# PhD Course in KTH - Sparse Signal Processing
# Slide 3
# Discussion Topic - Pursuit Algorithms

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# Approaches

$$(P_0): \quad \underset{\mathbf{x}\in\mathbb{R}^m}{\arg\min} \; \|\mathbf{x}\|_0 \text{ subject to } \mathbf{b} = \mathbf{A}\mathbf{x}. \tag{1}$$

1. Finding support: Estimation of support is a discrete problem. As the support is discrete in nature, algorithms that seek it are discrete as well. This line of reasoning leads to greedy algorithms.
2. Smoothing penalty: Most talked about algo is $l_1$ norm minimization.

# Greedy Algorithm

1. Core Idea: Suppose $spark(\mathbf{A}) > 2$ and the $value(P_0) = 1$. So $\mathbf{b}$ is a scalar multiplication of a column of $\mathbf{A}$.
   Explain more.
2. Generalization: If $spark(\mathbf{A}) > 2k_0$ and the $value(P_0) = k_0$, then we can chheck all the combinations. But this is enormous computation - NP hard.

Greedy Strategy: A greedy strategy abandons exhaustive search in favor of a series of locally optimal single-term updates.

# Orthogonal matching pursuit

1. Explain the original algorithm
2. Why the name 'Orthogonal matching pursuit'?
3. Discuss about the Sweep stage: Show that quest for small error is nothing but the quest for largest (in absolute value) inner product between the residual $\mathbf{r}^{k-1}$ and the normalized vectors of $\mathbf{A}$
4. Write the alternate simple form of OMP algorithm

# Other greedy methods

Other greedy methods:

1. Mathcing pursuit (MP), Weak MP, LS-OMP
2. CoSaMP, Subspace pursuit (SP)

A question?

Normalization: Is there any difference between using $\mathbf{A}$ and its normalized version $\tilde{\mathbf{A}}$ where each column $l_2$ norm is one?

We can express $\tilde{\mathbf{A}} = \mathbf{AW}$ where $\mathbf{W} = diag\{\frac{1}{\|\mathbf{a}_i\|_2}\}$

Theorem

*The greedy algorithms (OMP, MP and Weak MP) produce the same solution using either original matrix* $\mathbf{A}$ *or its normalized version* $\tilde{\mathbf{A}}$.

Proof: We work out

Question: Is this theorem holds true for other greedy algorithms? Such as CoSaMP and SP? Check it out.

# Smoothing Penalty

1. FOCUSS, Iteratively reweighted least squares
   Difficult to analyze
2. Using $l_1$ norm:

$$(P_1): \quad \arg\min_{\mathbf{x} \in \mathbb{R}^m} \|\mathbf{x}\|_1 \text{ subject to } \mathbf{b} = \mathbf{A}\mathbf{x}. \tag{2}$$

   Candes and Tao: If **A** holds Restricted Isometry Property (RIP), then we need $n = \mathcal{O}(\|\mathbf{x}\|_0 \log m)$ for perfect recovery.

# Evaluating Some Algorithms

1. Write the codes for some algorithms and experiment with them. Find by your own that it really works.
2. Design a formal experiment setup and evaluate the algorithms.
3. The project is assigned through the course website.