# System planning, EG2050 Simulation of electricity markets – L11

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## Aim of power market simulation

- Estimate how the market will work in the future based on
  - Production and transmission resources
  - Load
  - Rules for power trading
- Selected simulation model depends on aim of study
- The result considers what **could** happen, not what **will** happen



## What is the ”knowledge” of future?

- About available resources: Some estimation of capacities and availability
- About consumption: Some estimation of possible levels in different areas or nodes

**Consequence:**

- A probabilistic modelling is needed



## About the **past** and the **future**

The **past** is about **statistics** while the **future** is about **probabilistics**

For the **past** there is **information**
While for the **future** an **estimation**

# Probability distribution

## Density function

**Definition C.1.** The probability that an observation of $X$ belongs to a given set $\mathcal{X}$ is given by the density function $f_X(x)$, i.e.,

$$P(X \in \mathcal{X}) = \int_{\mathcal{X}} f(x)dx.$$

## For descrete random variables

$$f_X(x) = P(X=x)$$



# Distribution functions

## Distribution function

**Definition C.2.** The probability that $X \leq x$ is given by the distribution function $F_X(x)$, i.e., $F_X(x) = P(X \leq x)$.

## Duration curve

**Definition C.3.** The probability that $X > x$ is given by the duration curve $\tilde{F}_X(x)$, i.e., $\tilde{F}_X(x) = P(X > x)$.



# Probability distribution relations

**Theorem C.4.** The density function, the distribution function and the duration curve are related through

$$\tilde{F}_X(x) = 1 - F_X(x) = 1 - \int_{-\infty}^{x} f_X(t)dt = \int_{x}^{\infty} f_X(t)dt.$$



# Convolution

**Theorem C.5.** The density function of the sum, $Z$, of two independent random variables $X$ and $Y$ can be obtained by a convolution formula. For independent discrete random variables the convolution formula is written as

$$f_Z(x) = \sum_i f_X(i) f_Y(x-i)$$

## Expectation value and variance

***Definition C.6.*** The expectation value of a random variable $X$ is given by

$$E[X] = \int_{-\infty}^{\infty} x f_X(x)dx.$$

***Definition C.7.*** The variance of a random variable $X$ is equal to the expected square of the deviation from the average, i.e.,

$$Var[X] = E[(X - E[X])^2] = \int_{-\infty}^{\infty} (x - E[x])^2 f_X(x)dx.$$

Another common measure is the standard deviation, which simply is the square root of the variance.



## Covariance and correlation

***Definition C.11.*** The covariance of two random variables $X$ and $Y$ is given by

$$Cov[X, Y] = E[(X - E[X])(Y - E[Y])].$$

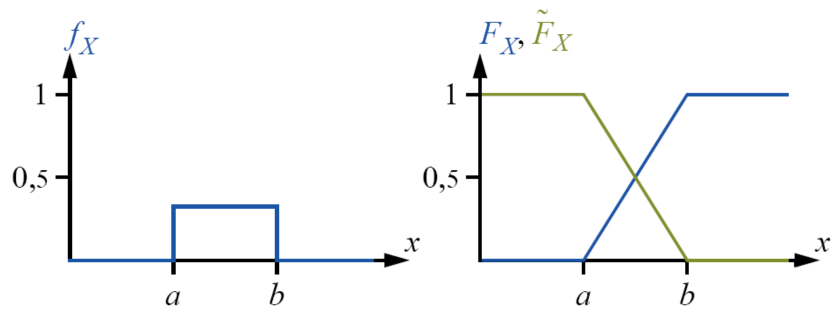
***Definition C.12.*** The correlation coefficient of two random variables $X$ and $Y$ is given by

