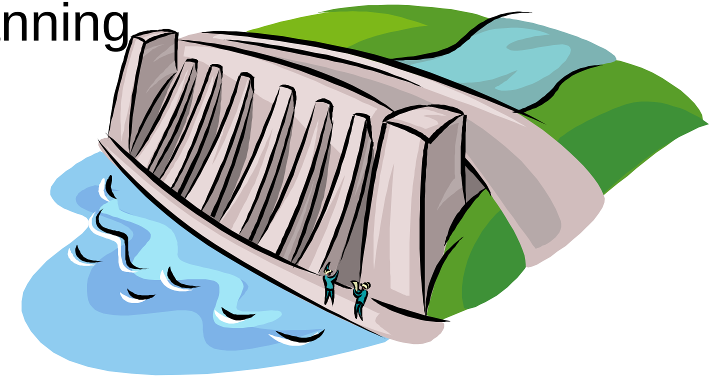
System Planning 2014

Lecture 8, L8:

Short-term planning of hydro power systems

System planning 2014

- Short-term planning of hydro power systems
- Chapter 5.1-5.2.4 (actually 5.2.1-5.2.4)
- Content:
  - Generally about short-term planning
  - Generally about hydropower
  - Electricity Production
  - Hydrological coupling
  - Hydro power planning



Generally about hydro power planning (1/3)

- What is short-term planning?
  - Timeframe: 24 hours – 1 week
  - In this course: Hourly planning
  - Minimize costs, maximize profit
- Results:
  - Operation schedules for the power plants
  - Trade on the electricity market
- Limiting factors planning the operation:
  - Technological/Physical
  - Economical/Juridical

Generally about hydro power planning (2/3)

- A generally formulated short-term planning program:
- Maximize
  - Incomes during the studied period
  - (Expected) future incomes
- Minimize
  - Costs during the studied period
  - (Expected) future costs
- Continued …

Generally about hydro power planning (3/3)

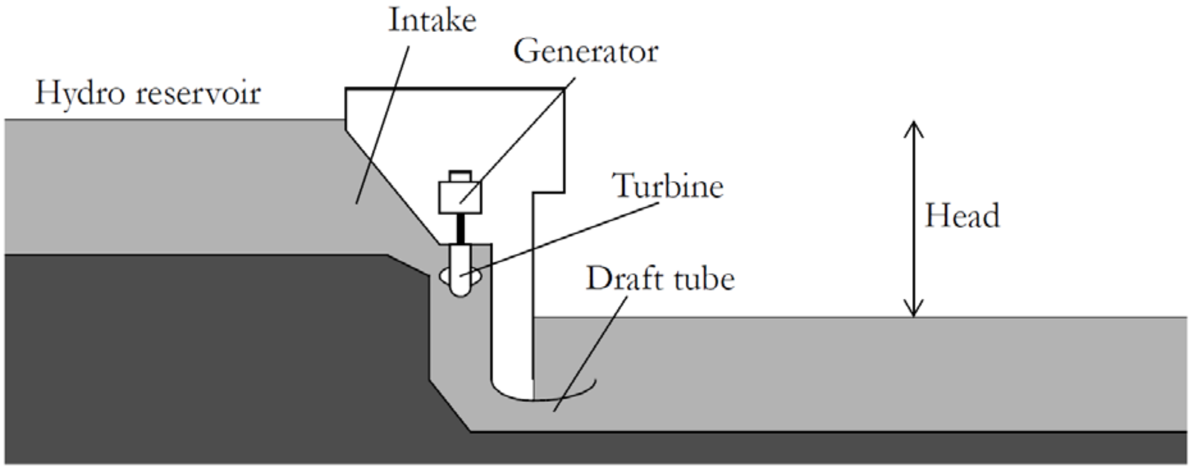
- Continues …
- Subject to
  - Physical constraints,
  - Juridical constraints, e.g.
    - Water-right judgements
    - Emission certificates
- This results in an optimization problem
- This course: deterministic models

Generally about hydro power (1/5)

- Produces electricity by potential energy differences
- Reservoirs common – also for streaming water

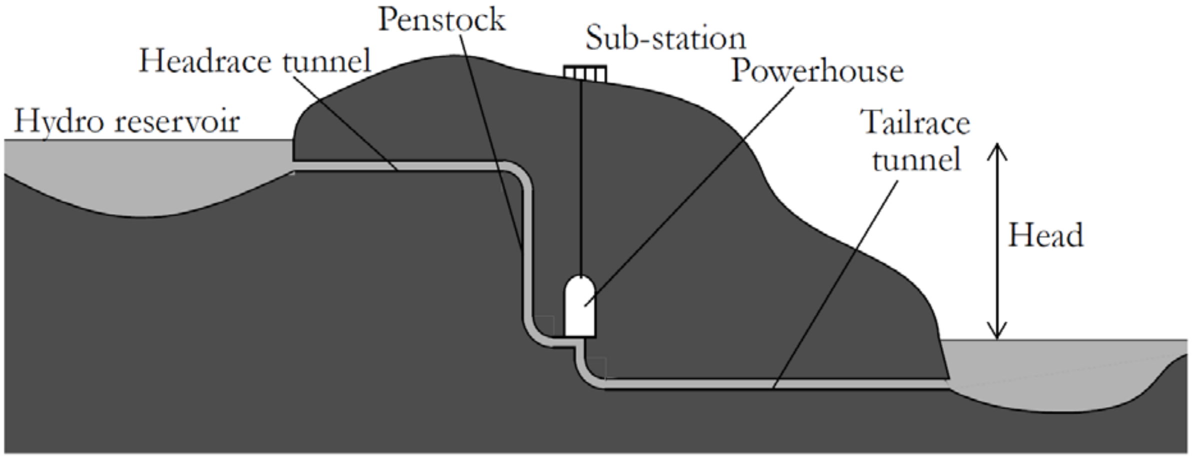
Generally about hydro power (2/5)

Typical construction with low head



Generally about hydro power (3/5)

Typical construction with high head



Generally about hydro power (4/5)

- Necessary variables in hydro power planning
  - Discharge, Q
  - Spillage, S
  - Reservoir content, M
  - Generated power, $H(Q)$
- Variables expressed in ”hour equivalents”, HE (SE: TE)
  - 1 HE means:
  - Discharge & spillage: **flow** 1 $\frac{\mathrm{m}^3}{\mathrm{s}}$ during 1 h
  - Reservoir content: **volume** corresponding to flow 1 $\frac{\mathrm{m}^3}{\mathrm{s}}$ during 1 h

Generally about hydro power (5/5)

Uddby Hydro Power Plant, Tyresö

Turbine

- Maximal discharge: $3.5\ \frac{\mathrm{m}^3}{\mathrm{s}}$
- Minimal spillage: $0.15\ \frac{\mathrm{m}^3}{\mathrm{s}}$

