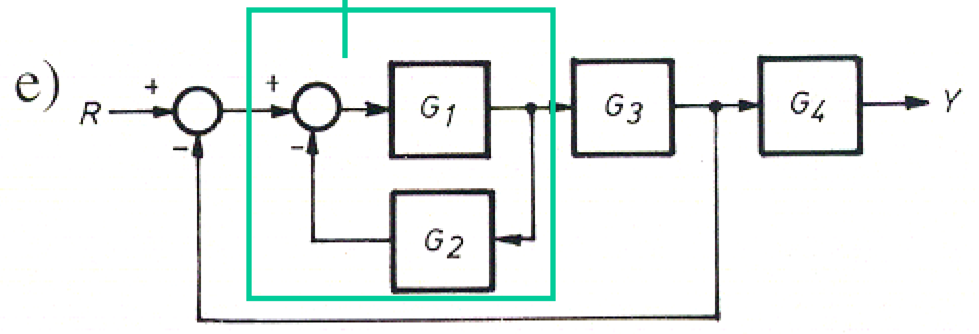
e)



8.5 e Blockschemareduktion

negative feedback

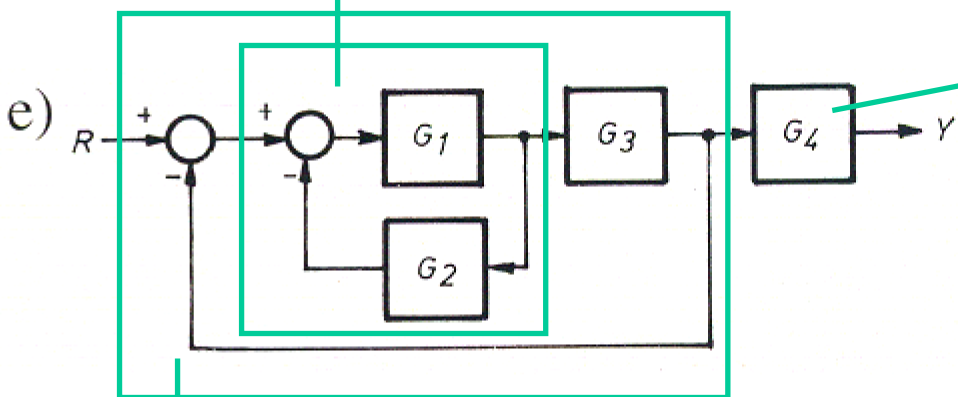
$$\frac{G_1}{1+G_1G_2}$$



8.5 e Blockschemareduktion

$$\frac{G_1}{1+G_1G_2}$$

serie



$$\frac{G_1G_3 \cdot G_4}{1+G_1G_2+G_1G_3}$$

negative feedback

$$\frac{\dfrac{G_1G_3}{1+G_1G_2}}{1+\dfrac{G_1G_3}{1+G_1G_2}} = \frac{G_1G_3}{1+G_1G_2+G_1G_3}$$

