Indirect Instantaneous Car-Fuel Consumption Measurements

Isaac Skog Member, IEEE and Peter Händel, Senior Member, IEEE

Abstract—A method to estimate the instantaneous fuel consumption of a personal car, using speed and height data recorded by a global positioning system (GPS) receiver and vehicle parameters accessible via national vehicle registers and databases on the world wide web, is proposed. The method is based upon a physical model describing the relationship between the dynamics of the car, the engine speed, and the energy consumption of the system. An evaluation of the proposed method is done by comparing the estimated instantaneous fuel consumption with that measured by the car’s on-board diagnostics (OBD) data bus. The results of three tests with different cars driven in mixed highway and urban conditions, indicate that the instantaneous fuel consumption may be estimated with a root mean square error of about 0.3 \( [g/s] \); in terms of a normalized mean square error, that corresponded to slightly less than 10 percent. One application of the proposed method is in the development of smartphone applications that educate drivers to drive more fuel efficiently.

I. INTRODUCTION

The number of cars in the world is predicted to double by the year 2035, according to World Energy Outlook 2011 [1]. The emissions of those predicted 1.7 billion cars will induce large environmental challenges. Consequently, there are intense research and development activities in the areas of non-fossil energy sources and low emission vehicles. These are also the areas where in the long run, the largest contributions to the overall reduction in emissions and fuel consumption of the car fleet are forecasted. However, in the short run, while waiting for tomorrow’s technology, traffic management and a change in driving behaviors may significantly contribute to lowering fuel consumption and emissions [2].

One way to teach a driver about energy-efficient driving is through eco-driving courses, something which today is mandatory in order to obtain a driving licence in several European countries. However, the effect of an eco-driving course on a driver’s behavior diminishes as time goes by, and regular monitoring and feedback is required to maintain the effects of the eco-driving training [3], [4]. Thus, a system that can give the driver immediate and constructive feedback about how their actions affect the fuel consumption is needed, to generate a long-term change in the behavior of a driver. In [2], such a feedback system is presented and their test results show an up to 23% reduction in fuel consumption. As a key component for generating the feedback, they use a fuel consumption map,\(^1\) together with sensors to observe the speed of the car, the engine speed, gear position, etc. However, if such a system should have the potential to reach a large group of drivers, it must be cheap and easy to use. This means that the system should be able to run on already existing computational platforms, e.g., a smartphone, and be based upon indirect measurements of the fuel consumption so that the trouble of connecting the system to the vehicle’s onboard computer is avoided. Another important application is found in the developing nations where the fuel cost is one of the major costs of running a vehicle fleet. Many vehicles, because of their age or the down-stripping of the electronics by the manufacturers, do not provide the relevant data for calculating the actual fuel consumption via the on board diagnostic outlet. In this case, a smartphone can play the role of a measurement device to track the fuel consumption to reduce not only the direct fuel consumption, but also to detect fraud or tampered pumps at filling stations.

Work on indirect measurements appear quite frequently in the instrumentation and measurement literature, since from a cost, practical, or safety perspective, it is often undesirable to connect and interact with the system undergoing testing. To give a few examples: a method to indirectly estimate the timing and duration of the manual gear shift in a personal car using accelerometer measurements and a piecewise linear model of the car’s acceleration were considered in [5]; a method to measure car performance in terms of quantities such as elapsed time and speed during drag racing activities.

\(^1\)A fuel consumption map is a three dimensional plot of the specific fuel consumption versus the engine rotation speed and brake torque.
using accelerometer measurement and a model of the chassis squat during acceleration were considered in [6]; a method to indirectly measure railroad-curvature by fusing GPS-receiver, gyroscope, and speed measurements were considered in [7]; pedestrian activity classification using measurements from chest-mounted inertial measurement units and a model for a set of pre-defined gait activities were considered in [8]. The idea behind all the mentioned examples is to, via model based signal processing relate a measured quantity to another quantity that cannot be directly measured, i.e., to perform indirect measurements.

One approach to design a system to instantaneously estimate the fuel consumption of a car indirectly, is thus to, from physical laws, deduce a model that relates the motion dynamics of a car to its fuel consumption. In [9] and [10], such physical models are deduced and used for emission modeling in microscale traffic simulations. However, these models assume that several car model specific parameters are known, and that not only the motion dynamics of the car are observed, but also the engine speed. These models can thus, not without modification and prior knowledge about the technical parameters of the car, be used to estimate the fuel consumption of the car, purely from its motion dynamics.