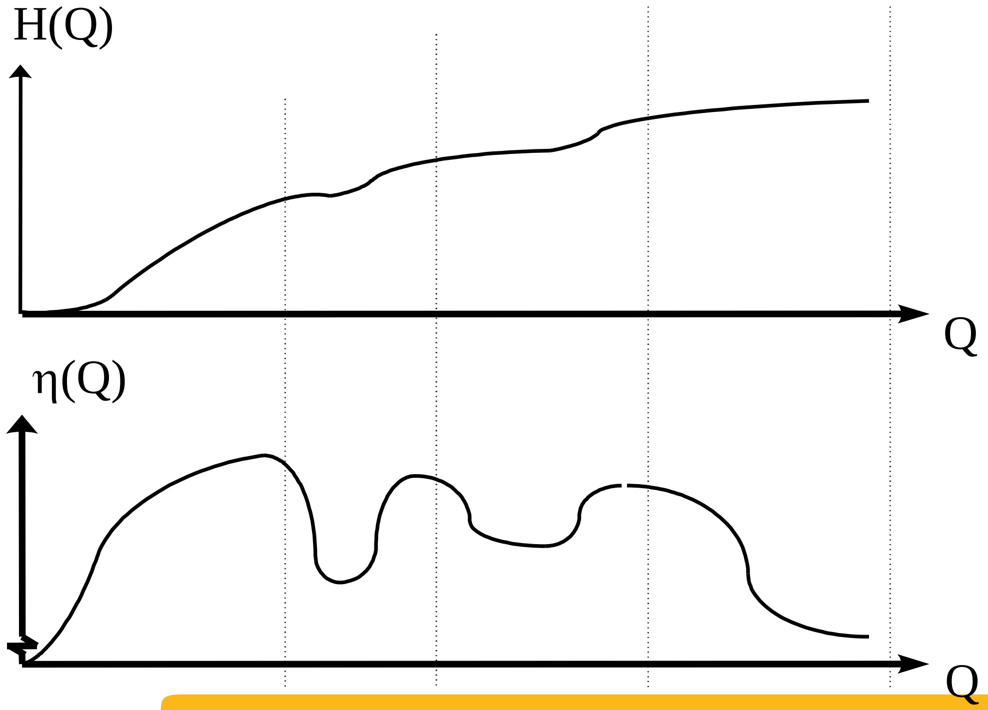
Hydro Power Electricity Production

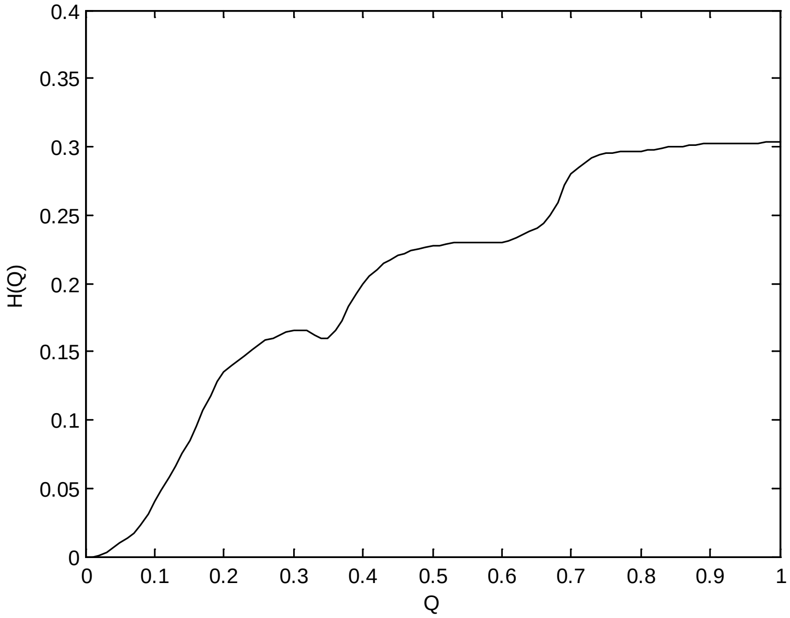
- Q, discharge
- $H(Q)$, power production function of discharge
- $\gamma(Q) = \frac{H(Q)}{Q}$, production equivalent (efficiency)
- $\mu(Q) = \frac{dH(Q)}{dQ}$, marginal production equivalent
  - Marginal production change for altered discharge

Hydro Power Electricity Production

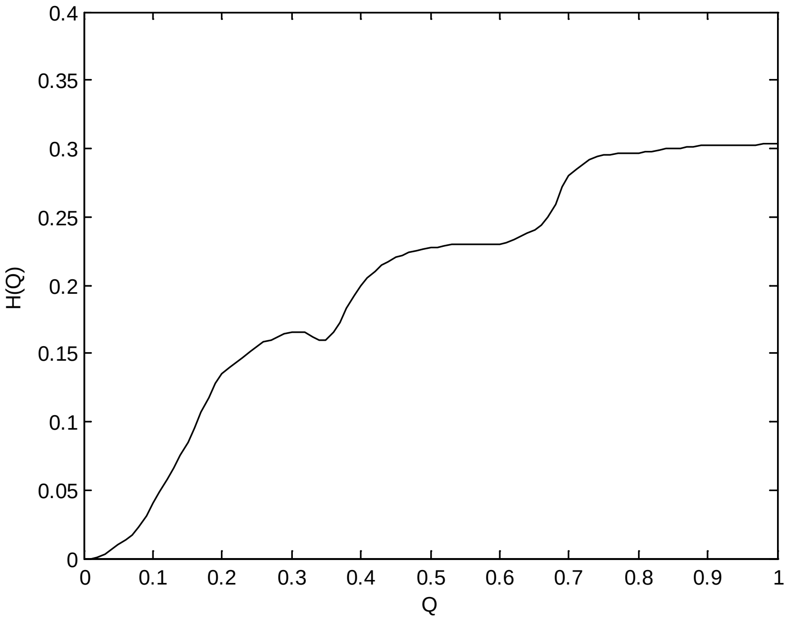
- $\eta(Q) = \frac{\gamma(Q)}{\gamma_{\max}}$, relative efficiency,
    - normalized production equivalent
    - $\gamma_{\max} = \max_{Q} \gamma(Q)$

Hydro Power Electricity Production



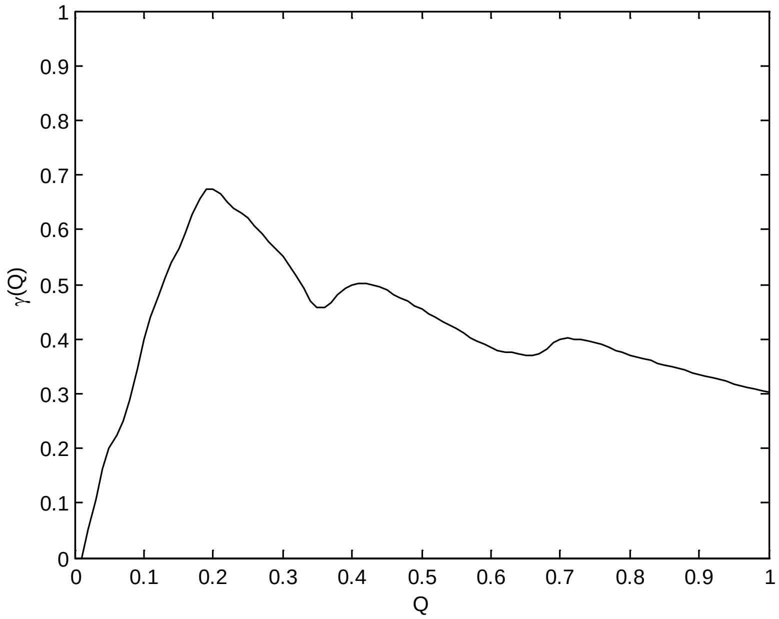


Hydro Power Electricity Production



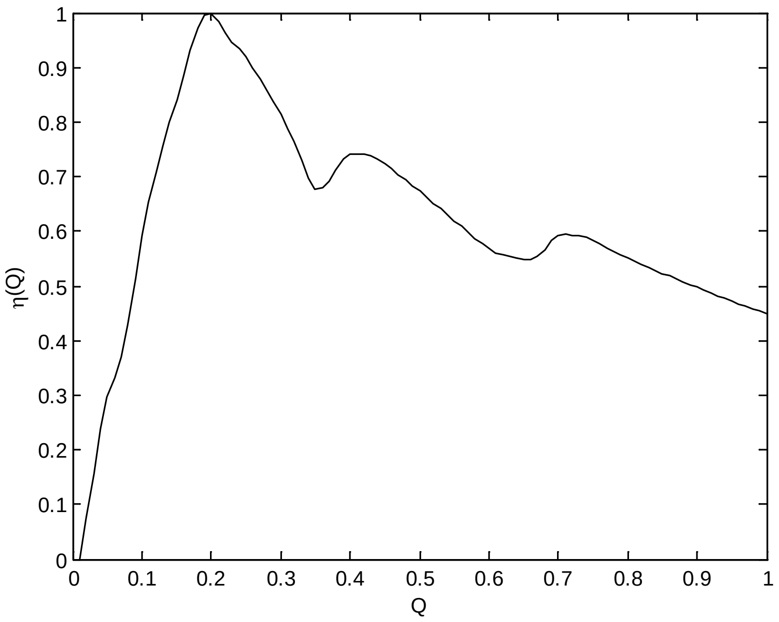
- $H(Q)$, power production function of discharge
- Not linear
- What to do?