$$G\left(\frac{Y}{R}\right) = \frac{G_1G_3G_4}{1+G_1G_2+G_1G_3}$$

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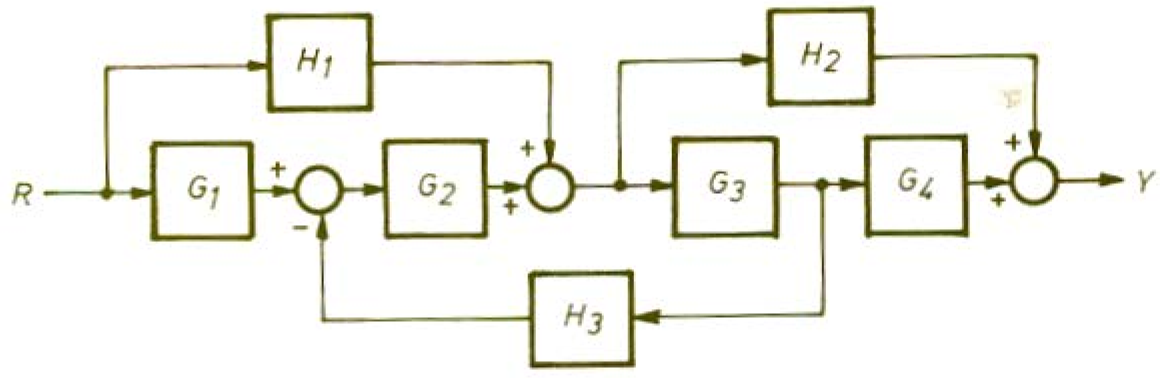
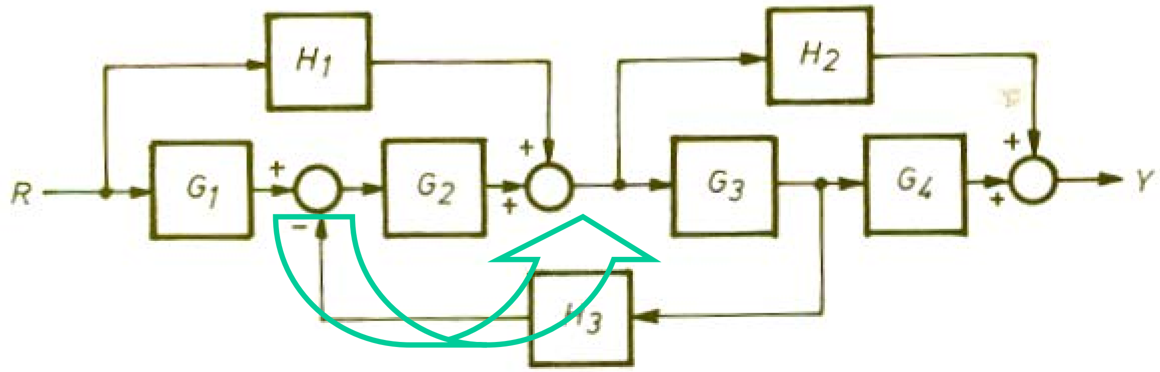
8.5 f Blockschemareduktion

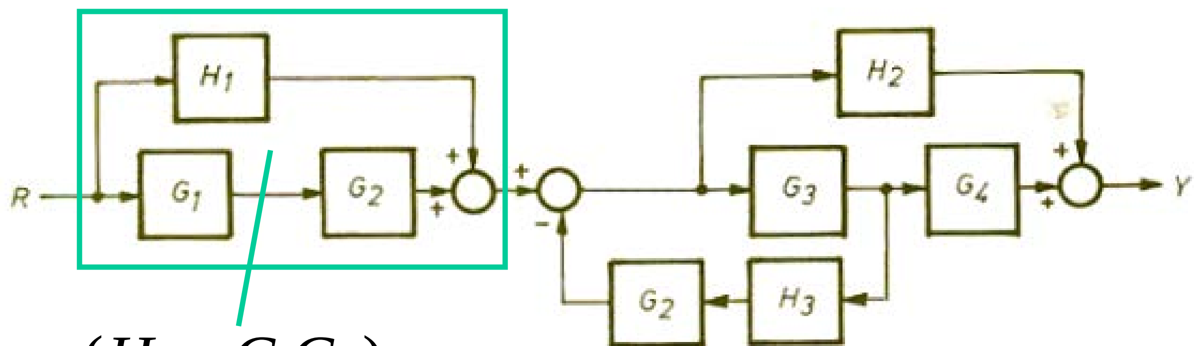
f)



8.5 f Blockschemareduktion

f)