Therefore, in this paper we investigate the possibility of extending the physical model in [9], with a model for the engine speed, and to extract the necessary model parameters from the Swedish Traffic Registry\(^2\), and thereby estimate the fuel consumption of a car solely from GPS-receiver recordings of its speed and height dynamics. The accuracy of the proposed fuel consumption estimation method is evaluated by comparing the estimated fuel consumption with the fuel consumption readout from the car’s on-board diagnostics (OBD) port, see Fig. 1. The results from three test drives in mixed highway and urban conditions, with three different car models, indicate that the proposed method can estimate the fuel consumption with a root mean square error of the order of 0.2-0.4 \([g/s]\); in terms of normalized mean square error, that corresponds to slightly less than 10 percent.

II. POWER SOURCES AND LOADS

To develop a model for the instantaneous fuel consumption of the vehicle, we need models for the various power sources and loads in the vehicle, as well as a model for the power flow between them. Hence, we will start by introducing a simple power flow model. Thereafter, we will present models for the different power sources and loads. We will develop the models to resemble the properties of cars with manual transmissions and electronically controlled injection gasoline engines. However, the models will be sufficiently generic to be easily adapted to cars with automatic transmissions and those that run on diesel, E85, etc.

\(^2\)The Swedish Traffic Registry is a publicly accessible register that contains information about the current keeper of the vehicle and vehicle data such as brand, model, vehicle class, engine capacity, wheel diameters, etc. Similar registers also exist in many other countries, e.g., in Denmark, Norway, and Finland.

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<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho_a )</td>
<td>kg/m(^3)</td>
<td>1.225</td>
<td>Mass density of air</td>
</tr>
<tr>
<td>( g )</td>
<td>m/s(^2)</td>
<td>9.806</td>
<td>Standard gravity</td>
</tr>
</tbody>
</table>

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**TABLE I: Physical constants and car model independent parameters used in the fuel consumption estimator.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Q_L )</td>
<td>J/kg</td>
<td>4.44 \cdot 10^8</td>
<td>Lower heating value of gasoline</td>
</tr>
<tr>
<td>( \eta_t )</td>
<td>-</td>
<td>0.96</td>
<td>Transmission efficiency</td>
</tr>
<tr>
<td>( \eta_c )</td>
<td>-</td>
<td>0.98</td>
<td>Combustion efficiency</td>
</tr>
<tr>
<td>( C_r )</td>
<td>N/m</td>
<td>0.013</td>
<td>Roll resistance</td>
</tr>
<tr>
<td>( c_1 )</td>
<td>N/m((rev/s))</td>
<td>9.7 \cdot 10^4</td>
<td>Total friction mean effective pressure coefficient</td>
</tr>
<tr>
<td>( c_2 )</td>
<td>(N/m)((rev/s))</td>
<td>900</td>
<td>Total friction mean effective pressure coefficient</td>
</tr>
<tr>
<td>( c_3 )</td>
<td>(N/m)((rev/s))</td>
<td>18</td>
<td>Total friction mean effective pressure coefficient</td>
</tr>
<tr>
<td>( P_a )</td>
<td>W</td>
<td>1500</td>
<td>Power consumption of the accessories in the car</td>
</tr>
<tr>
<td>( \nu_{idle} )</td>
<td>rev/s</td>
<td>13</td>
<td>Idle engine speed</td>
</tr>
</tbody>
</table>

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A. Power flow model

Let \( P_t [W] \) denote the instantaneous tractive power needed at the wheels for the car to obey the motion change commanded by the driver. Further, let \( P_{t_f} [W] \) denote the power required to overcome the total engine friction, and \( P_a [W] \) the power needed to drive accessories such as the air-conditioner, etc. Next, assuming that the driver never has the clutch engaged when ever \( P_t < 0 \), then all power generated from the wheels will be dissipated through the brakes.\(^3\) We cannot model the required instantaneous gross indicated power, \( P_{ig} [W] \) as

\[
P_{ig} = \frac{1}{\eta_t} \max(P_t, 0) + P_{t_f} + P_a. \tag{1}
\]

Here, \( \eta_t [-] \) denotes the efficiency of the transmission (including the final drive). The efficiency of the transmission depends on several parameters, such as engine speed, torque, gear-ratio, temperature, etc., and thus varies between different car models and with the driving conditions [11], [12]. In [12], the calculated average efficiency for a five-speed manual transmission, varied from 92%-97% depending upon the gear. In our model, we will use a value on the upper end of the scale, i.e., \( \eta_t = 0.96 \). The values of all physical constants and parameters used in the fuel consumption estimator are summarized in Table I.