Hydro Power Electricity Production



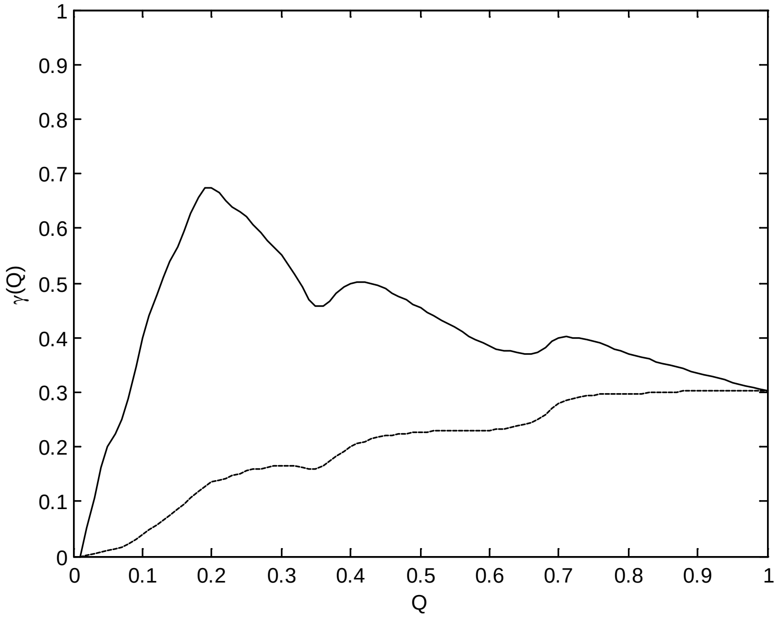
$\gamma(Q) = \frac{H(Q)}{Q}$, production equivalent (efficiency)

Hydro Power Electricity Production



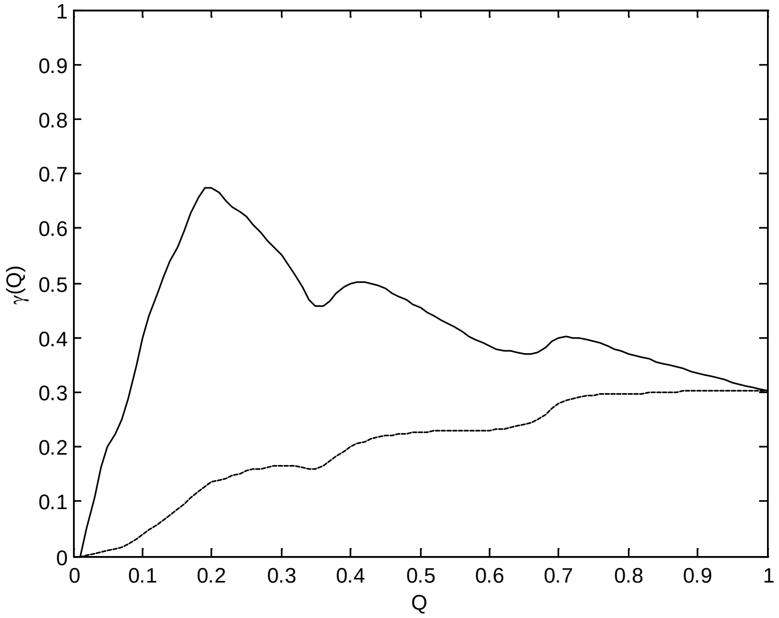
$\eta(Q) = \frac{\gamma(Q)}{\gamma_{\max}}$, relative efficiency,

Hydro Power Electricity Production



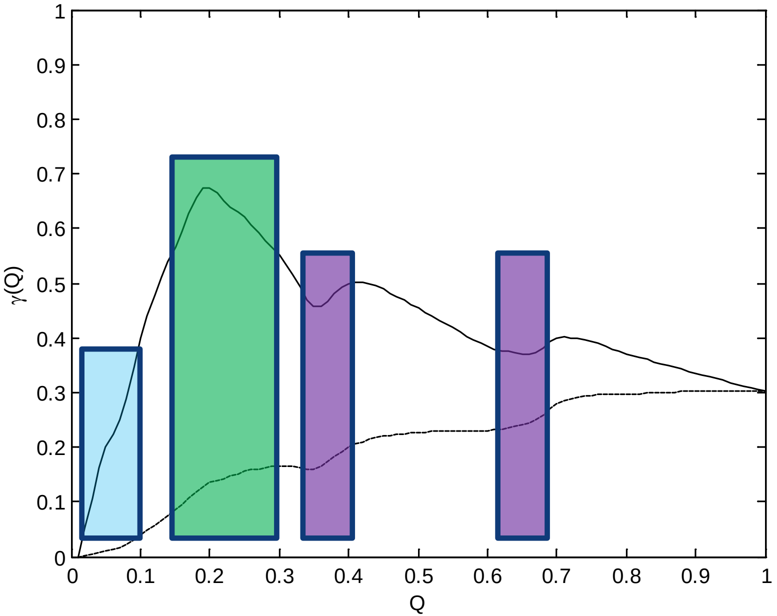
$\gamma(Q) = \frac{H(Q)}{Q}$, production equivalent (efficiency)

Hydro Power Electricity Production



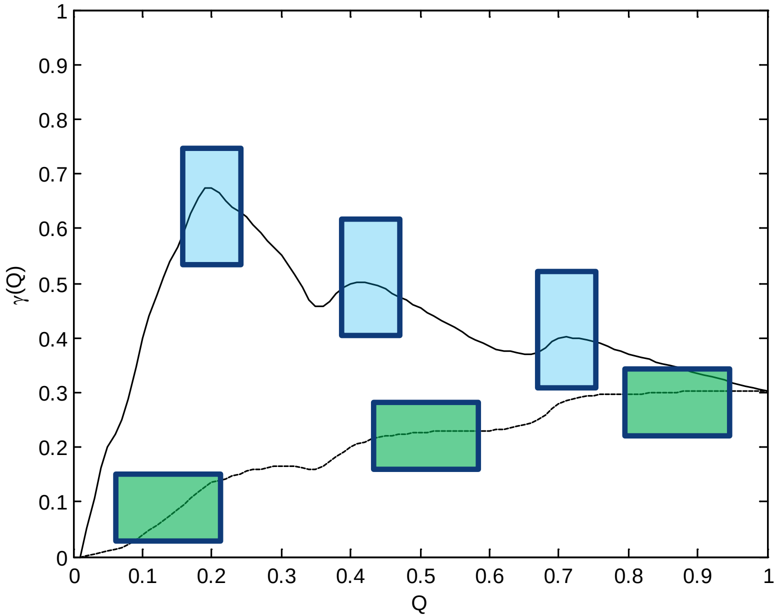
- $\gamma(Q) = \frac{H(Q)}{Q}$, production equivalent (efficiency)
- Why is this?
- Why not equally high peaks?

