PROBLEM 1

Calculate the flux of the vector field:

$$\operatorname{grad}\left(\frac{1}{\sqrt{(x-3)^2+(y+1)^2+z^2}}+xy^3\right)$$

On a sphere with radius 3 and centre in the point (2,1,1) = $\bar r_s$

PROBLEM 2

A quadrupole in the origin produce the vector field:

$$\bar A = \frac{3\cos^2\theta - 1}{r^4}\hat e_r + \frac{\sin 2\theta}{r^4}\hat e_\theta$$

Use the Gauss' theorem to calculate the flux on the cylinder:

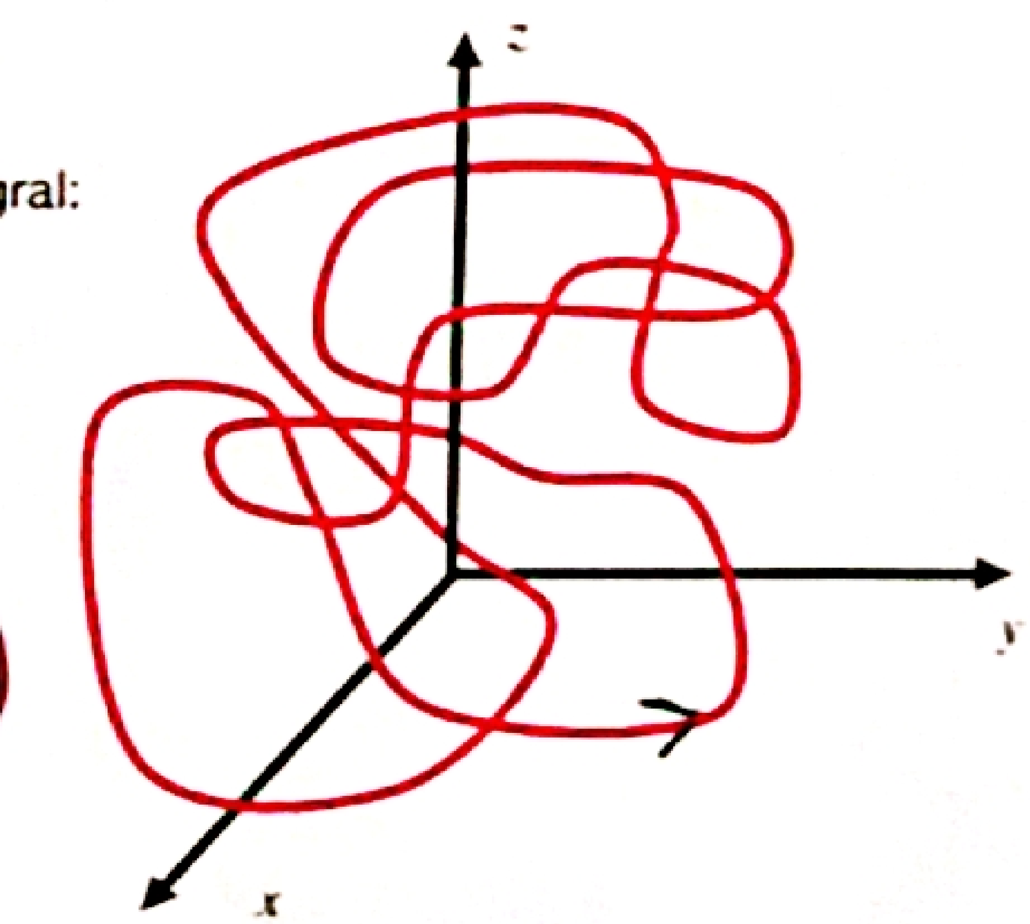
$$\begin{cases} x^2+y^2 \le 9 \\ -1 \le z \le 2 \end{cases}$$

PROBLEM 3

Calculate the following line integral:

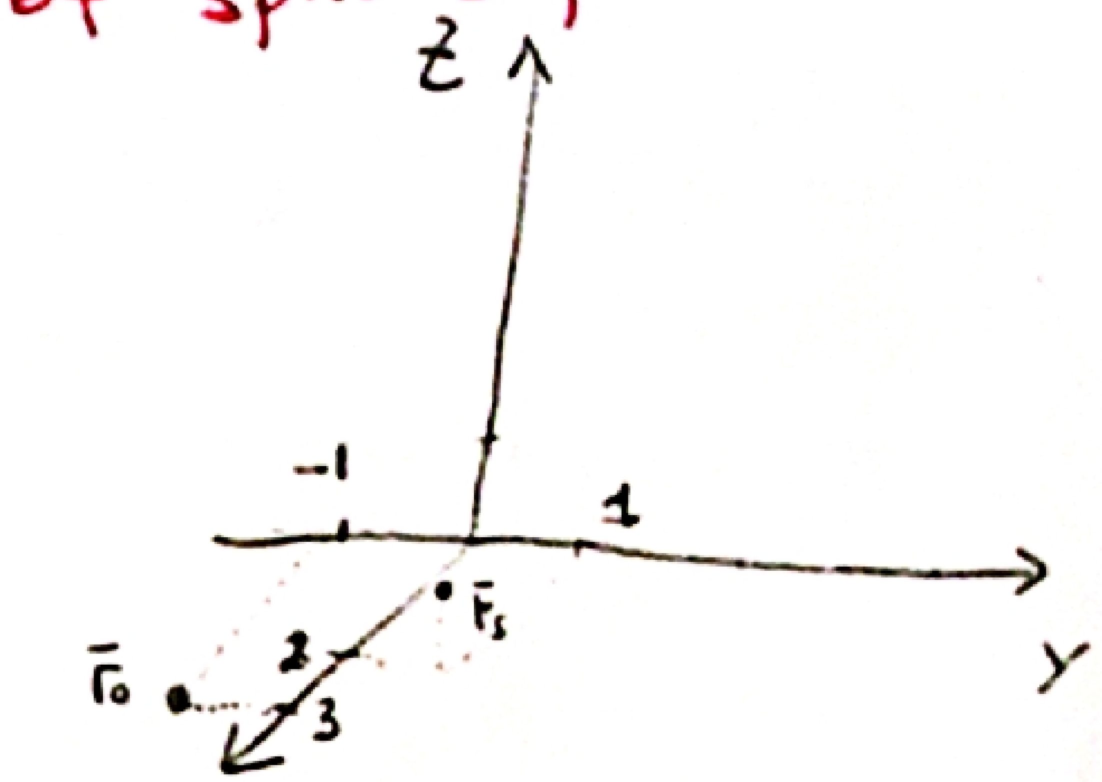
$$\oint_L \frac{2}{\rho}\hat e_\varphi \cdot d\bar r$$

(Använd sats 11.2)



(Distance from (0,0,0) to centre of sphere)

① $|\bar r_s| = \sqrt{2^2+1^2+1^2} = \sqrt6$



$\bar A = \text{grad}\,\phi$

$\phi = \dfrac{1}{|\bar r - \bar r_0|} + xy^3$

with $\bar r_0 = (3,-1,0)$

$|\bar r_0 - \bar r_s| = \sqrt{1^2+2^2+1^2} = \sqrt6 <$ Radius sphere $= 3$ ⇒ The source is inside the sphere!!

$$\bar A = \text{grad}\left(\frac{1}{|\bar r-\bar r_0|}+xy^3\right) = -\underbrace{\frac{(\bar r-\bar r_0)}{|\bar r-\bar r_0|^3}}_{\text{vector field from a point source NOT in the origin}} + (y^3, 3xy^2, 0)$$

$$\iint_S \bar A\cdot d\bar S = \underbrace{\iint_S -\frac{(\bar r-\bar r_0)}{|\bar r-\bar r_0|^3}\,d\bar S}_{=-4\pi\ \text{(Theorem 11.1)}} + \underbrace{\iint_S (y^3, 3xy^2, 0)\cdot d\bar S}_{\text{Gauss}}$$

