Instructor Assigned Project for the Course "Sparse Signal Processing"

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I. Sparse Undetermined Setup

We address the following problem

$$(P_0): \quad \arg\min_{\mathbf{x}\in\mathbb{R}^m} \; \|\mathbf{x}\|_0 \text{ subject to } \mathbf{b} = \mathbf{A}\mathbf{x}, \tag{1}$$

where $\mathbf{b} \in \mathbb{R}^n$ and $\mathbf{x}$ is sparse. We must have $\|\mathbf{x}\|_0 = K < n < m$. We will seek the solutions (design algorithms) for the above problem.

II. Project Problem

Project Problem: Performance evaluation for some Algorithms: Matching pursuit (MP), Orthogonal matching pursuit (OMP), Subspace pursuit (SP) and Basis pursuit (BP). Simulate the algorithms and evaluate them.

III. Experiments and Results

The BP is the l_1 norm minimization based convex algorithm. Either you design the BP algorithm by CVX toolbox or download simulation code (matlab) from the l_1-magic toolbox [1]. Now, we first discuss the reconstruction performance measures and experimental setups.

A. Performance measures and experimental setups

We will use signal-to-reconstruction-noise ratio (SRNR) as the performance measure. SRNR is defined as

$$\text{SRNR} = \frac{\mathcal{E}\{\|\mathbf{x}\|_2^2\}}{\mathcal{E}\{\|\mathbf{x} - \hat{\mathbf{x}}\|_2^2\}}, \tag{2}$$

where $\hat{\mathbf{x}}$ is the reconstructed signal vector. Note that our objective is to achieve a higher SRNR.

Now we discuss experimental setups. In a sparse setup, all sparse signal vectors are expected to be exactly reconstructed if the number of measurements is more than a certain threshold [2]. However, the computational complexity to test this uniform reconstruction ability is very high. Instead, for empirical testing, we can devise a strategy that can compute the performance measures for random measurement matrix ensemble. Let us define the fraction of measurements (FoM)

$$\alpha = \frac{n}{m}. \tag{3}$$

Using α, steps of the testing strategy are listed as follows:

1) For given values of the parameters K and m, choose α such that the number of measurements n is an integer.
2) Randomly generate a sensing matrix $\mathbf{A} \in \mathbb{R}^{n \times m}$ where the components are drawn independently from a Gaussian source (i.e., $a_{i,j} \sim \mathcal{N}\left(0, \frac{1}{n}\right)$) and then scale the columns of $\mathbf{A}$ to unit-norm.
3) Randomly generate a set of K-sparse data $\mathbf{x}$ where the support set $\mathcal{S}$ is chosen uniformly over the set $\{1, 2, \ldots, m\}$. Let we denote the size of data as D (i.e., the number of signal vectors $\mathbf{x}$ is D). The non-zero components of $\mathbf{x}$ are independently drawn from a standard Gaussian source.
4) For each data, compute the measurement $\mathbf{b} = \mathbf{A}\mathbf{x}$ and apply the reconstruction methods independently.
5) Repeat steps 2-4 for a given times (let T times). Then evaluate the performance evaluation measure (by averaging over DT data).
6) Repeat steps 1-5 for a new α.

This test can be performed for any chosen K and m.

Choose $m = 500$ and $K = 20$. Also choose $D = T = 20$. So we perform averaging over 400 data. Vary α from 0.1 to 0.5 in a step size of 0.01. Write a full simulation code that can handle the situation.

Plot α versus SRNR (in dB). Note that there should be a thresholding effect where the performance goes suddenly high after a value of α.

References

[1] E. Candes and J. Romberg, *l_1-MAGIC: Recovery of sparse signal via convex programming.* A document in l_1-Magic toolbox shared online, http://www.acm.caltech.edu/l1magic/, 2005.

[2] W. Dai and O. Milenkovic, "Subspace pursuit for compressive sensing signal reconstruction," *Information Theory, IEEE Transactions on*, vol. 55, pp. 2230 –2249, may 2009.