Hydro Power Electricity Production



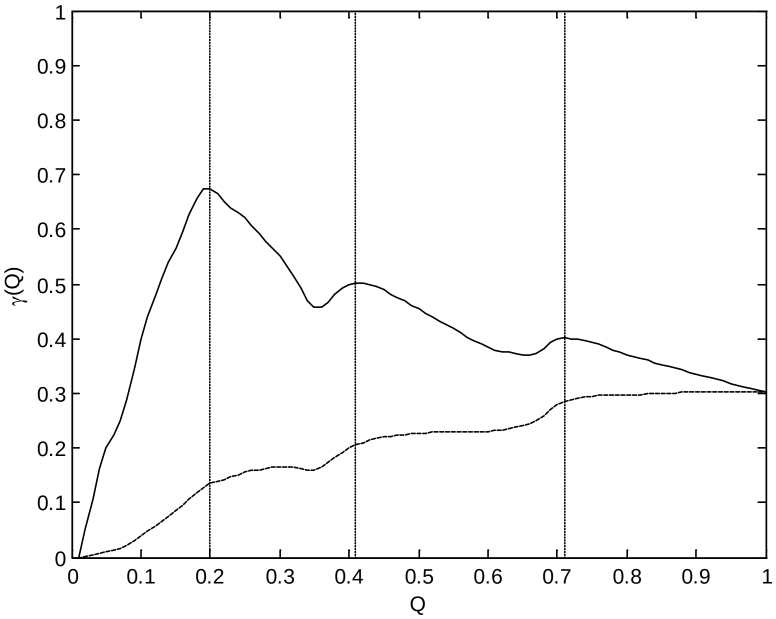
- $\gamma(Q) = \frac{H(Q)}{Q}$, production equivalent (efficiency)
- Why is this?
- Why not equally high peaks?
- Turbines
  - Needs some discharge
  - Peak efficiency a design parameter
  - Next turbine starts
  - (Thomas Sandberg, KTH)

Hydro Power Electricity Production



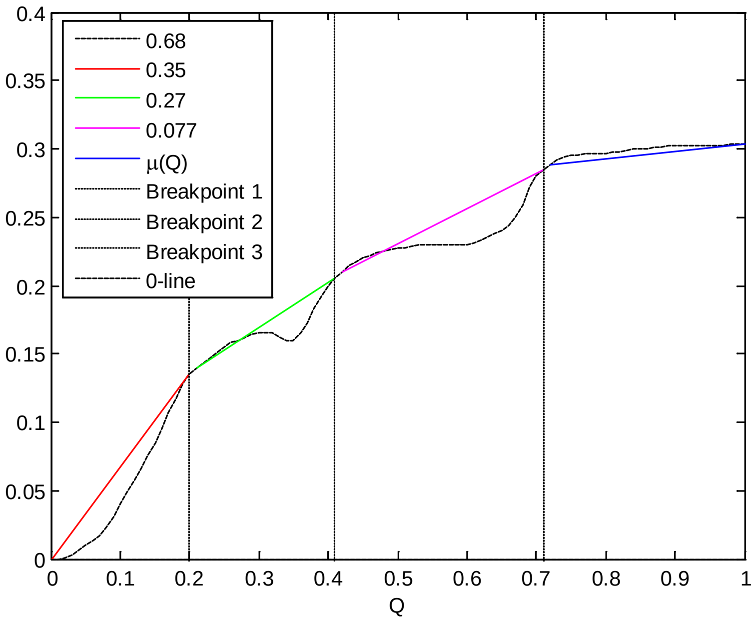
- $\gamma(Q) = \frac{H(Q)}{Q}$, production equivalent (efficiency)
- Why is this?
- Why not equally high peaks?
- Friction losses in tunnels
    - Headrace
    - Tailrace
    - $P_L \propto Q^2$
    - (Anders Wörman, KTH)
- But, $P_G \propto Q$

Hydro Power Electricity Production



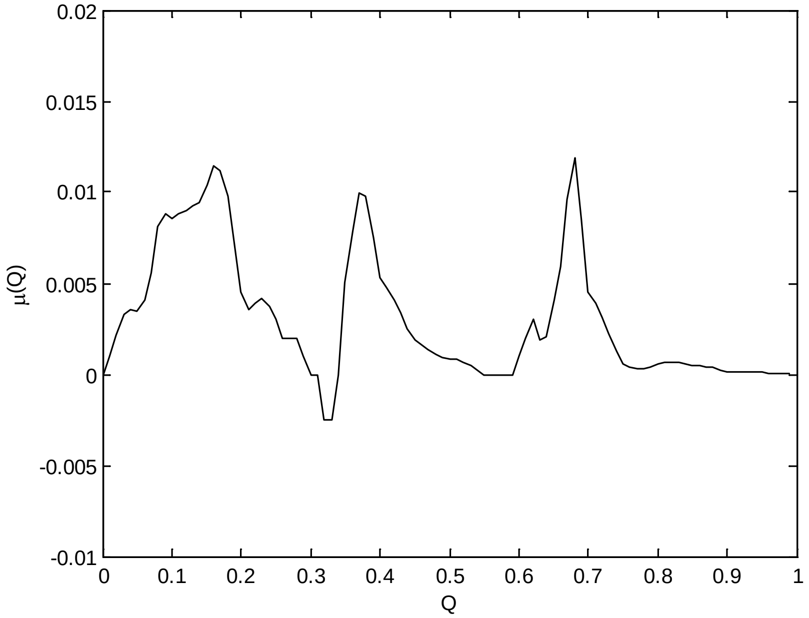
- $\gamma(Q) = \frac{H(Q)}{Q}$, production equivalent (efficiency)
- $H(Q)$, power production function of discharge
- Not linear
- What to do?
- Use efficiency peaks for segmentation

Hydro Power Electricity Production



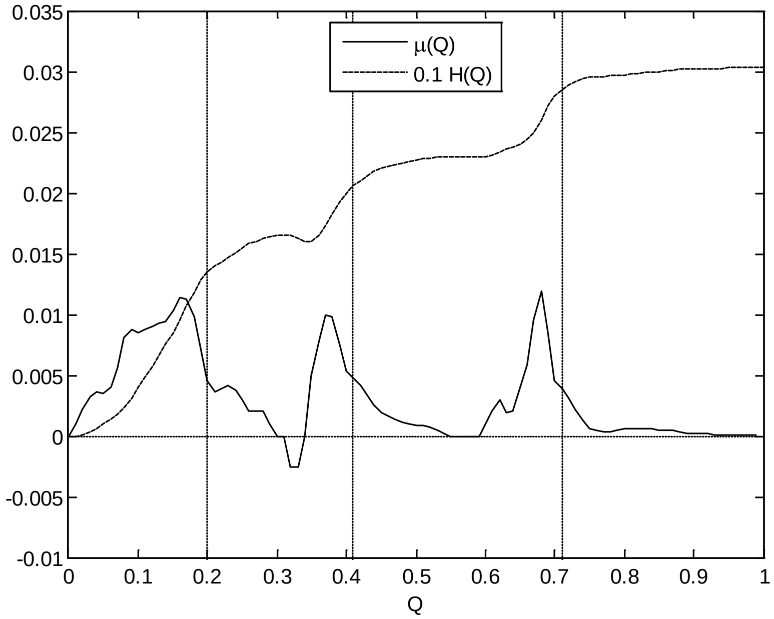
- $\gamma(Q) = \frac{H(Q)}{Q}$, production equivalent (efficiency)
- Use peaks for segmentation
- Piecewise linear approximation
- Each segment constant marginal production equivalent (slopes)
- Validity?

