# DD2476 Search Engines and Information Retrieval Systems Lecture 4: Scoring, Weighting, Vector Space Model


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## Assignment 1

- Thus far, Boolean queries:

  BRUTUS AND CAESAR AND NOT CALPURNIA

- Good for:
  - Expert users with precise understanding of their needs and the collection
  - Computer applications
- Not good for:
  - The majority of users

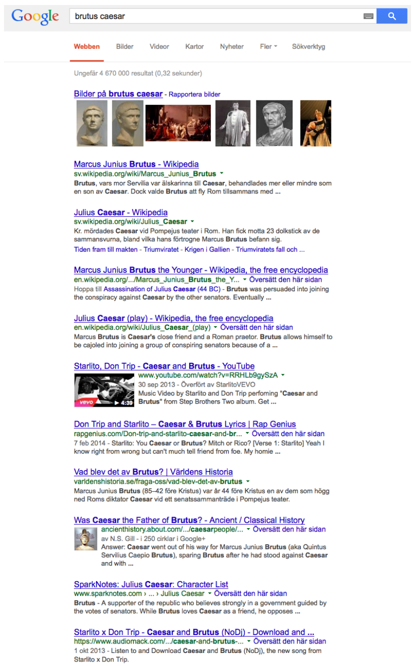


## Problem with Boolean Search: Feast or Famine

- Experienced maybe in Task 1.5?
- Boolean queries often result in either too few (=0) or too many (1000s) results.
  - Query 1: "*standard user dlink 650*": 200,000 hits
  - Query 2: "*standard user dlink 650 no card found*": 0 hits
- It takes a lot of skill to come up with a query that produces a manageable number of hits.
  - AND gives too few; OR gives too many



## Ranked Retrieval



## Feast or Famine: Not a Problem in Ranked Retrieval

- Large result sets no issue
  - Show top $K$ ( ≈ 10) results
  - Option to see more results
- Premise: the ranking algorithm works well enough



## Today

- Tf-idf and the vector space model (Manning Chapter 6)
  - Term frequency, collection statistics
  - Vector space scoring and ranking
- Efficient scoring and ranking (Manning Chapter 7)
  - Speeding up vector space scoring and ranking



# Tf-idf and the Vector Space Model

(Manning Chapter 6)

## Scoring as the Basis of Ranked Retrieval

- Wish to return in order the documents most likely to be useful to the searcher
- Rank-order documents with respect to a query
- Assign a score – say in [0, 1] – to each document
  - Measures how well document and query "match"

## Query-Document Matching Scores

- One-term query:

  BRUTUS

- Term not in document: score 0
- More appearances of term in document: higher score



## Recall (Lecture 1): Binary Term-Document Incidence Matrix

| | Antony and Cleopatra | Julius Caesar | The Tempest | Hamlet | Othello | Macbeth |
|---|---|---|---|---|---|---|
| ANTONY | 1 | 1 | 0 | 0 | 0 | 0 |
| BRUTUS | 1 | 1 | 0 | 1 | 0 | 0 |
| CAESAR | 1 | 1 | 0 | 1 | 1 | 1 |
| CALPURNIA | 0 | 1 | 0 | 0 | 0 | 0 |
| CLEOPATRA | 1 | 0 | 0 | 0 | 0 | 0 |
| MERCY | 1 | 0 | 1 | 1 | 1 | 1 |
| WORSER | 1 | 0 | 1 | 1 | 1 | 0 |

- Document represented by binary vector $\in \{0,1\}^{|V|}$



## Term-Document Count Matrix

| | Antony and Cleopatra | Julius Caesar | The Tempest | Hamlet | Othello | Macbeth |
|---|---|---|---|---|---|---|
| ANTONY | 157 | 73 | 0 | 0 | 0 | 0 |
| BRUTUS | 4 | 157 | 0 | 1 | 0 | 0 |
| CAESAR | 232 | 227 | 0 | 2 | 1 | 1 |
| CALPURNIA | 0 | 10 | 0 | 0 | 0 | 0 |
| CLEOPATRA | 57 | 0 | 0 | 0 | 0 | 0 |
| MERCY | 2 | 0 | 3 | 5 | 5 | 1 |
| WORSER | 2 | 0 | 1 | 1 | 1 | 0 |

- Document represented by count vector $\in \mathbb{N}^{v}$



## Bag of Words Model

- Ordering of words in document not considered:
  - "John is quicker than Mary" ≅ "Mary is quicker than John"
- This is called the bag of words model
- In a sense, step back: The positional index was able to distinguish these two documents
- Assignment 2 Ranked Retrieval: Back to Bag-of-Words