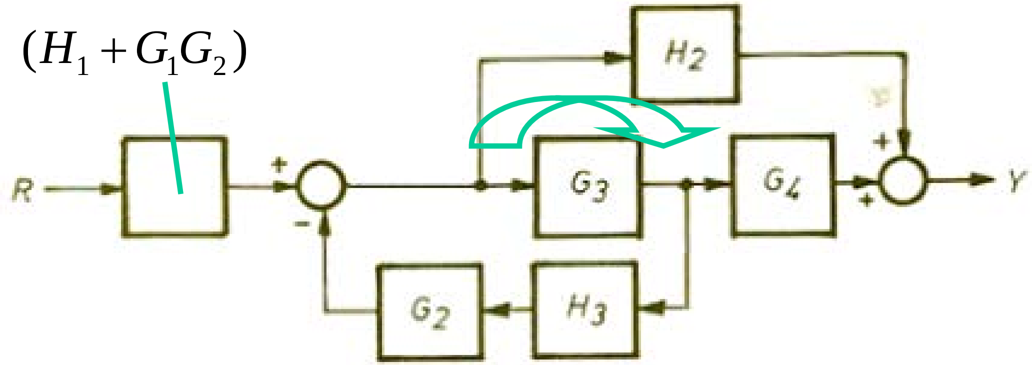


$(H_1 + G_1 G_2)$

parallell

8.5 f Blockschemareduktion

$(H_1 + G_1 G_2)$

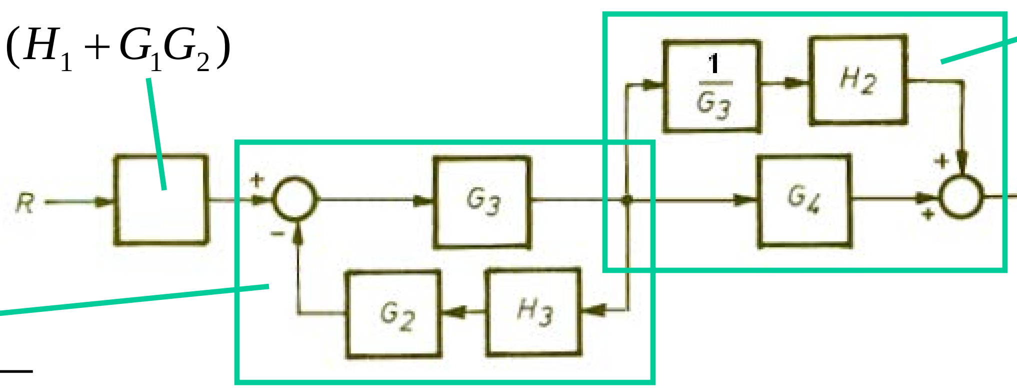


parallell

$$(G_4 + \frac{H_2}{G_3})$$

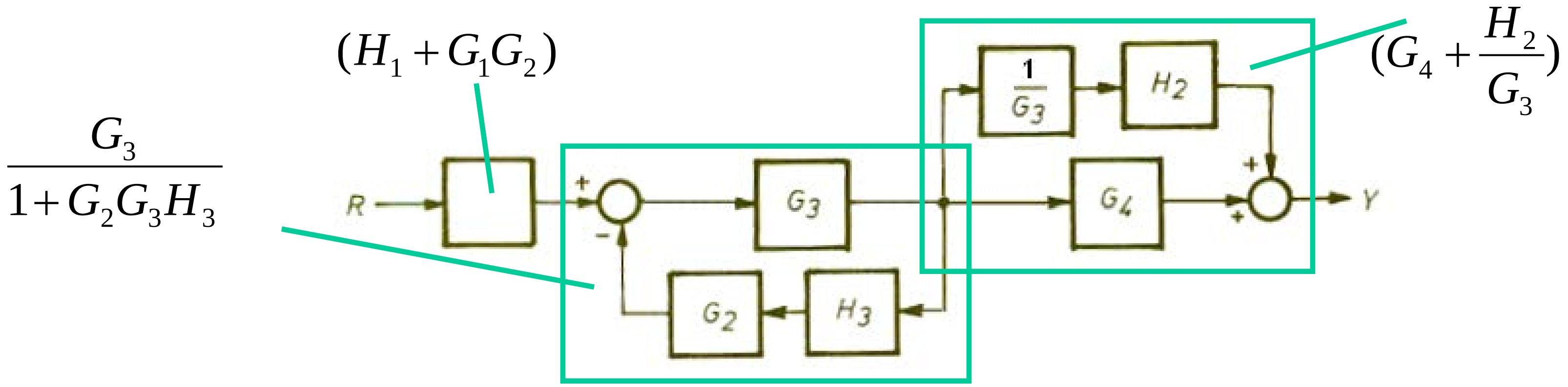
$(H_1 + G_1 G_2)$

Negative feedback



$$\frac{G_3}{1 + G_2 G_3 H_3}$$

8.5 f Blockschemareduktion

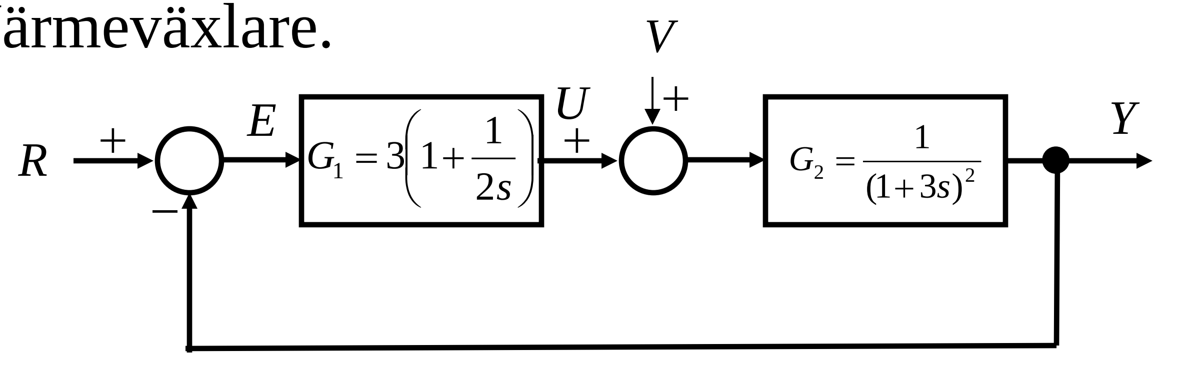


$$(H_1 + G_1G_2) \cdot \frac{G_3}{1+G_2G_3H_3} \cdot (G_4 + \frac{H_2}{G_3}) =$$

