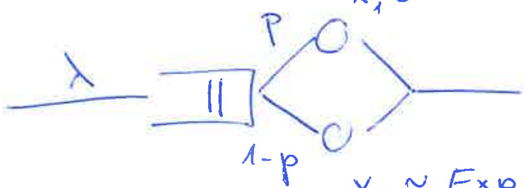
Recitation - Chapter 10 $\quad M/G/1$

10.1.

$x_1 = 0 \Rightarrow E[x_1]=0, \; E[x_1^2]=0, \; V[x_1]=0$



$x_2 \sim Exp(p) \Rightarrow E[x_2]=\frac{1}{p}, \; E[x_2^2]=\frac{2}{p^2}, \; V[x]=\frac{1}{p^2}$

a) $b(x) = p\cdot 0 + (1-p)\cdot p \cdot e^{-px}$

$E[x] = pE[x_1] + (1-p)E[x_2] = 0 + (1-p)\cdot\frac{1}{p} = \frac{1-p}{p}$

$E[x^2] = pE[x_1^2] + (1-p)E[x_2^2] = \ldots \frac{2(1-p)}{p^2}$

$V[x] = E[x^2] - E[x]^2 = \ldots = \frac{1-p^2}{p^2}$

b) $W = \frac{\lambda E[x^2]}{2(1-\rho)} = \ldots = \frac{\rho}{p(1-\rho)}$

$\underbrace{S^*(s) = B^*(s)}_{\text{different notation for the same thing.}} \triangleq \int_0^\infty e^{-sx} b(x)dx = \int_0^\infty p\,\delta(x)e^{-sx} + \int_0^\infty (1-p)\underbrace{pe^{-px}e^{-sx}}_{exp}dx =$

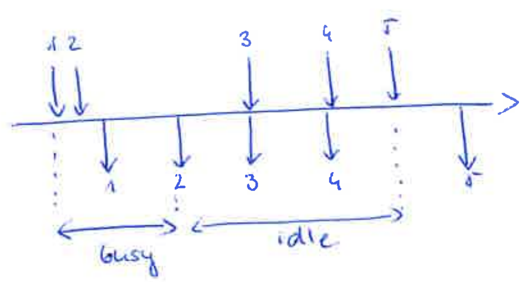
$= p + (1-p)\frac{p}{p+s} = \ldots = \frac{p(1+s)}{p+s}$

$W^*(s) = \frac{s(1-\rho)}{s-\lambda+\lambda S^*(s)} = \ldots = \frac{(1-\rho)(s+p)}{s+p(1+\rho)}$

c) utilization $\triangleq \frac{\lambda_{eff}E[x]}{m} = \lambda E[x] = \rho = \frac{\lambda(1-p)}{p}$

$\lambda_{max}: \; \rho < 1 \Rightarrow \lambda_{max} < \frac{p}{1-p}$

Idle and busy period length



- Only type 2 packets move the system to busy state $\Rightarrow$
- Arrival of type 2: Poisson $(1-p)\lambda$

$\Rightarrow \bar{\tau}_{idle} = \frac{1}{(1-p)\lambda}$, $\quad \rho = \frac{\bar{\tau}_{busy}}{\bar{\tau}_{busy}+\bar{\tau}_{idle}}$, $\quad \bar{\tau}_{busy} = \frac{\rho}{1-\rho}\cdot\frac{1}{(1-p)\lambda} = \ldots = \frac{1}{p-\lambda(1-p)}$

10.2.

$M/M/1 \quad N_q = 8.1$

$M/E_5/1$, same $\rho \rightarrow N_q = ?$

~~$\Rightarrow$ Wecht to calculate~~

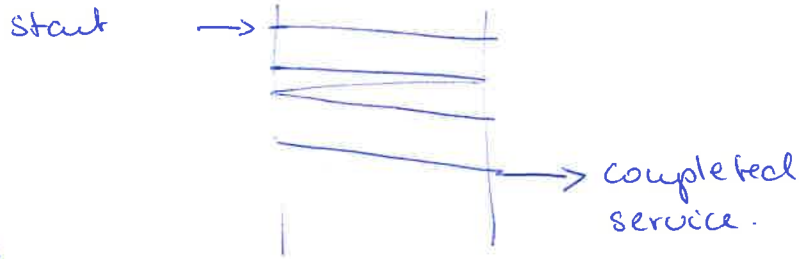
a) $M/G/1$: $\bar{N_q} = \lambda W = \frac{\lambda \rho E[x]}{2(1-\rho)}(1+C_x^2) = \frac{\rho^2}{2(1-\rho)}(1+C_x^2)$