Hydro Power Electricity Production



$\mu(Q) = \frac{dH(Q)}{dQ}$, marginal production equivalent

Hydro Power Electricity Production



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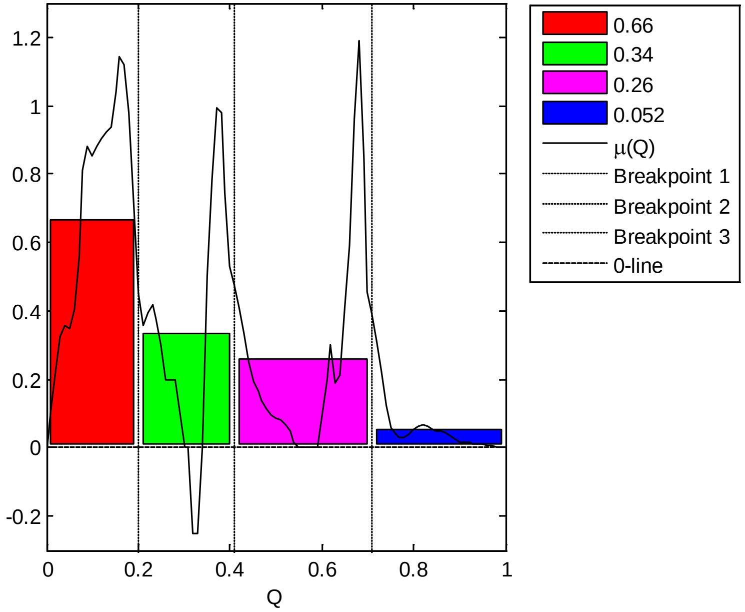
Hydro Power Electricity Production



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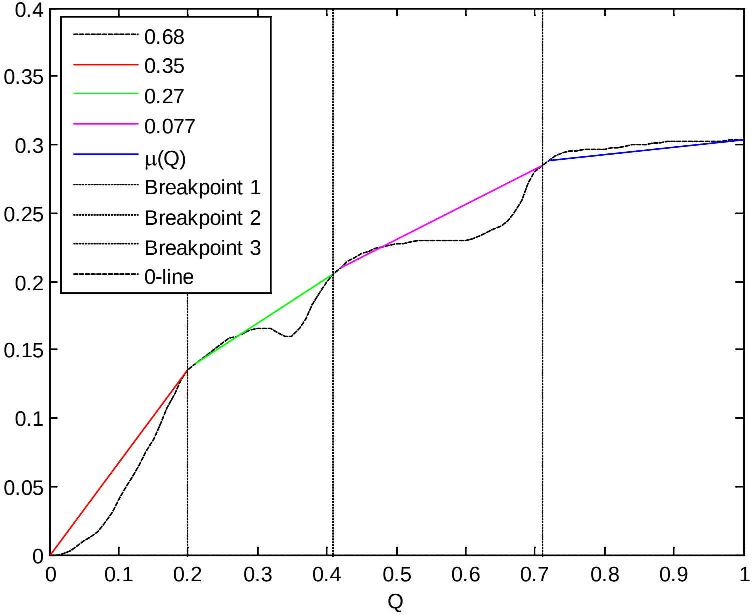
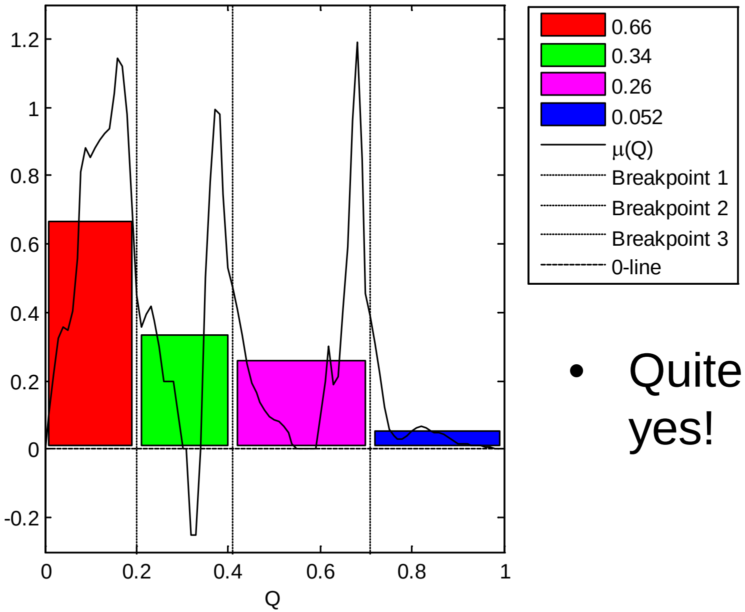
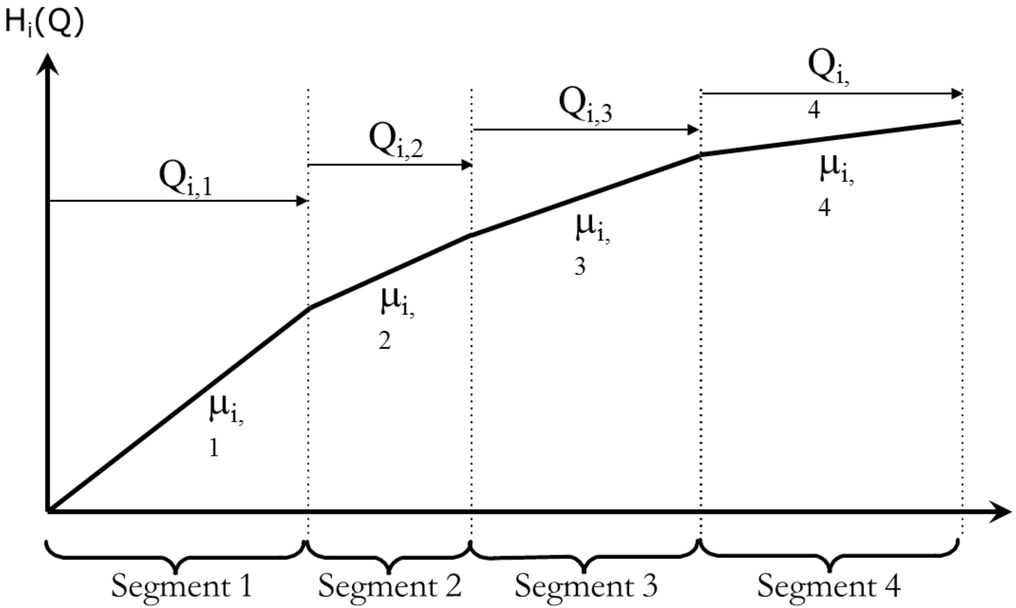
Hydro Power Electricity Production





- Quite valid – yes!