## Term Frequency tf

| | Antony and Cleopatra |
|---|---|
| ANTONY | 157 |

frequency = count in IR

- Term frequency $\mathrm{tf}_{t,d}$ of term $t$ in document $d$ ≅ number of times that $t$ occurs in $d$



## Log-Frequency Weighting

- Raw term frequency is a bit overestimated:
  - A document with 10 occurrences of the term is more relevant than a document with 1 occurrence of the term
  - But arguably not 10 times more relevant
- Alternative is log-frequency weight of term $t$ in document $d$

$$w_{t,d} = \begin{cases} 1 + \log_{10} \mathrm{tf}_{t,d}, & \text{if } \mathrm{tf}_{t,d} > 0 \\ 0, & \text{otherwise} \end{cases}$$



## Simple Query-Document Score

- Queries with >1 terms
- Score for a document-query pair: sum over terms $t$ in both $q$ and $d$:
  score $= \sum_{t \in q \cap d} \mathrm{tf}_{t,d}$
- The score is 0 if none of the query terms is present in the document
- What is the problem with this measure?



## Document Frequency

- Rare terms are more informative than frequent terms
- Example: rare word ARACHNOCENTRIC
  - Document containing this term is very likely to be relevant to query ARACHNOCENTRIC
    → High weight for rare terms like ARACHNOCENTRIC
- Example: common word THE
  - Document containing this term can be about anything
    → Very low weight for common terms like THE
- We will use document frequency (df) to capture this.

# idf Weight

- $df_t$ is the document frequency of term $t$: the number of documents that contain $t$
  - $df_t$ is an inverse measure of the informativeness of $t$
  - $df_t \leq N$

- Informativeness idf (inverse document frequency) of $t$:

$$\mathrm{idf}_t = \log_{10}(N/\mathrm{df}_t)$$

  - log ($N$/$df_t$) instead of $N$/$df_t$ to "dampen" the effect of idf.
    - Mathematical reasons later in the course



# Exercise 2 Minutes

- Suppose $N$ = 1,000,000 $\mathrm{idf}_t = \log_{10}(N/\mathrm{df}_t)$

| term | $df_t$ | $idf_t$ |
|---|---|---|
| calpurnia | 1 | |
| animal | 100 | |
| sunday | 1,000 | |
| fly | 10,000 | |
| under | 100,000 | |
| the | 1,000,000 | |

- Fill in the $idf_t$ column



# Effect of idf on Ranking

- Does idf have an effect on ranking for one-term queries, like IPHONE?

- Only effect for >1 term
  - Query CAPRICIOUS PERSON: idf puts more weight on CAPRICIOUS than PERSON.



# Collection vs. Document Frequency

- Collection frequency of $t$: total number of occurrences of $t$ in the collection, counting multiple occurrences

- Example:

| Word | Collection frequency | Document frequency |
|---|---|---|
| *insurance* | 10440 | 3997 |
| *try* | 10422 | 8760 |

- Which word is a better search term (and should get a higher weight)?

# tf-idf Weighting

- tf-idf weight of a term: product of tf weight and idf weight

$$w_{t,d} = \mathrm{tf}_{t,d} \times \log_{10}(N/\mathrm{df}_t)$$

- Best known weighting scheme in information retrieval
  - Note: the "-" in tf-idf is a hyphen, not a minus sign!
  - Alternative names: tf.idf, tf x idf
- Increases with the number of occurrences within a document
- Increases with the rarity of the term in the collection



# Binary → Count → Weight Matrix

| | Antony and Cleopatra | Julius Caesar | The Tempest | Hamlet | Othello | Macbeth |
|---|---|---|---|---|---|---|
| ANTONY | 5.255 | 3.18 | 0 | 0 | 0 | 0.35 |
| BRUTUS | 1.21 | 6.1 | 0 | 1 | 0 | 0 |
| CAESAR | 8.59 | 2.54 | 0 | 1.51 | 0.25 | 1 |
| CALPURNIA | 0 | 1.54 | 0 | 0 | 0 | 0 |
| CLEOPATRA | 2.85 | 0 | 0 | 0 | 0 | 0 |
| MERCY | 1.51 | 0 | 1.9 | 0.12 | 5.25 | 0.88 |
| WORSER | 1.37 | 0 | 0.11 | 4.15 | 0.25 | 1.95 |

- Document represented by tf-idf weight vector $\in \mathbb{R}^{V}$



# Documents as Vectors

- So we have a |V|-dimensional vector space
  - Terms are axes/dimensions
  - Documents are points in this space
- Very high-dimensional
  - Order of $10^7$ dimensions when for a web search engine
- Very sparse vectors - most entries zero