$$= \frac{(H_1 + G_1G_2)(G_4G_3 + H_2)}{1+G_2G_3H_3}$$

8.7 Styrsignaler

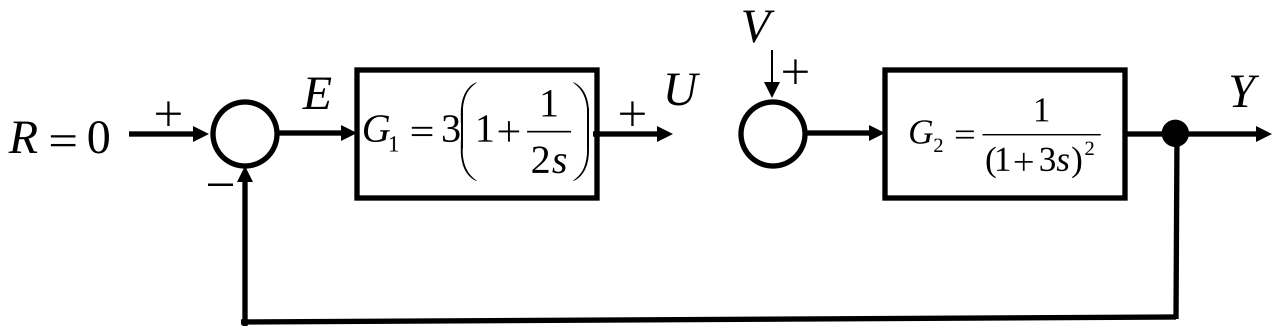
Värmeväxlare.



Hur stora styrsignaler U krävs vid olika störningar V ?
För att svara på det behöver man överföringsfunktionen
från $V \rightarrow U$.

$$G\left(\frac{U}{V}\right) = ?$$

8.7 Styrsignaler



$$G\left(\frac{U}{V}\right) = \frac{1}{(1+3s)^2} \cdot (-1) \cdot 3\left(1 + \frac{1}{2s}\right) =$$