Hydro Power Electricity Production



Hydro Power Electricity Production

- Total discharge, power station i, hour t

$$Q_{i,t} = \sum_{j=1}^{n_i} Q_{i,j,t}$$

- $Q_{i,j,t}$, discharge, station i, segment j, hour t

Hydro Power Electricity Production

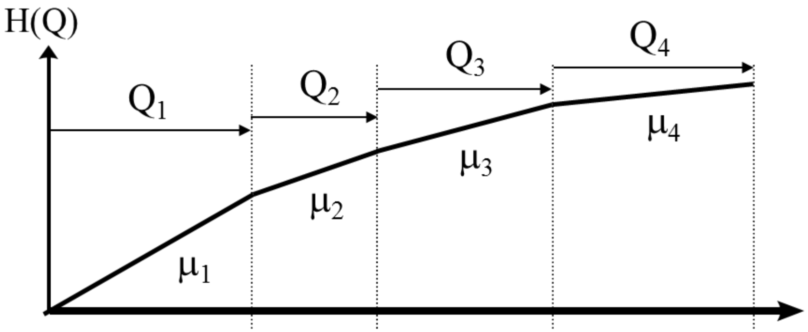
- Total power production, power station i, hour t

$$H_{i,t} = \sum_{j=1}^{n_i} \mu_{i,j} Q_{i,j,t}$$

- $\mu_{i,j}$, marginal production equivalent, station i, segment j

Hydro Power Electricity Production

- How to assure ordered segment activation?
- Not an issue here
- $\mu_{i,j} > \mu_{i,j+2}$
- We maximize income
- Last lecture we treated consumption
- Sometimes binaries not needed!

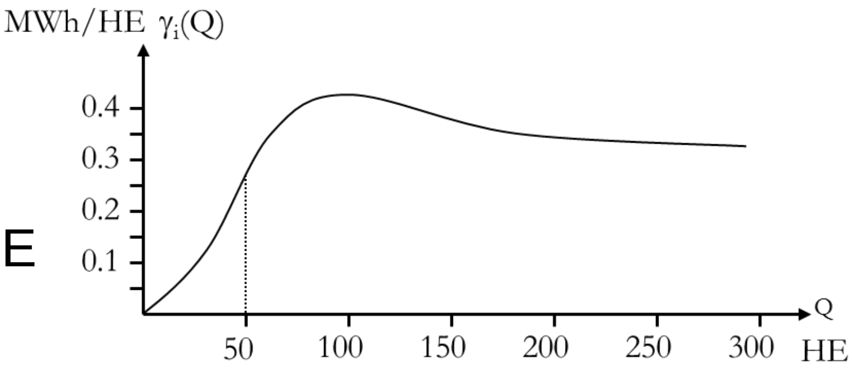


Forbidden discharges (1/5)

- Small discharges – low efficiency
- Not considered in piecewise linear model
- *Primarily* a problem in first segment
- Avoid low discharges
- Introduce binary variables

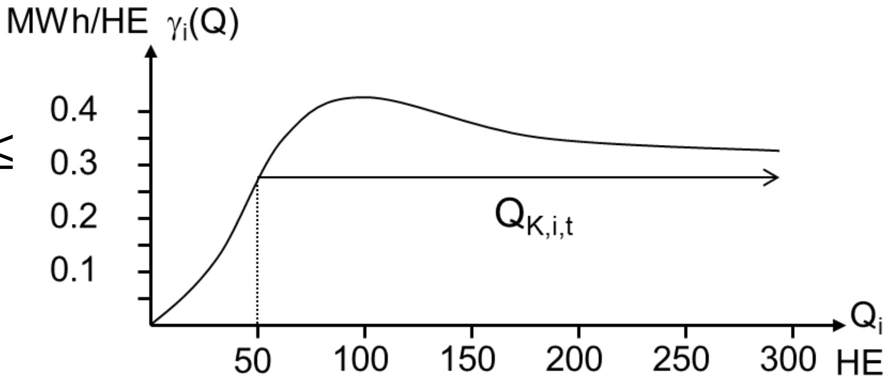
Forbidden discharges (2/5)

- Production equivalent according to figure
- Compare with slide 16
- Low efficiency below 50 HE
- Either above 50 HE or nothing
- Piecewise linear model



Forbidden discharges (3/5)

- Two segments
  - The origin
  - Discharges for $50\ \mathrm{HE} \leq Q \leq 300\ \mathrm{HE}$
- Assume (for simplicity) constant marginal production equivalent, μ
- Average of remaining allowed interval
- Question: Higher or lower than before?



Forbidden discharges (4/5)

- Binary variables, $z_{i,t}$, each segment and time-step
- $z = \begin{cases} 0, & Q = 0 \text{ HE} \\ 1, & Q \geq 50 \text{ HE} \end{cases}$
- Total discharge and production given by:

$$Q_{i,t} = 50 z_{i,t} + Q_{K,i,t}$$

$$H_{i,t} = 50 \mu_{K,i,t} z_{i,t} + \mu_{K,i} Q_{K,i,t}$$

Forbidden discharges (5/5)

- Constraints ensuring no discharge when $z = 0$
- $Q_{K,i,t} \leq \overline{Q_{K,i}} z_{i,t}$
- Variable limits:

$$0 \leq Q_{K,i,t}$$

$$z_{i,t} \in \{0,1\}$$

The planning problem (1/3)

- Maximize
  - Incomes during period
  - Assets after end of period
- Minimize
  - Production costs during period
- Subject to
  - Hydrological coupling
  - Laws and regulations
  - (Other) physical limitations

The planning problem (2/3)

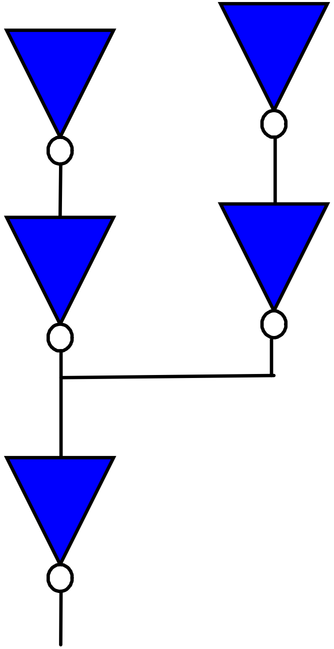
- Production costs neglectable
  - Very small
  - Remember from electricity pricing?
- Incomes during period
  - Sales on spot market
  - Bilateral sales to customer
  - $\sum_{t=1}^{T} \lambda_t H_{i,t}$
  - Where, λ_t, denotes the price

The planning problem (3/3)

- Assets after end of period
  - Stored water
  - Expected future price
  - Expected future production
- Value of stored water at station i
  - $B_i(M_{i,T}) = \lambda_e M_{i,T} \sum_{j \in N_i} \gamma_j$
  - λ_e, expected future price
  - $M_{i,T}$, reservoir content at end of period
  - N_i, set of indices for downstream stations
  - γ_j, expected future production equivalent

Hydrologic Coupling (1/5)

- Hydropower stations in a river dependent
- Operation of a station affect others
- The interrelations need to be considered

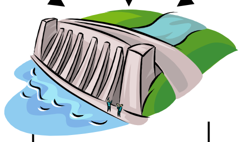


Hydrologic Coupling (2/5)

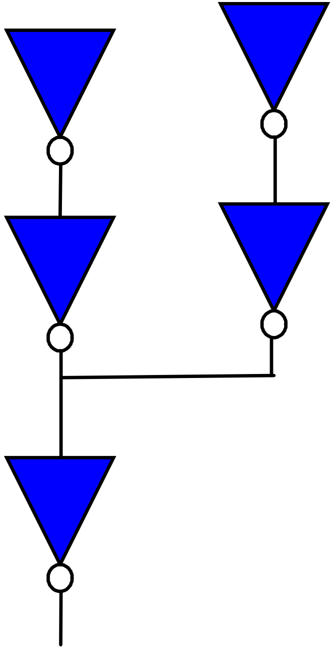
Spillage from stations *directly* upstream