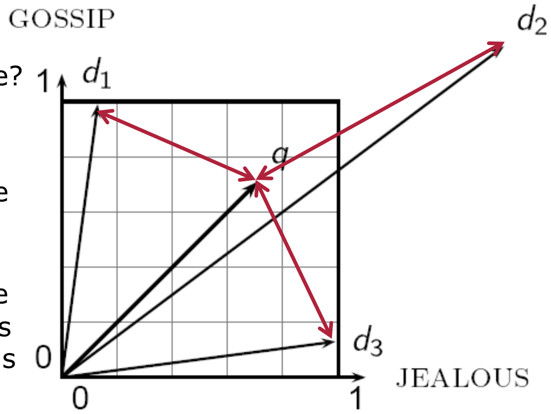


# Queries as Vectors

- Key idea 1: Represent queries as vectors in same space
- Key idea 2: Rank documents according to proximity to query in this space
  - proximity = similarity of vectors
  - proximity ≈ inverse of distance
- Recall:
  - Get away from Boolean model
  - Rank more relevant documents higher than less relevant documents

## Formalizing Vector Space Proximity

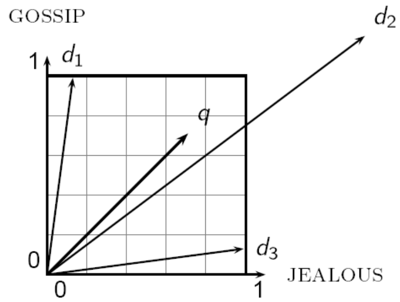
- First cut:
  Euclidean distance?
  $$\left\| q - d_n \right\|_2$$
- Euclidean distance is a bad idea . . .
- . . . because Euclidean distance is large for vectors of different lengths
  - What determines length here?






## Exercise 5 Minutes

- Euclidean distance bad for
  - vectors of different length (documents with different #words)
  - high-dimensional vectors (large dictionaries)




- Discuss in pairs:
  - Can you come up with a better difference measure?



## Use Angle Instead of Distance

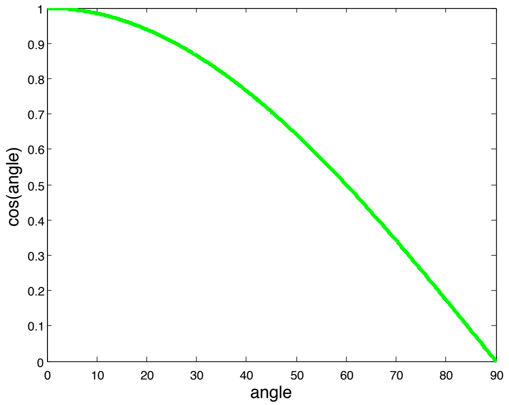
- Thought experiment: take a document d and append it to itself. Call this document d’
- “Semantically” d and d’ have the same content
- The Euclidean distance between the two documents can be quite large
- The angle between the two documents is 0, corresponding to maximal similarity
- Key idea:
  - Length unimportant
  - Rank documents according to angle from query



## Problems with Angle

- Angles expensive to compute – arctan
- Find a computationally cheaper, equivalent measure
  - Give same ranking order ≅ monotonically increasing/ decreasing with angle
- Any ideas?

# Cosine More Efficient Than Angle




Monotonically decreasing



# Length Normalization

- Computing cosine similarity involves length-normalizing document and query vectors
- $L_2$ norm: $$\|\vec{x}\|_2 = \sqrt{\sum_i x_i^2}$$
- Dividing a vector by its $L_2$ norm makes it a unit (length) vector (on surface of unit hypersphere)
- Recall:
  - Length unimportant
  - Rank documents according to angle from query



# Cosine Similarity

dot / scalar / inner product

$$\cos(\vec{q},\vec{d}) = \frac{\vec{q}}{\|\vec{q}\|_2} \bullet \frac{\vec{d}}{\|\vec{d}\|_2} = \frac{\vec{q} \bullet \vec{d}}{\|\vec{q}\|_2 \|\vec{d}\|_2} = \frac{\sum_{i=1}^{|V|} q_i d_i}{\sqrt{\sum_{i=1}^{|V|} q_i^2}\sqrt{\sum_{i=1}^{|V|} d_i^2}}$$

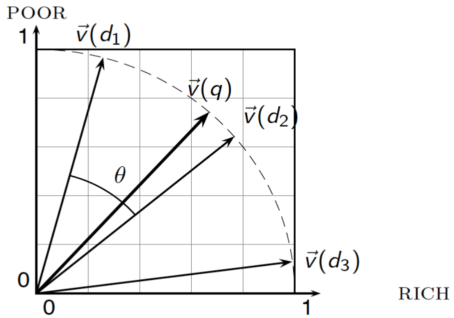
- $q_i$ is the tf-idf weight of term $i$ in the query
- $d_i$ is the tf-idf weight of term $i$ in the document

- $\cos(\vec{q},\vec{d})$ is the cosine similarity of $\vec{q}$ and $\vec{d}$ = the cosine of the angle between $\vec{q}$ and $\vec{d}$.