B. Gross indicated power versus fuel mass flow rate

We can relate the gross indicated power in (1) to the fuel mass flow rate (fuel consumption) \( y [g/s] \) via \[13\]

\(^3\)Note that, even though power is a positive quantity, the idea of negative tractive power is used throughout the paper to mathematically model the fact that when the car is decelerating, the wheels may be used as a power source.
Here, \( Q_L \), \( \eta_c \), and \( \eta_g \) denote the lower heating value of the fuel, the combustion efficiency, and the gross indicated efficiency, respectively. The lower heating value of the fuel varies with the fuel composition. The combustion efficiency and the gross indicated efficiency varies with parameters such as the equivalence ratio and sparking time [13, 14]; parameters that cannot be deduced from the data output by the GPS-receiver. We will therefore, use the following constant values for the parameters, \( Q_L = 44.4 \cdot 10^3 [J/kg] \), \( \eta_c = 0.98 \), and \( \eta_g = 0.31 [15, p.155] \).

**C. Motion dynamics versus tractive power**

Next, we will derive a simple model for the tractive power \( P_t \), needed for the car to obey the driver’s commanded motion change. The model is similar to the models presented in [9], [10], and [16], and is deduced from Newton’s second law of motion and the major forces acting upon the car. The major forces acting upon the car are the tractive force, the air drag force, the roll-resistance force, and the gravity force. We have illustrated these forces and their directions in Fig. 2. The change in velocity, \( \mathbf{v} [m/s] \), of a vehicle with the mass, \( m [kg] \), as a function of the sum of these forces is

\[
m \frac{d\mathbf{v}^p}{dt} = \mathbf{f}_t^p - \mathbf{f}_a^p - \mathbf{f}_r^p - \mathbf{f}_g^p. \tag{3}
\]

Here, \( \mathbf{f}_t [N] \), \( \mathbf{f}_a [N] \), \( \mathbf{f}_r [N] \), and \( \mathbf{f}_g [N] \) denote the tractive force, the air drag force, the roll-resistance force, and the gravity force, respectively. The superscript \( p \) denotes that quantity is expressed with respect to the vehicle’s (platform) coordinate system. The tractive power, \( P_t [W] \), needed to induce the velocity change is

\[
P_t = \mathbf{f}_t^p \cdot \mathbf{v}^p = (m \frac{d\mathbf{v}^p}{dt} + \mathbf{f}_a^p + \mathbf{f}_r^p + \mathbf{f}_g^p) \cdot \mathbf{v}^p \tag{4}
\]

\[
= (m \frac{d\mathbf{v}^p}{dt} + \mathbf{f}_a^p + \mathbf{f}_r^p) \cdot \mathbf{v}^p + \mathbf{f}_g^p \cdot \mathbf{v}^n.
\]

Here, \( \mathbf{a} \cdot \mathbf{b} \) denotes the dot-product between the vectors \( \mathbf{a} \) and \( \mathbf{b} \), and the superscript \( n \) denotes that a quantity is expressed with respect to the local geodetic coordinate system (north, east, and down). Next, note that \( \mathbf{f}_g^p = [0 0 -m \mathbf{g}] \), where \( \mathbf{g} [m/s^2] \) is the magnitude of the local gravity vector. Further, assume that the car experiences no sideslip, i.e., \( \mathbf{v}^p = [v^p_x 0 0] \), where \( v^p_x [m/s] \) is the along-track speed of the car. Then, (4) simplifies to

\[
P_t = \left( m \frac{d\mathbf{v}^p}{dt} + \mathbf{f}_a^p + \mathbf{f}_r^p \right) v^p_x - m \mathbf{g}[\mathbf{v}^n]_z. \tag{5}
\]

Here, \( \mathbf{u}_i, i = \{x, y, z\} \), denotes the \( i \)-th element of the vector \( \mathbf{u} \). The roll resistance force can be modeled as

\[
[f^r_p]_x = C_r m \mathbf{g} \cos(\theta) \cos(\phi), \tag{6}
\]

where \( C_r [-] \) is the roll resistance coefficient of the tires. Further, \( \theta \) and \( \phi \) are the slope of the road in the longitude and lateral directions, respectively. In general, the slope of the road is less than 10 percent, and we will approximate the roll resistance force as

\[
[f^r_p]_x \approx C_r m \mathbf{g}. \tag{7}
\]

The air-drag force can be modeled as

\[
[f^a_p]_x = \frac{\rho_a}{2} A_C (v^p_x)^2, \tag{8}
\]

where \( C_a [-] \) is the air resistance coefficient, \( \rho_a [kg/m^3] \) the mass density of the air, and \( A [m^2] \) the cross area section of the vehicle. Inserting (7) and (8) into (5), we get that the tractive power needed to induce the change in velocity can be described by

\[
P_t = m a^p_x v^p_x + C_r m \mathbf{g} v^p_x + \frac{\rho_a}{2} A C_a (v^p_x)^3 - m \mathbf{g} v^p_n, \tag{9}
\]

where \( a^p_x \triangleq \frac{dv^p_x}{dt} \) and \( \mathbf{v}^n \triangleq [\mathbf{v}^n]_z \). Thus, if the vehicle specific parameters, mass \( m \), air resistance coefficient \( C_a \), and cross area section \( A \), and the environmental and tire dependent roll resistance coefficient \( C_r \), are known, then the tractive power \( P_t \) can be estimated from the motion dynamics \( a^p_x, v^p_x \), and \( v^p_n \). An illustration of the magnitude of the different components in (9) are shown in Fig. 3.