$$\rho(X, Y) = \frac{Cov[X, Y]}{\sqrt{Var[X]Var[Y]}}.$$



## Probability distributions - Equal distributed

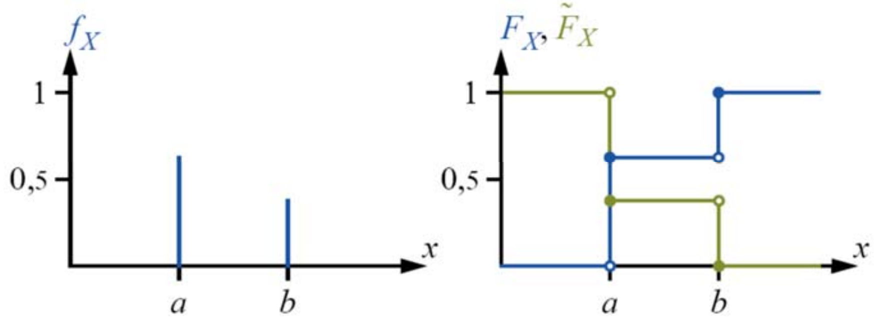
$X \in U(a, b) \Rightarrow$





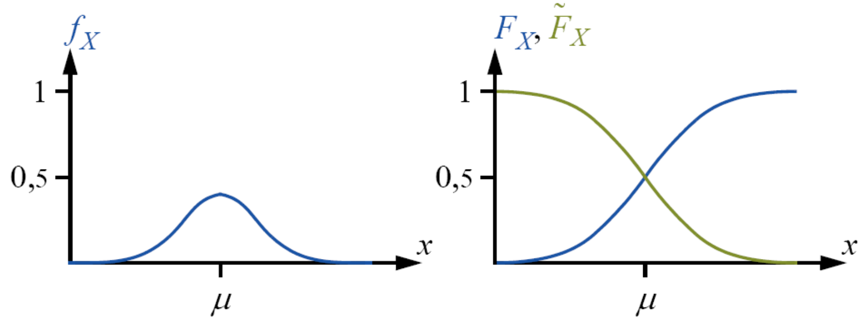

## Probability distributions - Two point distribution

$X$ can have two discrete values



## Probability distributions
## - Normal (Gaussian) distribution

$X \in N(\mu, \sigma) \Rightarrow$

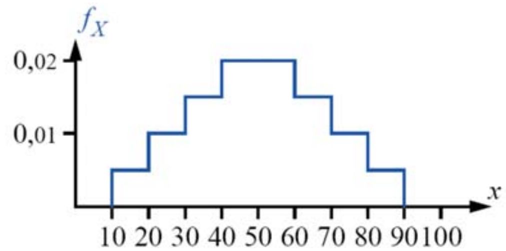





## Probability distributions
## - Example

Are the following statements true or false?

- X is a discrete variable
- E[X]=50
- P(X<20)=0,5%

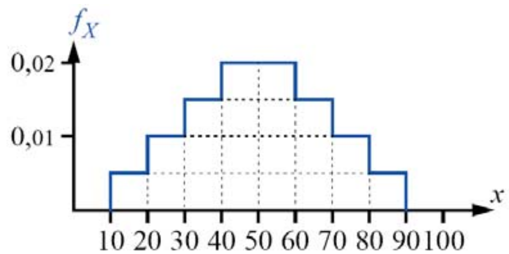





## Probability distributions
## - Example

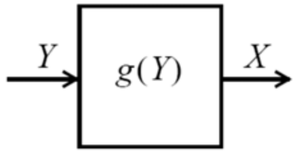
Are the following statements true or false?

- X is a discrete variable. FALSE!
- E[X]=50, TRUE!
- P(X<20)=0,5% FALSE!





