KTH Electrical Engineering

Exam in EG2050 System Planning, 17 March 2014, 14:00–19:00, Q33, Q34, Q36, V01, V12

Allowed aids

In this exam you are allowed to use the following aids:

- Calculator without information relevant to the course.
- One **handwritten**, **single-sided** A4-page with **your own** notes (original, not a copy), which should be handed in together with the answer sheet.

PART I (MANDATORY)

Write all answers on the answer sheet provided. Motivations and calculations do not have to be presented.

Part I can yield 40 points in total. The examinee is guaranteed to pass if the score is at least 33 points. If the result in part I is at least 31 points, then there will be a possibility to complement for passing the exam with the grade E.

Problem 1 (4 p)

Answer the following theoretical questions by choosing *one* alternative, which you find correct.

a) (2 p) A balance responsible player has the following responsibilities: I) Physical responsibility that the system continuously is supplied as much power as consumed by the customers of the player, II) Economical responsibility that the system continuously is supplied as much power as consumed by the customers of the player, III) Economical responsibility that the system during each trading period (for example one hour) is supplied as much energy as consumed by the customers of the player.

1. None of the statements is true.
2. Only I is true.
3. Only II is true.
4. Only III is true.
5. I and II are true but not III.

b) (1 p) We use the notion "real-time trading" to describe all the trading which occurs during the hour of delivery (or any other trading period). Which of the following contracts can be traded in a real-time market?

1. Balance power, i.e., when a balance responsible player is selling any surplus in their balance to the system operator, or when a balance responsible player is buying from the system operator to cover for any deficit in their balance.
2. Firm power, i.e., the customer buys the same amount of energy in each trading period as long as the contract is valid.
3. Regulation power, i.e., when a player at request from the system operator is supplying more power to the system (up-regulation) or when a player at request from the system operator is supplying less power to the system (down-regulation).

c) (1 p) What does a take-and-pay contract mean?

1. The customer must in advance notify the supplier about how much the customer will consume during each trading period.
2. The customer buys the same amount of energy in each trading period as long as the contract is valid.
3. During the time the contract is valid, the customer is allowed to consume as much energy they want each trading period, provided that the maximal power is not exceeded.

Problem 2 (6 p)

Assume that the electricity market in Land has perfect competition, perfect information and that there are neither capacity, transmission nor reservoir limitations. Data for the power plants in Land are shown in table 1. The variable operation costs are assumed to be linear within the intervals; the production is zero if the price is on the lower price level and the production is maximal at the higher price level.

Table 1 Data for the electricity producers in Land.

Power source	Production capability [TWh/year]	Variable costs [¤/MWh]
Hydro power	66	5
Nuclear power	60	90–100
Biofuel	20	200–400
Fossil fuels	10	300–500

a) (3 p) What will the electricity price be in Land if the electricity consumption is not price sensitive and amounts to 142 TWh/year?

b) (1 p) Assume that in addition to the power plants in table 1 there are 8 TWh wind power with a negligible variable cost. What will the electricity price be in Land if the electricity consumption is still 142 TWh/year (and not price sensitive)?

c) (2 p) Asssume that the fixed costs of 8 TWh wind power are 4 240 M¤/year. How large subsidies are necessary if the wind power should not be making a loss? (Answer 0 if the wind power is making a profit at the electricity price from problem b.)

Problem 3 (6 p)

Consider a power system divided in five areas. At a certain occasion there is balance between production and consumption in the system and the frequency is exactly equal to 50 Hz. Data for the primary control in the system are given in table 2. Data for the transmission lines between the countries are shown in table 3. Each transmission line is equipped with a protection system which after a short time delay disconnects the line if the power flow exceeds the maximal capacity of the line. The power flow on the HVDC line are not affected by the frequency of the system, but can only be controlled manually..

Table 2 Data for the primary control.

Area	Gain (available between 49.9 and 50.1 Hz) [MW/Hz]
A	2 000
B	2 000
C	1 000
D	500
E	500

Table 3 Data for the interconnections.

Connection	Type	Current transmission [MW]	Maximal capacity [MW]
A ↔ B	Alternating current	1 000 MW from A to B	2 000
A ↔ C	Direct current (HVDC)	600 MW from A to C	600
A ↔ D	Direct current (HVDC)	400 MW from A to D	400
B ↔ D	Direct current (HVDC)	450 MW from B to D	500
B ↔ E	Alternating current	2 000 MW from B to E	2 500
C ↔ D	Alternating current	1 000 MW from C to D	1 500

a) (3 p) At this occasion, the load in area A is decreased by 45 MW. What will the frequency be in area A when the primary control has restored the balance between generation and consumption?

b) (1 p) What will the frequency be in area C after the event in area A?

c) (1 p) What will the frequency be in area D after the event in area A?

d) (1 p) What will the frequency be in area E after the event in area A?

Problem 4 (12 p)

Stads energi AB owns a thermal power plant with three blocks. Assume that the company has formulated their short-term planning problem as a MILP problem and that the following symbols have been introduced:

Indices for the power plants: Block I - 1, Block II - 2, Block III - 3.