Out of these parameters, only the vehicle’s curb weight\(^4\) information is available in the Swedish Traffic Registry. However, the product \( C_a A \) varies little between cars in the same class (segment). Therefore, given the class the car belongs to (information that can be found in the Swedish vehicle register), we approximate \( C_a A \) with the average value given in Table II. The rolling resistance of a car’s tires depends on several factors, their rotation speed, their temperature, and the texture of the road [17]. For modern car tires, the rolling resistance ranges, approximately from 0.008 on a smooth-textured surface, to 0.018 on rough-textured surface [18]. In our model, we will therefore assume that \( C_r = 0.013 \).

**D. Engine friction versus engine rotation speed**

The total friction mean effective power \( P_{fr} [W] \), needed to do the pumping of the fuel in the cylinders, to run the engine consumables, a full tank of fuel, and a 75 kg heavy driver.

\(^4\)Total weight of a vehicle with standard equipment, all necessary operating consumables, a full tank of fuel, and a 75 kg heavy driver.
TABLE II: Engine volume, peak engine power, gear-ratio, and air resistance specifications for seven gasoline engine cars in the C segment.

<table>
<thead>
<tr>
<th>Model</th>
<th>$V_d$ [cm$^3$]</th>
<th>$P_{\text{peak}}^k$ (r$^<em>$) [kW] @ (r$^</em>$ [rev/s])</th>
<th>$G_L$</th>
<th>$G_F$</th>
<th>$G_F G_L$</th>
<th>$L$</th>
<th>$C_a A$ [m$^2$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.6 Liters</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Audi A3 1.6 FSI (2005)</td>
<td>1598</td>
<td>85 (100)</td>
<td>0.71:1</td>
<td>4.53:1</td>
<td>3.22:1</td>
<td>6</td>
<td>0.70</td>
</tr>
<tr>
<td>Kia Cerra Hatch 1.6 (2011)</td>
<td>1591</td>
<td>91 (105)</td>
<td>0.73:1</td>
<td>4.27:1</td>
<td>3.11:1</td>
<td>6</td>
<td>–</td>
</tr>
<tr>
<td>Citroën C4 1.6i 16v (2004)</td>
<td>1587</td>
<td>80 (97)</td>
<td>–</td>
<td>–</td>
<td>5</td>
<td>–</td>
<td>0.68</td>
</tr>
<tr>
<td>Ford Focus 1.6i (2007)</td>
<td>1596</td>
<td>85 (100)</td>
<td>0.88:1</td>
<td>4.06:1</td>
<td>3.57:1</td>
<td>5</td>
<td>0.73</td>
</tr>
<tr>
<td>Seat Toledo 1.6 (2004)</td>
<td>1596</td>
<td>75 (93)</td>
<td>0.85:1</td>
<td>4.53:1</td>
<td>3.85:1</td>
<td>5</td>
<td>–</td>
</tr>
<tr>
<td>Skoda Octavia 1.6 FSI (2004)</td>
<td>1598</td>
<td>84 (100)</td>
<td>0.78:1</td>
<td>4.53:1</td>
<td>3.53:1</td>
<td>5</td>
<td>–</td>
</tr>
<tr>
<td>VW Golf 1.6 FSI (2003)</td>
<td>1598</td>
<td>84 (100)</td>
<td>0.71:1</td>
<td>4.53:1</td>
<td>3.22:1</td>
<td>6</td>
<td>0.71</td>
</tr>
<tr>
<td>Average values</td>
<td>–</td>
<td>83 (99)</td>
<td>0.78:1</td>
<td>4.41:1</td>
<td>3.42:1</td>
<td>-</td>
<td>0.71</td>
</tr>
<tr>
<td>2.0 Liters</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Audi A3 2.0 FSI (2003)</td>
<td>1984</td>
<td>110 (100)</td>
<td>0.82:1</td>
<td>3.65:1</td>
<td>2.99:1</td>
<td>6</td>
<td>0.66</td>
</tr>
<tr>
<td>Kia Cerra Hatch 2.0 (2011)</td>
<td>1998</td>
<td>115 (103)</td>
<td>0.76:1</td>
<td>4.19:1</td>
<td>3.18:1</td>
<td>6</td>
<td>–</td>
</tr>
<tr>
<td>Citroën C4 2.0i 16v (2007)</td>
<td>1997</td>
<td>103 (100)</td>
<td>–</td>
<td>–</td>
<td>5</td>
<td>–</td>
<td>0.70</td>
</tr>
<tr>
<td>Ford Focus 2.0 (2011)</td>
<td>1999</td>
<td>119 (108)</td>
<td>0.81:1</td>
<td>3.82:1</td>
<td>3.09:1</td>
<td>5</td>
<td>–</td>
</tr>
<tr>
<td>Seat Toledo 2.0 FSI (2004)</td>
<td>1984</td>
<td>110 (100)</td>
<td>0.87:1</td>
<td>3.84:1</td>
<td>3.43:1</td>
<td>5</td>
<td>–</td>
</tr>
<tr>
<td>Skoda Octavia 2.0 FSI (2004)</td>
<td>1984</td>
<td>110 (100)</td>
<td>0.82:1</td>
<td>3.65:1</td>
<td>2.99:1</td>
<td>6</td>
<td>–</td>
</tr>
<tr>
<td>VW Golf 2.0 FSI (2003)</td>
<td>1984</td>
<td>110 (100)</td>
<td>0.82:1</td>
<td>3.65:1</td>
<td>2.99:1</td>
<td>6</td>
<td>0.71</td>
</tr>
<tr>
<td>Average values</td>
<td>–</td>
<td>111 (102)</td>
<td>0.82:1</td>
<td>3.82:1</td>
<td>3.11:1</td>
<td>-</td>
<td>0.69</td>
</tr>
</tbody>
</table>