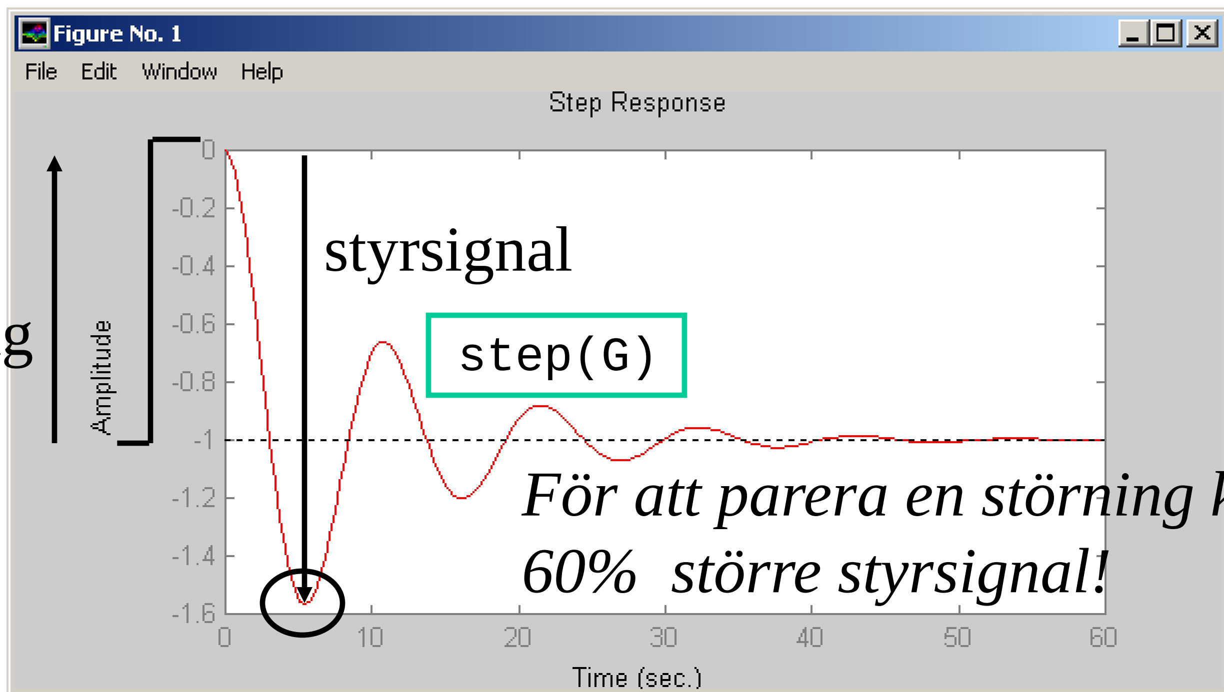
$$= \frac{-3(2s+1)}{(1+3s)^2 2s} = \frac{-(6s+3)}{18s^3 + 12s^2 + 8s + 3}$$

8.7 Styrsignaler med Matlab

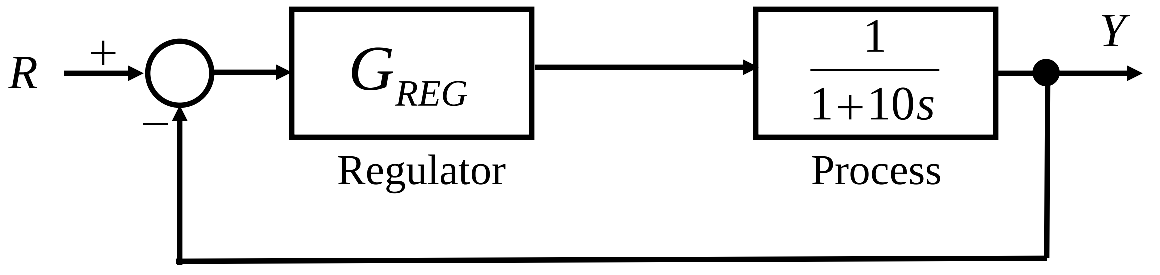
$$G\left(\frac{U}{V}\right) = \frac{-(6s+3)}{18s^3 + 12s^2 + 8s + 3}$$

```
G=tf([-6,-3],[18,12,8,3])
Transfer function:
        -6 s - 3
-------------------------
18 s^3 + 12 s^2 + 8 s + 3
```

steg-
störning



8.8 Regulatorer



a) $G\left(\frac{Y}{R}\right) = ?\quad G_{REG} = 5$ P-regulator

b) $G\left(\frac{Y}{R}\right) = ?\quad G_{REG} = 3\left(2 + \frac{1}{10s}\right)$ PI-regulator

8.8 a med Matlab, P-Regulator

a) $G\left(\frac{Y}{R}\right) = ?$ $G_{REG} = 5$

P-regulator

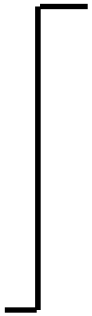
```
Greg=tf(5)
Transfer function:
5
static gain
G=Greg*Gp
Transfer function:
    5
--------
10 s + 1
```

