I. Sparse Undetermined Setup

We address the following problem \((P_0)\): 
\[
\arg \min_{x \in \mathbb{R}^m} \|x\|_0 \quad \text{subject to} \quad b = Ax, 
\]
where \(b \in \mathbb{R}^n\) and \(x\) is sparse. We must have \(\|x\|_0 = K < n < m\). We will seek the solutions (design algorithms) for the above problem.

II. Project Problem

Project Problem: Performance evaluation for some Algorithms: Matching pursuit (MP), Orthogonal matching pursuit (OMP), Subspace pursuit (SP) and Basis pursuit (BP). Simulate the algorithms and evaluate them.

III. Experiments and Results

The BP is the \(l_1\) norm minimization based convex algorithm. Either you design the BP algorithm by CVX toolbox or download simulation code (matlab) from the \(l_1\)-magic toolbox [1]. Now, we first discuss the reconstruction performance measures and experimental setups.

A. Performance measures and experimental setups

We will use signal-to-reconstruction-noise ratio (SRNR) as the performance measure. SRNR is defined as
\[
\text{SRNR} = \frac{\mathcal{E}\{\|x\|^2\}}{\mathcal{E}\{\|x - \hat{x}\|^2\}},
\]
where \(\hat{x}\) is the reconstructed signal vector. Note that our objective is to achieve a higher SRNR.
Now we discuss experimental setups. In a sparse setup, all sparse signal vectors are expected to be exactly reconstructed if the number of measurements is more than a certain threshold [2]. However, the computational complexity to test this uniform reconstruction ability is very high. Instead, for empirical testing, we can devise a strategy that can compute the performance measures for random measurement matrix ensemble. Let us define the fraction of measurements (FoM)

\[
\alpha = \frac{n}{m}.
\]

Using \(\alpha\), steps of the testing strategy are listed as follows:

1) For given values of the parameters \(K\) and \(m\), choose \(\alpha\) such that the number of measurements \(n\) is an integer.
2) Randomly generate a sensing matrix \(A \in \mathbb{R}^{n \times m}\) where the components are drawn independently from a Gaussian source (i.e., \(a_{i,j} \sim \mathcal{N}(0, \frac{1}{n})\)) and then scale the columns of \(A\) to unit-norm.
3) Randomly generate a set of \(K\)-sparse data \(x\) where the support set \(S\) is chosen uniformly over the set \(\{1, 2, \ldots, m\}\). Let we denote the size of data as \(D\) (i.e., the number of signal vectors \(x\) is \(D\)). The non-zero components of \(x\) are independently drawn from a standard Gaussian source.
4) For each data, compute the measurement \(b = Ax\) and apply the reconstruction methods independently.
5) Repeat steps 2-4 for a given times (let \(T\) times). Then evaluate the performance evaluation measure (by averaging over \(DT\) data).
6) Repeat steps 1-5 for a new \(\alpha\).

This test can be performed for any chosen \(K\) and \(m\).

Choose \(m = 500\) and \(K = 20\). Also choose \(D = T = 20\). So we perform averaging over 400 data. Vary \(\alpha\) from 0.1 to 0.5 in a step size of 0.01. Write a full simulation code that can handle the situation.

Plot \(\alpha\) versus SRNR (in dB). Note that there should be a thresholding effect where the performance goes suddenly high after a value of \(\alpha\).

REFERENCES