Fig. 3: Illustration of the magnitude of the different terms in the fuel consumption model for a vehicle with a 1.6 [dm$^3$] four-stroke engine, mass $m = 1000$ [kg], and an air drag factor $C_a A = 0.7$ [m$^2$]. Remaining model parameters are given in Table I.

Further, $n_R$ denotes the ratio between the number of strokes of the pistons to the number of rotations of the crankshaft, i.e., $n_r = 1$ for a two stroke engine and $n_R = 2$ for a four stroke engine. A simple model for the total motored friction mean effective pressure $k_{tmf}(r) [N/m^2]$ is given by [14], [19]

$$k_{tmf}(r) = c_1 + c_2 r + c_3 r^2. \quad (11)$$

In [14, pp.722-723], the coefficients that best fit (11) with the measured total motorized friction mean effective pressure for several four stroke engines with the displacement volumes ranging from 845-2000 [cm$^3$] were identified as $c_1 = 9.7 \times 10^4$, $c_2 = 900$, and $c_3 = 18$. In our fuel consumption estimator, we will assume that $k_{if}(r) \equiv k_{tmf}(r)$.

E. Accessories power consumption

Generally, the most power demanding accessory in a modern car is the air-conditioner, which, for a sedan at peak power, can induce a load of 5-6 [kW] on the engine [20]. The power consumed by the air-conditioner to generate a climate that satisfies the driver, depends on a number of factors such as the solar radiation, ambient air temperature, air humidity, and number of people in the car. As these factors are obviously not deducible from the data from the GPS-receiver, we will assume a constant value, $P_{ac} = 1.5$ [kW], for the power consumption of the accessories in the car.

III. SYNTHESIZING THE ENGINE ROTATION SPEED

In the previous section models for the power sources and load in vehicle were presented. The magnitude of one of them, the power needed to overcome the engine friction, depends on engine speed. Since the engine speed is not directly measurable using a GPS-receiver, we will in this section present a way to synthesize the engine speed using knowledge about the speed of the vehicle, the vehicles engine size, etc.
The speed of the car is related to the rotation speed of the engine, \( r \, [\text{rev/s}] \), according to
\[
\nu_x^p = \phi_w \frac{r}{G_F G_L} , \quad \ell = 1, \ldots, L ,
\]
where \( \phi_w \, [\text{m/rev}] \), \( G_F \, [-] \), and \( G_L \, [-] \) denote the circumference of the wheels, the final-drive gear-ratio, and the transmission gear-ratio at gear \( \ell \).

Let, \( L \) be the number of the highest gear (top gear), then a lower bound on the engine speed is given by
\[
r \geq r_{\text{speed}} , \quad r_{\text{speed}} = G_F G_L \frac{\nu_x^p}{\phi_w} .
\] (13)

Another lower bound on the engine speed may be found from the tractive (wheel) power \( P_t^{\text{max}}(r) \) curve, that specifies the maximum power that can be delivered at the wheels at a given engine speed. Since \( P_t^{\text{max}}(r) \) must be greater than or equal to the needed tractive power \( P_t \), we have that
\[
r \geq r_{\text{power}} , \quad r_{\text{power}} = \text{arg min}_r ( P_t^{\text{max}}(r) \geq P_t ) .
\] (14)

By combining (13) and (14), we get that
\[
r \geq r_{\text{min}} , \quad r_{\text{min}} = \text{max}(r_{\text{speed}}, r_{\text{power}}, r_{\text{idle}}) ,
\]
where \( r_{\text{idle}} \) is the engine rotation speed at idle.