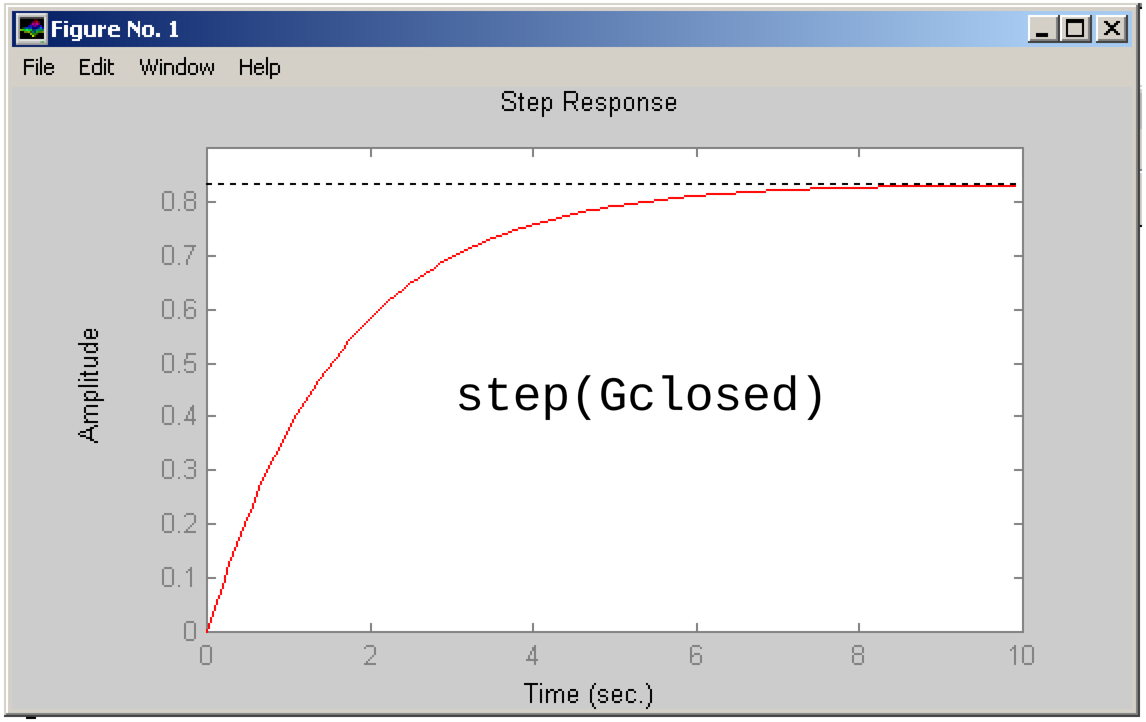
```
Gp=tf([1],[10,1])
Transfer function:
   1
--------
10 s + 1
```

```
Gclosed=feedback(G,1,-1)
Transfer function:
    5
--------
10 s + 6
```

8.8 a med Matlab, P-Regulator

```
Gclosed=feedback(G,1,-1)
Transfer function:
    5
--------
10 s + 6
```





8.8 b med Matlab, PI-Regulator

b) $G\left(\frac{Y}{R}\right) = ?$ $G_{REG} = 3\left(2 + \frac{1}{10s}\right)$

PI-regulator

$$G_{REG} = 3\left(2 + \frac{1}{10s}\right) = \frac{60s + 3}{10s}$$

```
Gp=tf([1],[10,1])
Transfer function:
    1
--------
10 s + 1
```

```
Greg=tf([60,3],[10,0])
Transfer function:
60 s + 3
--------
  10 s
G=Greg*Gp
Transfer function:
   60 s + 3
--------------
100 s^2 + 10 s
```

```
Gclosed=feedback(G,1,-1)
Transfer function:
     60 s + 3
------------------
100 s^2 + 70 s + 3
```

8.8 b med Matlab, PI-Regulator

```
Gclosed=feedback(G,1,-1)
Transfer function:
    60 s + 3
------------------
100 s^2 + 70 s + 3
```

Visst kan en väl inställd regulator göra nytta?
(Ingenjörer kan göra skillnad)