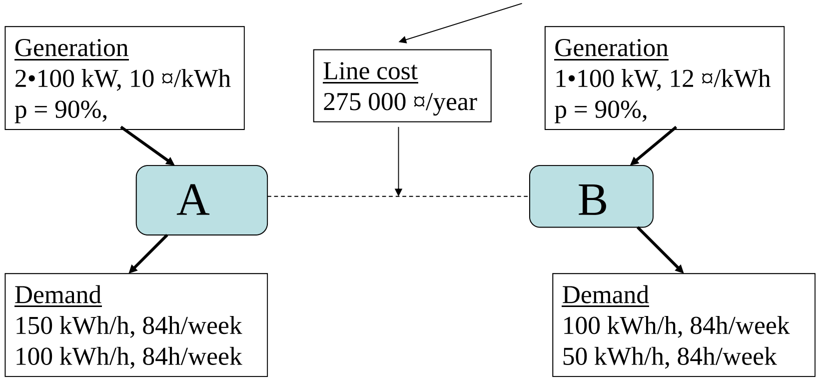

## Power market simulation
## - Structure




- Scenario parameter, Y
  - Vector with stochastic variables with known parameters
- Market model, g
  - Mathematical model of the power market
- Result variable, X
  - Vector with stochastic variables with unknown distribution

## Power market simulations - Value of an investment

Example 6.1: Value of transmission line






## Power market simulation - Structure

- The aim of the simulations is to estimate the probability distribution of the result variables
- In many cases it is enough to study some key values (system indices)
- A system index is a statistical measure (often the expected value) of a result variable



## Power market simulation - System index - 1

- ***Definition 6.1.*** The Total Operation Cost, *TOC*, is the sum of the operation cost in all power plants, as well as any other operation costs (as for example compensation for consumers who have been involuntarily disconnected).
- ***Definition 6.2.*** The Expected Total Operation Cost, *ETOC*, is the expected sum of the operation cost in all power plants, as well as any other operation cost, i.e., $ETOC = E[TOC]$.



## Power market simulation - System index - 2

- ***Definition 6.3.*** The result variable *LOLO* (Loss of Load Occasion) is a binary variable which is equal to one if load shedding occurs (i.e., if at least one consumer has been involuntarily disconnected due to capacity limitations in the system) and zero otherwise.
- ***Definition 6.4.*** The risk of power deficit, *LOLP* (Loss of Load Probability) is equal to the probability that at least one consumer is involuntarily disconnected due to capacity limitations in the system, i.e., $LOLP = E[LOLO]$.

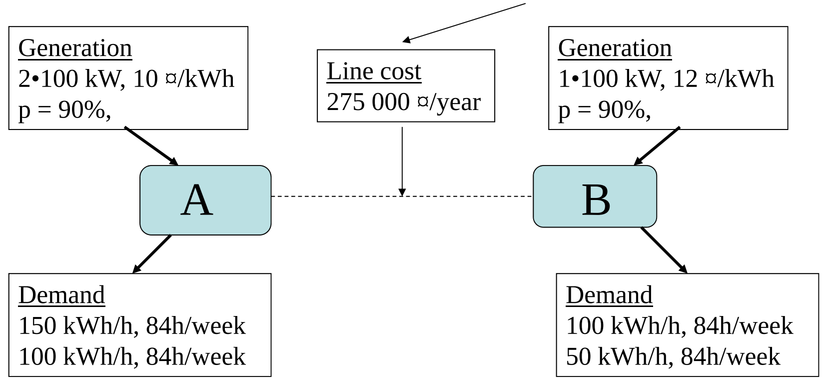
# Power market simulation - System index - 3

- ***Definition 6.5.*** The unserved energy, *ENS* (Energy Not Served) is the energy that could not be delivered due to capacity limitations in the power system.
- ***Definition 6.6.*** The expected unserved energy, *EENS*, is the expected energy that could not be delivered due to capacity limitations in the power system, i.e., $EENS = E[ENS]$.



# Power market simulations - Value of an investment

Example 6.1: Value of transmission line






# Power market simulations - Value of an investment

Example 6.1

**Table 6.1** System index with and without the transmission line.

| System index | No transmission line | With the transmission line |
|---|---|---|
| *ETOC* [¤/h] | 2 002.50 | 1 972.80 |
| $LOLP_A$ [%] | 10 | 14.5 |
| $LOLP_B$ [%] | 10 | |



# Power market simulations - Value of an investment

<u>Example 6.1 (cont)</u>

- The investment cost for a new transmission line between region A and B is 275000 ¤/year
- The yearly operation cost decreases with 260000 ¤/year
- The risk for capacity deficit decreases from 19% to 14.5%, i.e. With about 394 h/year

# Power market simulations - Capacity credit of a plant

- **Definition:** Capacity credit means the possibility of a power plant to increase the reliability (decrease in LOLP) of the system.
- **Question:** Is there any capacity credit for wind power?