The Swedish vehicle register only holds information regarding the circumference of the wheels and the engine’s peak power \( P_{\text{peak}} \, [\text{kW}] \), and (15) therefore, cannot directly be used to lower bound the rotational velocity of the engine. However, as seen from the vehicle data in Table II, the product \( G_F G_L \) varies little between cars in the same class. Further, the rotational velocity of the engine when at idle, is for most gasoline car engines between 10-16 \( [\text{revs/s}] \). Therefore, in our model we will use the average value in Table II for the product \( G_F G_L \), and let \( r_{\text{idle}} = 13 \, [\text{rev/s}] \). To find \( r_{\text{power}} \), we will assume that the power curve of the engine is approximately described by the following piecewise linear function
\[
P_t^{\text{max}}(r) = \begin{cases} \frac{2}{200} P_{\text{peak}}, & r \leq 80 \\ \frac{1}{200} P_{\text{peak}} r + \frac{1}{2} P_{\text{peak}}, & 80 < r < 100 \\ P_{\text{peak}}, & r \geq 100 \end{cases} . \] (16)

That is, between 0-80 \( [\text{rev/s}] \), the maximum power the engine can deliver increases linearly with the engine’s rotation velocity, until 90% of \( P_{\text{peak}} \). The difference between 80-100 \( [\text{rev/s}] \) the maximum power the engine can deliver increases at a much lower rate with the engine’s rotation velocity, until \( P_{\text{peak}} \). Above 100 \( [\text{rev/s}] \), the maximum power the engine can deliver is constantly \( P_{\text{peak}} \). In Fig. 4, the measured tractive (wheel) power curves for a Ford Fiesta 1.25 \( (P_{\text{peak}} = 55 \, [\text{kW}]) \), a Peugeot 307 1.6 \( (P_{\text{peak}} = 81 \, [\text{kW}]) \), an Audi A3 1.8T \( (P_{\text{peak}} = 110 \, [\text{kW}]) \), and a Volvo V70 2.4T \( (P_{\text{peak}} = 147 \, [\text{kW}]) \) are shown; the measured tractive power curves are based upon the data found in [21]. Also shown are the tractive (wheel) power curves estimated using the engine power model in (16), i.e., \( \bar{P}_t^{\text{max}}(r) = \eta_t P_t^{\text{max}}(r) \). The agreement between the measured and estimated curves shows that the model in (16) captures the main characteristics of the brake power curve of an engine, and that we may use it for evaluating the lower bound for the engine’s rotational speed. For more refined models for the power curve of an engine, see e.g., [22].

IV. CALCULATING THE MODEL INPUTS FROM GPS-DATA

The model for the fuel consumption (fuel mass flow) developed in the previous sections is driven by three inputs, the along track speed \( \nu_x^p \), the along track acceleration \( a_x^p \), and the vertical velocity \( \nu_w^p \). It may seem straightforward to calculate these quantities by simply differentiating the speed and height measurements of the GPS-receivers. However, automotive applications provide a challenging environment for the reception of GPS signals due to factors like antenna placement, potential interferences from embedded electronics, and signal multipath and obstruction, which may cause large errors in the measurements [23]. When differentiating the speed and height measurements, the high frequency error components in the measurements are amplified, and the calculated along track acceleration and vertical velocity may become very noisy or distorted by outliers [24].

One method that has successfully been used to address these problems, see e.g., [25], is the polynomial regression method, in which it is assumed that the speed and the height of the vehicle locally are described by polynomial models. That is, the \( k \)-th speed measurement provided by the GPS-receiver is modeled as
\[
\nu_{x,k}^p = \nu_{x,k}^p(t_k) + w_k ,
\] (17)

where
\[
\nu_{x,k}^p(t) = \alpha_0 t^2 + \alpha_1 t + \alpha_2 , \quad t \in [t_k - T/2, t_k + T/2] . \] (18)
Here, \( t_k \) [s] denotes the sample time of the \( k \)th measurement. Further, \( w_k \) [m/s] denotes the noise in the measurements and \( T \) [s] is the time period for which the polynomial model may be considered valid. The \( N \) measurements taken in the interval \([t_k - T/2, t_k + T/2]\) can then be modeled as

\[
\tilde{z}_k = H_k \theta + w_k, \tag{19}
\]