$$\iiint_V 6xy\,dV = \iiint 6(x'+2)(y'+1)\,dV = 12V$$

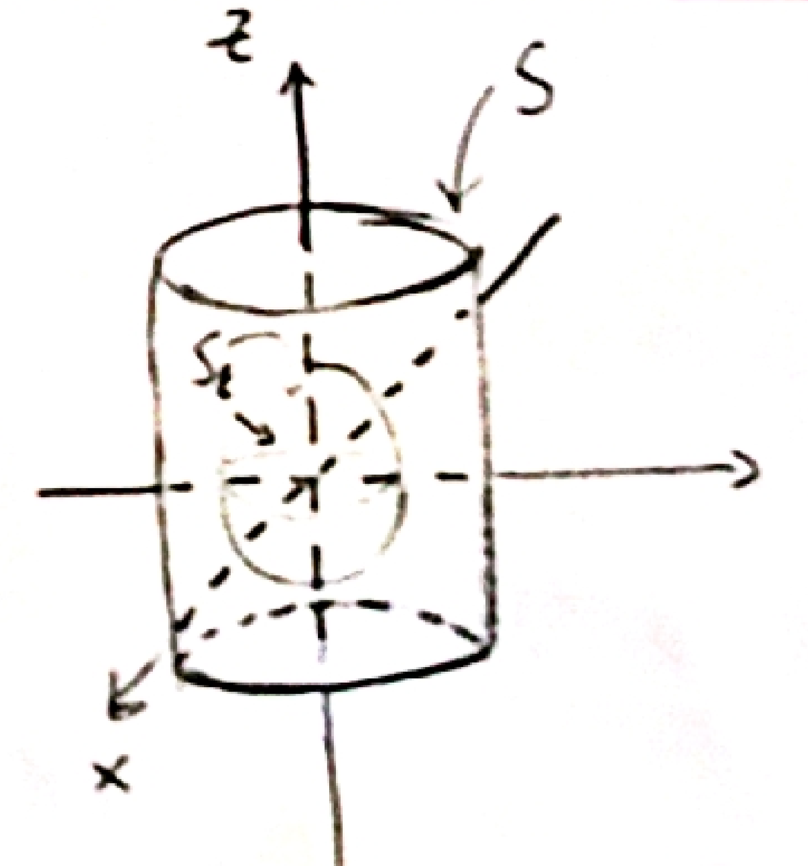
Coord. Transformation because the sphere is NOT in the origin.

(x' & y' odd fns) symm. Why we do NOT need big calculations!

Volume of the sphere

Final result $\boxed{428\pi}$

②



$$\iint_S \bar A\cdot d\bar S = \underbrace{\iint_{S+S_\varepsilon} \bar A\cdot d\bar S}_{\text{here we can apply Gauss}} + \iint_{-S_\varepsilon} \bar A\, d\bar S =$$

$$= \iint_{V_0} \text{div}\,\bar A\cdot dV - \iint_{S_\varepsilon} \bar A\cdot d\bar S = \longrightarrow$$

$V = V_0 + V_\varepsilon$

$\text{div}\,\bar A = 0 \longrightarrow 0$

Use spherical coord. because $\bar A = \bar A(r,\theta,\varphi)$

$$\rightarrow -\int_0^{\pi}\int_0^{2\pi}\left(\frac{3\cos^2\theta-1}{r^4}\,\hat{e}_r+\frac{\sin 2\theta}{r^4}\,\hat{e}_\theta\right)\cdot\hat{n}\,\underbrace{dS}_{=\,r^2\sin\theta\, d\theta d\varphi} = \int_0^{\pi}\int_0^{2\pi}\frac{3\cos^2\theta+1}{r^4}\,r^2\sin\theta\, d\theta d\varphi = \frac{1}{\varepsilon^2}\underbrace{\int_0^{\pi}\int_0^{2\pi}(3\cos^2\theta-1)\sin\theta\, d\theta\, d\varphi}_{=\,0} = 0$$

On S_ε $r=\varepsilon$

$$\hat{n}=-\hat{e}_r \Rightarrow \begin{array}{l}\hat{e}_r\cdot\hat{n}=-1\\ \hat{e}_\theta\cdot\hat{n}=0\end{array}$$