# Cosine Similarity

- In reality:
  - Length-normalize when document added to index: $$\vec{d} \leftarrow \frac{\vec{d}}{\|\vec{d}\|_2}$$
  - Length-normalize query: $$\vec{q} \leftarrow \frac{\vec{q}}{\|\vec{q}\|_2}$$
  - Fast to compute cosine similarity: $$\cos(\vec{q},\vec{d}) = \vec{q} \bullet \vec{d} = \sum_{i=1}^{|V|} q_i d_i$$



## Cosine Similarity Example

- How similar are the novels
  - SaS: Sense and Sensibility
  - PaP: Pride and Prejudice
  - WH: Wuthering Heights?

No idf weighting!

- Term frequency $\mathrm{tf}_t$

| term | SaS | PaP | WH |
|---|---|---|---|
| affection | 115 | 58 | 20 |
| jealous | 10 | 7 | 11 |
| gossip | 2 | 0 | 6 |
| wuthering | 0 | 0 | 38 |



## Cosine Similarity Example

- Log frequency weights $w_{t,d} = \begin{cases} 1+\log_{10} \mathrm{tf}_{t,d}, & \text{if } \mathrm{tf}_{t,d} > 0 \\ 0, & \text{otherwise} \end{cases}$

| term | SaS | PaP | WH |
|---|---|---|---|
| affection | 3.06 | 2.76 | 2.30 |
| jealous | 2.00 | 1.85 | 2.04 |
| gossip | 1.30 | 0 | 1.78 |
| wuthering | 0 | 0 | 2.58 |



## Cosine Similarity Example

- After length normalization $\vec{d} = [w_{1,d}, \ldots, w_{4,d}]$, $\vec{d} \leftarrow \frac{\vec{d}}{\|\vec{d}\|_2}$

| term | SaS | PaP | WH |
|---|---|---|---|
| affection | 0.789 | 0.832 | 0.524 |
| jealous | 0.515 | 0.555 | 0.465 |
| gossip | 0.335 | 0 | 0.405 |
| wuthering | 0 | 0 | 0.588 |

cos(SaS,PaP) ≈ 0.789*0.832 + 0.515*0.555 + 0.335*0 + 0*0 ≈ 0.94
cos(SaS,WH) ≈ 0.79
cos(PaP,WH) ≈ 0.69

- Why is cos(SaS,PaP) > cos(*,WH)?



## Computing Cosine Scores

```
CosineScore(q)
float Scores[N] = 0
float Length[N]
for each query term t
do calculate w_{t,q} and fetch postings list for t
    for each pair(d, tf_{t,d}) in postings list
    do Scores[d] += w_{t,d} × w_{t,q}
Read the array Length
for each d
do Scores[d] = Scores[d]/Length[d]
return Top K components of Scores[]
```

## Summary – vector space ranking

- Represent the query as a tf-idf vector
- Represent each document as a tf-idf vector
- Compute the cosine similarity score for the query vector and each document vector
- Rank documents with respect to the query by score
- Return the top *K* (e.g., *K* = 10) to the user



# Efficient Scoring and Ranking

(Manning Chapter 7)

## Efficient Cosine Ranking

- Find the *K* docs in the collection "nearest" to the query ⇒ *K* largest query-document cosine scores
- Up to now: Linear scan through collection
  - Did not make use of sparsity in term space
  - Computed all cosine scores
- Efficient cosine ranking:
  - Computing each cosine score efficiently
  - Choosing the K largest scores efficiently



## Computing Cosine Scores Efficiently

- Approximation:
  - Assume that terms only occur once in query

$$w_{t,q} \leftarrow \begin{cases} 1, & \text{if } w_{t,q} > 0 \\ 0, & \text{otherwise} \end{cases}$$

- Works for short querys (|q| << *N*)
- Works since ranking only relative

## Computing Cosine Scores Efficiently

```
FastCosineScore(q)
float Scores[N] = 0
for each d
do Initialize Length[d] to the length of doc d
for each query term t
do                    fetch postings list for t
   for each pair(d, tf_{t,d}) in postings list
   do add wf_{t,d} to Scores[d]
Read the array Length[d]
for each d
do Divide Scores[d] by Length[d]
return Top K components of Scores[]
```