where

\[
\tilde{z}_k = \begin{bmatrix} \tilde{v}^p_{x,k-[T]} & \cdots & \tilde{v}^p_{x,k} & \cdots & \tilde{v}^p_{x,k+[T]} \end{bmatrix}^\top, \tag{20}
\]

\[
H_k = \begin{bmatrix} 1 & (t_k-[T] - t_k) & (t_k-[T] - t_k)^2 \\ \vdots & \vdots & \vdots \\ 1 & 0 & 0 \\ \vdots & \vdots & \vdots \\ 1 & (t_k+[T] - t_k) & (t_k+[T] - t_k)^2 \end{bmatrix}, \tag{21}
\]

\[
w_k = \begin{bmatrix} w_{k-[T]} & \cdots & w_k & \cdots & w_{k+[T]} \end{bmatrix}^\top, \tag{22}
\]

and

\[
\theta = \begin{bmatrix} \alpha_0 & \alpha_1 & \alpha_2 \end{bmatrix}^\top. \tag{23}
\]

Here, \(( \cdot )^\top\) is used to denote the transpose operator. The coefficients of the polynomial model (18) can be estimated using the weight least squares method. That is,

\[
\hat{\theta}_k = (H_k^\top Q_k^{-1} H_k)^{-1} H_k^\top Q_k^{-1} \tilde{z}_k, \tag{24}
\]

where \( Q_k \) [(m/s)\(^2\)] denotes the covariance of the measurement error vector \( w_k \). Now, since \( a_x^p(t) = \frac{d^2 a(t)}{dt^2} = 2 \alpha_0 t + \alpha_1 \), then the acceleration of the vehicle at time \( t_k \) according to the polynomial model is given by \( \hat{a}^p_{x,k} = [\hat{\theta}_k]_2 \). Further, by the same reasoning, the speed \( \hat{v}^p_{x,k} = [\hat{\theta}_k]_3 \). The vertical velocity \( \hat{v}^p_z \) may be estimated in a similar way by assuming the height \( h_k \) of the car’s trajectory to locally be described by a polynomial model.

Also noteworthy is that if measurements of the GPS-receiver are uniformly sampled and the measurement noise covariance \( Q = c I \), where \( I \) denotes the identity matrix and \( c \) is a positive scalar, then the acceleration estimate of the polynomial regression is equivalent to the acceleration estimate obtained when filtering the speed signal through the minimum noise gain FIR filter differentiator derived in [26]. Thus, for a computational complexity sensitive application, the minimum noise gain FIR filter differentiator may be a good alternative. However, the benefit of the polynomial regression in (24) is that outliers in the measurements of the GPS-receiver may easily be detected by monitoring the magnitude of the residual of the fitted measurements.

If a measurement platform that in addition to a GPS-receiver also houses inertial sensors or wheel speed sensors is used, then it is advisable that instead of the polynomial regression method, a data fusion scheme such as that in e.g., [27] is used. In general, this will allow the inputs to the fuel consumption model to be calculated more accurately and at a higher rate. In [28] and [29][pp.435-462], surveys reviewing methods to track the motion dynamics of cars using a plurality of sensors can be found.

V. MODEL EVALUATION METHODOLOGY AND RESULTS

To evaluate the proposed fuel consumption estimation method, we tested it on three vehicles: a Peugeot 207, an Audi A3, and a Volvo XC70. In Table III, the vehicle parameters available in the Swedish Traffic Register for these three vehicles are shown. We drove these three vehicles along different trajectories of length 27-32 km; trajectories that consisted of a mixture of highway and urban roads. During these drives, we used the GPS and OBD data logger shown in Fig. 1 to record the engines’ rotation speed, air flow rate, and air-to-fuel ratio, as well as the speed and height measurements from the GPS-receiver. All the data was recorded at a rate of 5 Hz.

The data was then processed in accordance with the system illustrated in Fig. 5. That is, given the registration plate numbers of the cars, we downloaded the vehicle parameters available in the Swedish vehicle register, which together with the average parameter in Table I were used as parameters in our fuel consumption model. From the recorded GPS speed and height measurements, we then, using the polynomial regression method described in Section IV, estimated the along track acceleration, along track speed \( \hat{v}^p_{x,k} \), and vertical speed \( \hat{v}^p_z \). From these estimates, we then estimated the engine speed \( \hat{\omega}_{\text{engine}} \), using the lower bound described in Section III. We then used the estimated along track acceleration, along track speed, vertical speed, and engine speed to drive the fuel consumption model described in Section II.