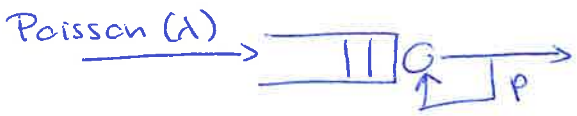
$$\frac{\bar{N}_{q,\,M/E_5/1}}{\bar{N}_{q\,M/M/1}} = \frac{1+\frac{1}{5}}{1+1} = \frac{\frac{6}{5}}{2} = \frac{6}{10} = \frac{3}{5}$$

$$\bar{N}_{q\,M/E_5/1} = \frac{3}{5} \cdot \bar{N}_{q\,M/M/1} = 4.86$$

b) We skipped the $M/H_2/1$, rather boring.

$\boxed{10.4.}$





Service? Sequence of tx attempts, each $\tau = 1$

a) Service time distribution?

$$P(\text{service time} = n) = P(n \text{ transmissions until success}) =$$

$$p^{n-1}(1-p) \Rightarrow N \sim Geom(1-p)$$

b) Message delay = waiting time + repeated transmission.

System: M / G / 1

$$E[x] = \sum_{n=1}^{\infty} n \cdot p^{n-1}(1-p) = (1-p)\sum_{n=1}^{\infty}(p^n)' = (1-p)\left[\sum_{n=1}^{\infty} p^n\right]'$$

$$= (1-p)\left[\frac{p}{1-p}\right]' = (1-p)\,\frac{(1-p)+p}{(1-p)^2} = \frac{1}{1-p}$$

$$E[x^2] = \sum_{n=1}^{\infty} n^2 p^{n-1}(1-p) = \dots \quad = \frac{1+p}{(1-p)^2}$$

$$T = E[x] + \frac{\lambda E[x^2]}{2(1-\rho)} = \frac{1}{1-p} + \frac{\lambda \frac{1+p}{(1-p)^2}}{2\left(1-\lambda\frac{1}{1-p}\right)} = \dots$$

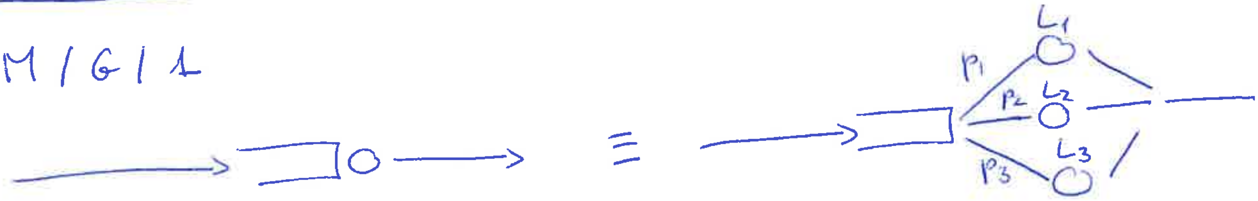
c) λ_{max}: $\rho < 1$

$$\lambda \cdot \frac{1}{1-p} < 1$$

$$\lambda < 1-p$$

Exam 9

M/G/1



$\lambda = 10^4$ packets/s

$C = 34 \cdot 10^6$ b/s

Packet size distribution:

$P(L = 40B) = 0.65$
$P(L = 595B) = 0.2$
$P(L = 1500B) = 0.15$
} deterministic

a) First calculate the respective transmission times:

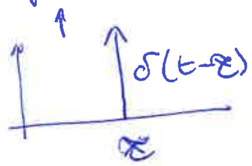
$\tau_1 = \frac{L_1 \cdot 8}{C} = \dots \quad , \tau_2 \quad , \tau_3$

$E[x] = 0.65\,\tau_1 + 0.2\,\tau_2 + 0.15 \cdot \tau_3 \quad \Rightarrow W$

$E[x^2] = 0.65\,\tau_1^2 + 0.2\,\tau_2^2 + 0.15\,\tau_3^2 \quad \Leftarrow$ Why?

$\Uparrow$ What is the second moment of a deterministic random variable? $P(x = \tau) = 1$

$E[x^2] = \int_0^\infty x^2 f(x)\,dx = \tau^2 \quad \Rightarrow V[x] = E[x^2] - E[x]^2 = 0.$



b) What will be $B^*(s)$?

Laplace transform $\delta(t-\tau) \Longleftrightarrow e^{-\tau s}$

$B^*(s) = \sum_{i=1}^{3} p_i\, e^{-\tau_i s}$

What distributions are often involved?

- Exponential
- Geometric
- Deterministic
- Erlang-r
- Hyperexp