Discharge from stations *directly* upstream

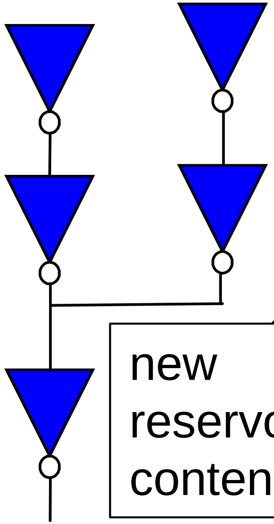
Inflow from surrounding area



Discharge Spillage



Hydrologic Coupling (3/5)



- Hydrologic balance in one reservoir
- $M_{i,t} = M_{i,t-1} - Q_{i,t} - S_{i,t} + \sum_{j \in K_i} Q_{j,t-\tau_{j,i}} + \sum_{j \in K_i} S_{j,t-\tau_{j,i}} + V_{i,t}$

$M_{i,t}$: new reservoir contents

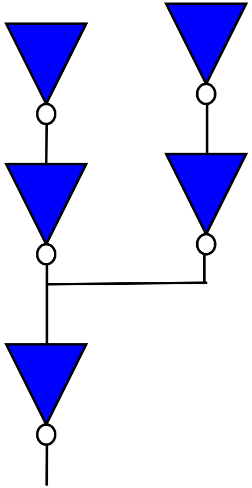
$M_{i,t-1}$: old reservoir contents

$Q_{i,t}$, $S_{i,t}$: water flowing out of reservoir

$\sum_{j \in K_i} Q_{j,t-\tau_{j,i}}$, $\sum_{j \in K_i} S_{j,t-\tau_{j,i}}$: water flowing into reservoir

$V_{i,t}$: Inflow from surrounding area

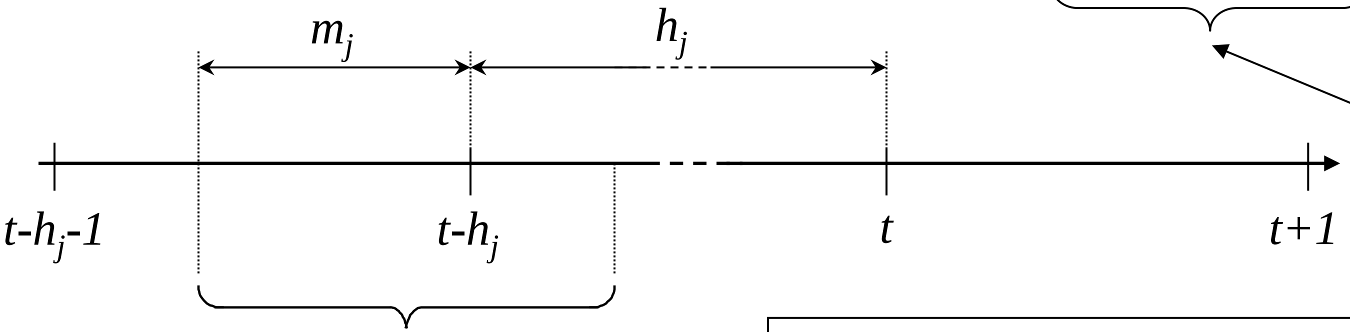
Hydrologic Coupling (4/5)



- Time consumed water flowing between stations j, i, $\tau_{j,i}$
- From j to closest downstream station i
- Complicated relations of $\tau_{j,i}$
  - Water flows
  - Reservoir levels
  - Etc.
- Assume constant water delay times, τ_j
  - h_j hours, m_j minutes

Hydrologic Coupling (5/5)

$$Q_{j,t-\tau_{j,i}} = \frac{m_j}{60} Q_{j,t-h_j-1} + \frac{60-m_j}{60} Q_{j,k-h_j}$$



Discharge & spillage this hour will arrive hour t

Weighted mean value of discharge and spillage h_j+1 and h_j hours earlier

Legal and physical constraints (1/2)

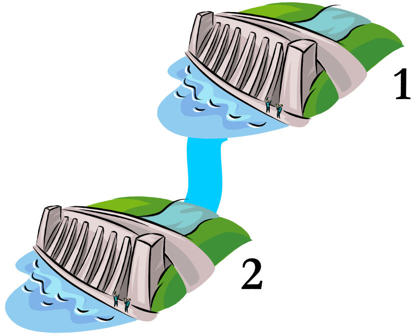
- Power station limitations
  - Laws and regulations
    - water-rights judgement
    - water-rights judgement
    - etc.
  - Physical limitations
- Variable limits
  - $\underline{Q}_i \leq Q_{i,t} \leq \bar{Q}_i$
  - $\underline{M}_i \leq M_{i,t} \leq \bar{M}_i$
  - $\underline{M}_{i,T} \leq M_{i,T} \leq \bar{M}_{i,T}$
  - $\underline{M}_{i,T}$ and $\bar{M}_{i,T}$ could be more constraining

Legal and physical constraints (2/2)

- Contracts with customer(s)
- $\sum_{i \in I} H_{i,t} \geq D_t$
- Equality also occurs
- $H_{i,t}$, production
- D_t, contracted load

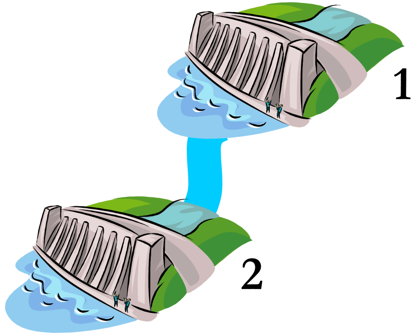
Planning problem – Example (1/7)

- Two hydropower station
  - Located after each other in river
  - All power sold on power exchange
  - Plan the 6 following hours
- Known
  - Expected price, λ_t, for $t = 1,2,\dots,6$
  - Stored water after $t = 6$ sold at λ_f
  - Reservoirs are half full at start



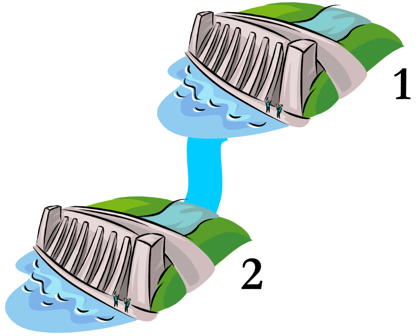
Planning problem – Example (2/7)

- Known
  - Installed capacity, $\overline{H}_i$, $i = 1{,}2$
  - Maximal discharge, $\overline{Q}_i$, $i = 1{,}2$
  - Maximal reservoir contents, $\overline{M}_i$, $i = 1{,}2$
  - Local hydro inflow, V_i, $i = 1{,}2$
- Assumptions
  - Constant efficiency, $\gamma(Q) = \frac{H(Q)}{Q} = \frac{\gamma Q}{Q} = \gamma$
  - Gives linear production function, $H(Q) = \gamma Q$



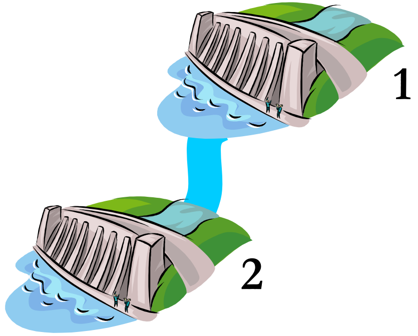
Planning problem – Example (3/7)

- Assumptions
  - Constant efficiency, $\gamma(Q) = \frac{H(Q)}{Q} = \frac{\gamma Q}{Q} = \gamma$
  - Gives linear production function, $H(Q) = \gamma Q$
  - Gives constant production equivalent, $\mu(Q) = \frac{d\gamma Q}{dQ} = \gamma = \mu$
  - Installed capacity reached at maximal discharge
  - Gives $\gamma_i = \frac{\overline{H}_i}{\overline{Q}_i}, i = 1,2$