Thereafter, we compared the estimated fuel consumption with the (measured) reference fuel consumption \( \hat{y}_k \) calculated from the recorded OBD data. The reference fuel consumption was calculated by dividing the air flow rate data with the air to fuel ratio data, and then low-pass filtering the result through a moving average filter with a 1 [Hz] bandwidth. The low pass filtering was done to make sure the measured fuel consumption had the same bandwidth as the motion dynamics estimated via the polynomial regression; the polynomial regression for the estimation of the along track acceleration and horizontal speed was done using window sizes \( N = 5 \) samples \( (T = 1 \text{ [s]}) \) and \( N = 50 \) samples \( (T = 10 \text{ [s]}) \), respectively.

In Fig. 6, scatter plots of the estimated versus measured fuel consumption for the three drives are shown. Both the estimated fuel consumption when using the true engine speed and the estimated engine speed are shown. The root mean square error and mean error are also included in the scatter plots. From the scatter plots it can be seen that on average, the
proposed instantaneous fuel consumption method works quite well, with a root mean square error on the order of 0.20-0.40 g/s, depending upon the size of the engine and car. Taking the magnitude of the fuel consumption of the car model under test into account, then in terms of normalized mean square error, i.e., $\frac{\sum (\hat{y}_k - \tilde{y}_k)^2}{\sum (\tilde{y}_k)^2}$, the estimation error of the proposed method for all three tests is slightly less than 10%. Further, from the scatter plots it can also be seen that the use of the estimated engine speed instead of the measured engine speed, mainly affects the estimation accuracy at time instances with low power demands (low fuel consumption). This is because at the time periods with moderate motion dynamics, the factor that has the largest effect on the power consumption is the engine speed, see Fig. 3.

To illustrate the behavior of the estimator during abrupt velocity changes, the measured and estimated fuel consumption when the Volvo XC 70 is accelerating from 0 to 100 [km/h] is shown in Fig. 7. Also shown is the measured and estimated engine speed. From the figure, we can see that the estimated fuel consumptions, both with the estimated and true engine speeds, agrees quite well with the measured fuel consumption; even though the engine speed estimator, most of the time, underestimates the engine’s speed (especially during the gear changes, at around 6, 10, and 13 [s] into the acceleration, when the clutch is disengaged and the engine speed peaks and the car momentarily stops accelerating). Clearly, during the acceleration, the needed tractive power dominates the total power consumption of the engine, and the power needed to overcome the engine friction only plays a minor role in the total fuel consumption.

VI. DISCUSSION AND CONCLUSIONS

We have proposed a method to estimate the instantaneous fuel consumption of a car only from the data recorded by a GPS-receiver onboard the car and the vehicle parameters accessible through the Swedish vehicle register. The proposed estimator is based on a physical model of the power sources and loads in a car, and is driven by the along-track acceleration, along-track speed, and vertical speed calculated via a polynomial regression of the data output by the GPS-receiver. We have tested the proposed method on three cars of different makes and sizes, that were driven on a mixture of highway and
urban roads. The results show that the fuel consumption can be estimated with a normalized mean square error of slightly less than 10 percent. We therefore, believe that the proposed method may be a useful tool in the development of eco-driving applications for smartphones, or in other situations where a car’s instantaneous fuel consumption needs to be monitored, but where it is not feasible or desirable to connect to the car’s internal sensors.

Indeed, an eco-drive meter based on the proposed fuel consumption estimation method has, together with other real-time feedback usable for, so called, usage based insurance (UBI), been implemented as a part of the virtual vehicle dashboard illustrated in Fig. 8; refer to [30] for details. UBI is a type of car insurance where the insurance fee is determined, not only from traditional factors such as age, gender, vehicle model, etc., but also from parameters such as distance traveled, time of the trip, location, or driver behavioral parameters such as smoothness and eco-ness; parameters monitored in real-time through a plurality of sensors. Accordingly, functionality such as the eco-drive meter does not only provide the driver with guidance towards a more eco-friendly driving style with savings in terms of fuel consumption, but also may imply a safer driving style with savings on insurance fees as a byproduct.

Fig. 6: Scatter plots of the estimated versus measured fuel instantaneous fuel consumptions, with the true engine speed and the lower bound on the engine speed feedback usable for, so called, usage based insurance (UBI), been implemented as a part of the virtual vehicle dashboard illustrated in Fig. 8; refer to [30] for details. UBI is a type of car insurance where the insurance fee is determined, not only from traditional factors such as age, gender, vehicle model, etc., but also from parameters such as distance traveled, time of the trip, location, or driver behavioral parameters such as smoothness and eco-ness; parameters monitored in real-time through a plurality of sensors. Accordingly, functionality such as the eco-drive meter does not only provide the driver with guidance towards a more eco-friendly driving style with savings in terms of fuel consumption, but also may imply a safer driving style with savings on insurance fees as a byproduct.

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