Speedup here



## Computing Cosine Scores Efficiently

- Downside of approximation: sometimes get it wrong
  - A document not in the top *K* may creep into the list of *K* output documents
- How bad is this?
- Cosine similarity is only a proxy (Task 1.5)
  - User has a task and a query formulation
  - Cosine matches documents to query
  - Thus cosine is anyway a proxy for user happiness
  - If we get a list of *K* documents "close" to the top *K* by cosine measure, should be ok



## Choosing K Largest Scores Efficiently

- Retrieve top *K* documents wrt query
  - Not totally order all documents in collection
- Do selection:
  - avoid visiting all documents
- Already do selection:
  - Sparse term-document incidence matrix, |d| << *N*
  - Many cosine scores = 0
  - Only visits documents with nonzero cosine scores (≥1 term in common with query)



## Choosing K Largest Scores Efficiently Generic Approach

- Find a set A of contenders, with *K* < |A| << *N*
  - A does not necessarily contain the top *K*, but has many docs from among the top *K*
  - Return the top *K* documents in A
- Think of A as pruning non-contenders
- Same approach used for any scoring function!
- Will look at several schemes following this approach

## Choosing K Largest Scores Efficiently
## Index Elimination

- Basic algorithm FastCosineScore only considers documents containing at least one query term
  - All documents have ≥1 term in common with query
- Take this further:
  - Only consider high-idf query terms
  - Only consider documents containing many query terms



## Choosing K Largest Scores Efficiently
## Index Elimination

- Example:

CATCHER IN THE RYE

- Only accumulate scores from CATCHER and RYE
- Intuition:
  - IN and THE contribute little to the scores – do not alter rank-ordering much
  - Compare to stop words
- Benefit:
  - Posting lists of low-idf terms have many documents → eliminated from set A of contenders



## Choosing K Largest Scores Efficiently
## Index Elimination

- Example:

CAESAR ANTONY CALPURNIA BRUTUS

- Only compute scores for documents containing ≥3 query terms

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| *Antony* | → | 3 | 4 | 8 | 16 | 32 | 64 | 128 |
| *Brutus* | → | 2 | 4 | 8 | 16 | 32 | 64 | 128 |
| *Caesar* | → | 1 | 2 | 3 | 5 | 8 | 13 | 21 | 34 |
| *Calpurnia* | → | 13 | 16 | 32 | | | | |



## Choosing K Largest Scores Efficiently
## Champion Lists

- Precompute for each dictionary term $t$, the $r$ documents of highest tf-idf$_{td}$ weight
  - Call this the champion list (fancy list, top docs) for $t$
- Benefit:
  - At query time, only compute scores for documents in the champion lists – fast
- Issue:
  - $r$ chosen at index build time
  - Too large: slow
  - Too small: $r < K$

# Exercise 5 Minutes

- Index Elimination: consider only high-idf query terms and only documents with many query terms
- Champion Lists: for each term $t$, consider only the $r$ documents with highest tf-idf$_{td}$ values

- Think quietly and write down:
  - How do Champion Lists relate to Index Elimination? Can they be used together?
  - How can Champion Lists be implemented in an inverted index?



# Choosing K Largest Scores Efficiently Static Quality Scores

- Develop idea of champion lists

- We want top-ranking documents to be both relevant and authoritative
  - Relevance – cosine scores
  - Authority – query-independent property

- Examples of authority signals
  - Wikipedia pages (qualitative)
  - Articles in certain newspapers (qualitative)
  - A scientific paper with many citations (quantitative)
  - PageRank (quantitative)

More in Lecture 5



# Choosing K Largest Scores Efficiently Static Quality Scores

- Assign query-independent quality score g($d$) in [0,1] to each document $d$

- net-score($q$,$d$) = g($d$) + cos($q$,$d$)
  - Two "signals" of user happiness
  - Other combination than equal weighting

- Seek top $K$ documents by net score



# Choosing K Largest Scores Efficiently Champion Lists + Static Quality Scores

- Can combine champion lists with *g(d)*-ordering

- Maintain for each term $t$ a champion list of the $r$ documents with highest *g(d)* + tf-idf$_{td}$

- Seek top $K$ results from only the documents in these champion lists

# Next

- Assignment 1 left?
  - Email Johan or Hedvig (away next week)

- Lecture 5 (March 4, 13.15-15.00)
  - B1
  - Readings: Manning Chapter 21
    Avrachenkov Sections 1-2

- Lecture 6 (March 7, 10.15-12.00)
  - B1
  - Readings: Manning Chapter 9, MAYBE MORE