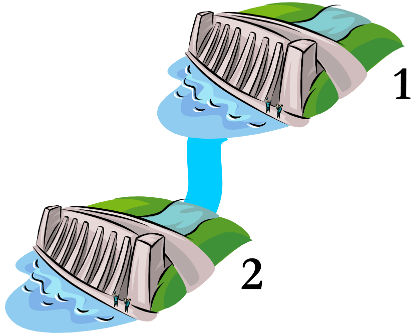
Planning problem – Example (4/7)

- Solution
  - Maximize
    - Income from sold power
    - Value of stored water
  - Subject to
    - Hydrological coupling



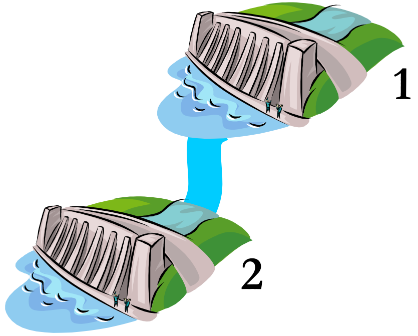
Planning problem – Example (5/7)

- Variables
  - Discharge, $Q_{i,t}, i \in \{1,2\}, t \in \{1,2,\dots,6\}$
  - Spillage, $S_{i,t}, i \in \{1,2\}, t \in \{1,2,\dots,6\}$
  - Reservoir content end of period t, $M_{i,t}, i \in \{1,2\}, t \in \{1,2,\dots,6\}$



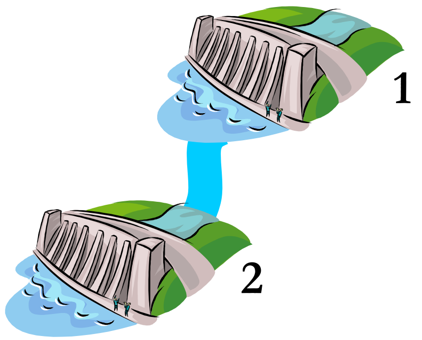
Planning problem – Example (6/7)

- Objective function
  - Sold electricity, $\sum_{t=1}^{6} \lambda_t \sum_{i=1}^{2} \gamma_i Q_{i,t}$
  - Stored water, $\lambda_e\left[(\gamma_1 + \gamma_2)M_{1,6} + \gamma_2 M_{2,6}\right]$
  - Altogether:
  - $\sum_{t=1}^{6} \lambda_t \sum_{i=1}^{2} \gamma_i Q_{i,t} +$
    $\lambda_e\left[(\gamma_1 + \gamma_2)M_{1,6} + \gamma_2 M_{2,6}\right]$



Planning problem – Example (7/7)

- Hydrologic constraints
  - $M_{1,t} = M_{1,t-1} - Q_{1,t} - S_{1,t} + V_1$
  - $M_{2,t} = M_{2,t-1} - Q_{2,t} - S_{2,t} + Q_{1,t} + S_{1,t} + V_2$
- Variable limits
  - $0 \leq Q_{i,t} \leq \bar{Q}_i$
  - $0 \leq S_{i,t}$
  - $0 \leq M_{i,t} \leq \bar{M}_i$
- Where, $M_{i,0} = 0.5\bar{M}_i$



Typical exam question, part 1 (1/2)

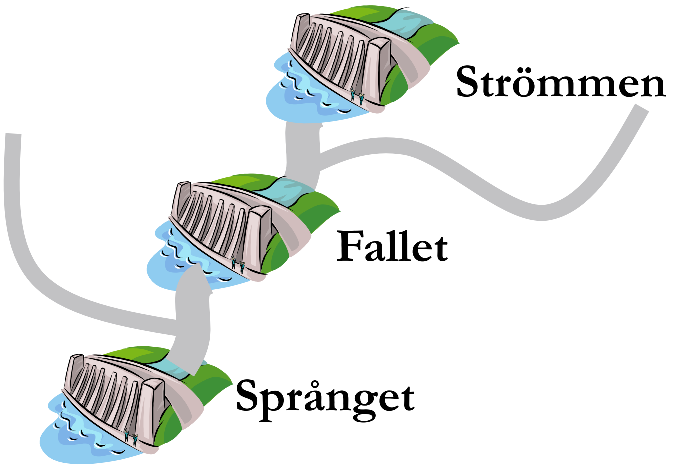
- Hydro station Språnget
  - Maximal discharge, 100 $\frac{m^3}{s}$
  - Best efficiency at discharge, 70 $\frac{m^3}{s}$
  - Maximal discharge, installed capacity generated, 20.8 MW
  - At best efficiency, 15.4 MW generated

Typical exam question, part 1 (2/2)

- Assume
  - Piecewise linear generation model needed
  - Two segments needed
  - Breaking point at best efficiency
- Compute:
  - Marginal production equivalent for each segment
  - Maximal discharge, each segment

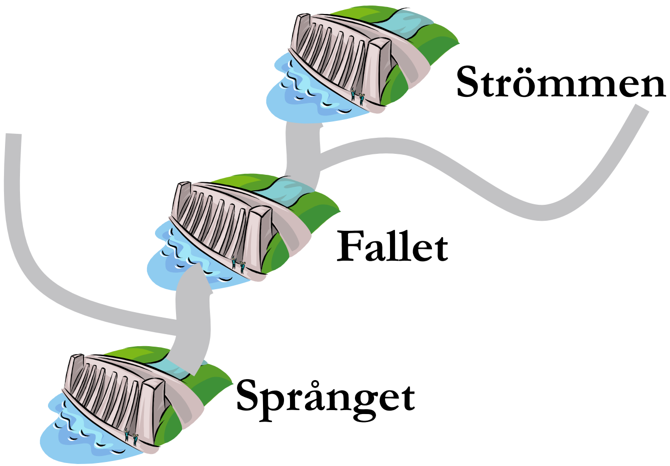
Typical exam question, part 2

- Power plant indices, $i \in \{1,2,3\}$
  - Strömmen 1,
  - Fallet 2,
  - Språnget 3
- Segment indices, $j \in \{1,2\}$
- Time (hour) indices, $i \in \{1,2,\ldots,24\}$

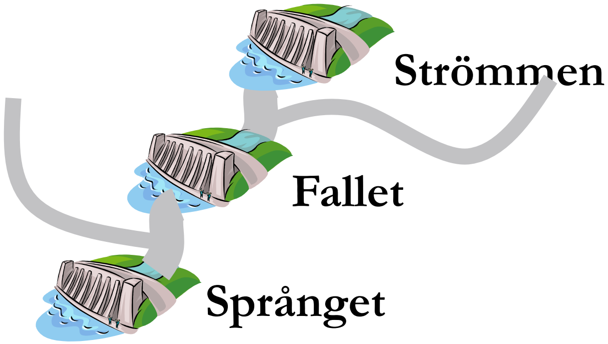


Typical exam question, part 2

- Given
  - $M_{i,0}$, reservoir content at start
  - $M_{i,t}$, reservoir content, end of period t
  - $Q_{i,j,t}$, discharge, plant i, segment j, during t



Typical exam question, part 2



- Given
  - $M_{i,0}$, reservoir content at start
  - $M_{i,t}$, reservoir content at end of t
  - $Q_{i,j,t}$, discharge, plant i, segment j, during t
  - $S_{i,t}$, spillage, reservoir i, during t
  - $V_{i,t}$, local inflow, reservoir i, during t
- Using given information
  - Formulate hydrological constraints
  - Neglect water delay times

End of lecture 8

Next time **thermal** short-term planning