Matrix Algebra
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Deadline: 2014–04–02, 10.00

Homework #1

Read Chapter 0 in "Matrix Analysis" and learn as much as possible.

1. Let $p(x)$ be a first order polynomial $p(x) = \alpha_1 + \alpha_2 x$ where $\alpha_i, x \in \mathbf{R}$. Let U be the vector space of real first order polynomials and let V be the vector space of real numbers. Define the mapping $T : U \to V$ such that $T(p(x)) = \alpha_2$ (i.e., the derivative of $p(x)$). Verify that T is a linear transformation. Choose a basis for U and one for V. What is the matrix representation of the transformation in terms of these bases? Choose another basis for U. What is the matrix representation in this case?

2. Show that $(AB)^T = B^T A^T$ for matrices of compatible dimensions.

3. Determine the range- and the null-spaces of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \end{bmatrix}$$

   What are the dimensions of these spaces? What is the rank of A?

4. Show that $\det(I+AB) = \det(I+BA)$ where A and B may be rectangular matrices of appropriate dimensions. (Hint: You may use the Schur complement determinantal formulae.)

5. Verify the statement that y_2 is orthogonal to z_1 in Section 0.6.4 Gram-Schmidt orthonormalization. Make a graph illustrating the first step of the procedure.

6. Prove the "push through rule:"

$$A(I_m + BA)^{-1} = (I_n + AB)^{-1} A$$

   where inverses are assumed to exist, I_n is an $n \times n$ identity matrix, $A \in M_{n,m}(\mathbf{F})$ and $B \in M_{m,n}(\mathbf{F})$.