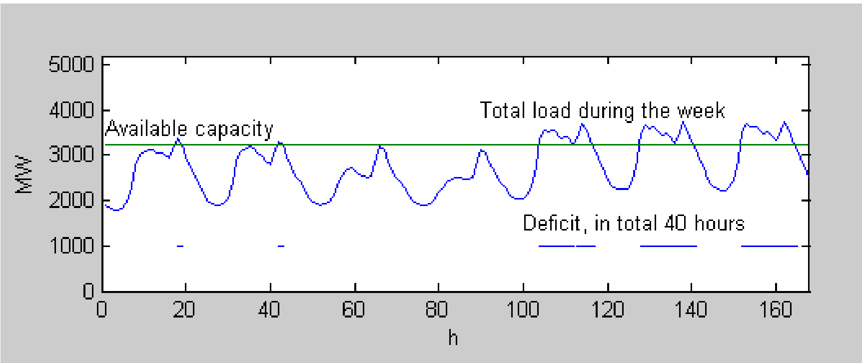


# Wind Power Capacity Credit
## (expressed as equivalent load increase)

How much can the consumption increase when the amount of wind power increases and the risk of power deficit is kept constant?



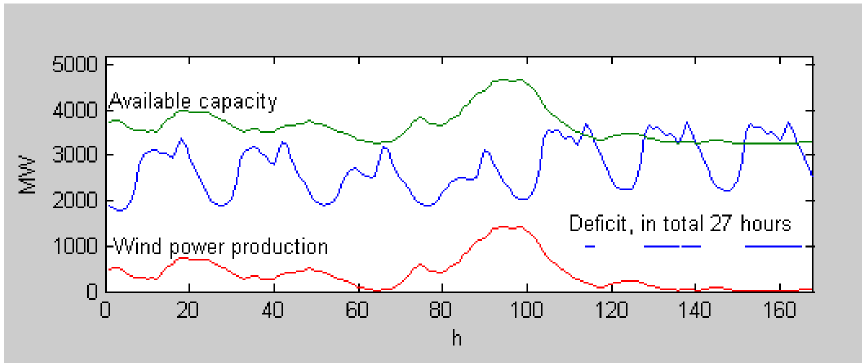
# Wind power capacity credit - 1




No wind power

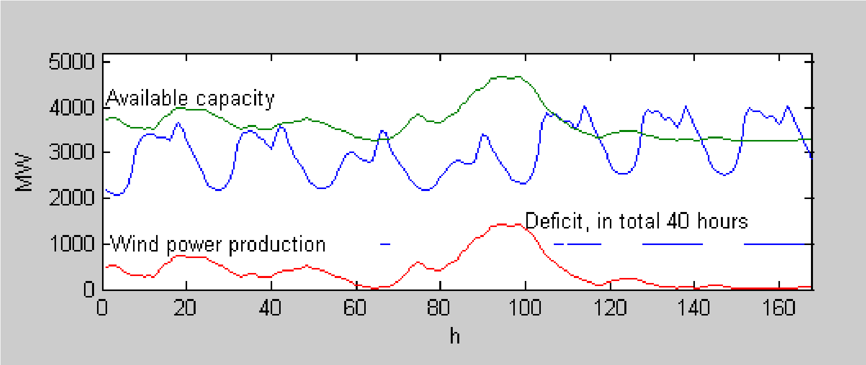


# Wind power capacity credit - 2




With wind power

# Wind power capacity credit - 3




With wind power, load + 300 MW



# Capacity credit of wind power

- **”True” value**: Considers the possibility of wind power increase the reliability of the power system, ≈ 20% of installed capacity.
- **Market value**: High market prices when there is a risk for power deficit.