β_{Gg} = variable operation cost in power plant g, g = 1, 2, 3,
C_g^+ = start-up cost in power plant g, g = 1, 2, 3,
D_t = contracted load during hour t, t = 1, …, 24,
$G_{g,t}$ = generation in power plant g, hour t, g = 1, 2, 3, t = 1, …, 24,
λ_t = expected electricity price at ElKräng hour t, t = 1, …, 24,
p_t = purchase from ElKräng hour t, t = 1, …, 24,
r_t = sales to ElKräng hour t, t = 1, …, 24,
$s_{g,t}^+$ = start-up variable for power plant g, hour t, g = 1, 2, 3, t = 1, …, 24,
$u_{g,0}$ = unit commitment of power plant g at the beginning of the planning period, g = 1, 2, 3,
$u_{g,t}$ = unit commitment of power plant g, hour t, g = 1, 2, 3, t = 1, …, 24.

a) (3 p) Which of the symbols above represent optimisation variables and parameters respectively?

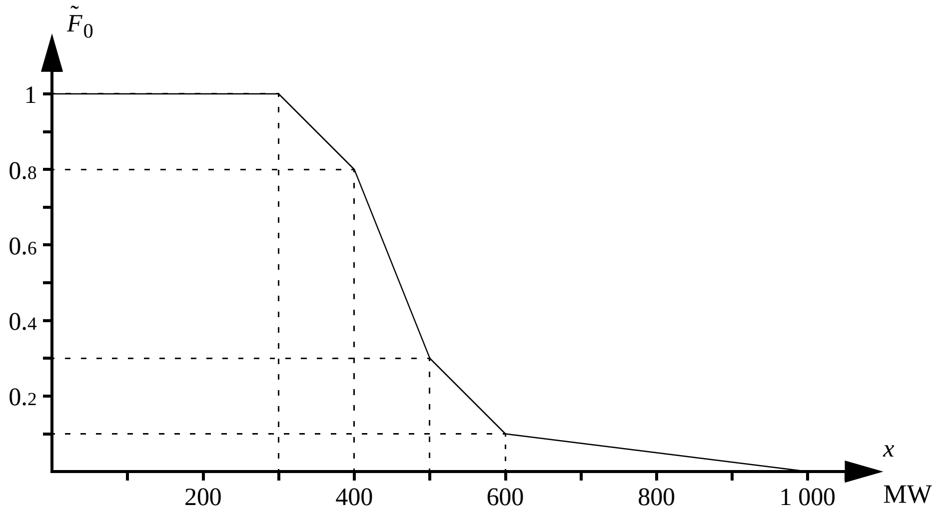
b) (4 p) Stads energi AB sells power to customers with firm power contracts, but the company also has the possibility to trade at the local power exchange ElKräng. Formulate the objective function if the aim of the planning problem is to maximise the income of sold electricity at ElKräng minus the costs of purchasing electricity from ElKräng and minus the costs of the thermal power plant. Use the symbols defined above.

c) (4 p) Formulate the constraint that sets the relation between $u_{g,t}$, $u_{g,t-1}$ and $s_{g,t}^+$ for hour t. Notice that the constraint must be formulated without using any other optimisation variables than those defined above!

d) (1 p) The best efficiency of the power plant Strömmen is achieved at the discharge 80 m^3/s. The power plant is then generating 30 MW. How large is the maximal production equivalent of the power plant?

Problem 5 (12 p)

The national grid in Nchi is supplied by three larger hydro power plants with a combined installed capacity of 700 MW and a 100 MW thermal power plant. The figure below shows the duration curve of the total load in Nchi.



a) (1 p) How large is the probability that the load in Nchi is larger than 800 MW?

b) (3 p) Assume that all power plants have 100% availability and that the variable operation cost is 10 ¤/MWh in the thermal power plant, whereas the variable operation cost in the hydro power plants is negligible. Use probabilistic production cost simulation to calculate the expected total operation cost per hour.

c) (2 p) Assume that the hydro power plants have 100% availability and that the thermal power plant has 90% availability. What is the risk of power deficit in Nchi?

d) (2 p) Assume that a Monte Carlo simulation is carried out of the power system in Nchi. The simulation uses a multi-area model, which takes into account the limitations in transmission capacity as well as the losses on the lines between the areas. 2 000 scenarios are generated in the simulation and the sum of the operation cost in all these scenarios is 240 000 ¤/h, i.e.

$$\sum_{i=1}^{n} toc_i = 240\ 000.$$

What is the estimate of *ETOC* from this simulation?

e) (2 p) Assume that complementary random numbers are used to improve the simulation of Nchi. What is the value of the complementary random number, D^*, if the total load of the system is randomised to D = 400 MW?

f) (2 p) The expectation value $E[X]$ is to be determined using a combination of a control variate and stratified sampling. Assume that L strata have been defined and let ω_h denote the stratum weight of stratum h. Introduce the symbol $x_{h,\,i}$ for the i:th observation of X from stratum h and let $z_{h,\,i}$ denote the i:th observation from stratum h of the control variate, Z. The total number of observations is n, and we use the symbol n_h to denote the number of observations from stratum h. How is the estimate m_X calculated?

1. $m_X = \frac{1}{n}\sum_{h=1}^{L}\sum_{i=1}^{n_h}\omega_h(x_{h,\,i} - z_{h,\,i}) + \frac{1}{n}\sum_{h=1}^{L}\sum_{i=1}^{n_h} z_{h,\,i}$.
2. $m_X = \frac{1}{n}\sum_{h=1}^{L}\sum_{i=1}^{n_h}\omega_h x_{h,\,i} + E[Z]$.
3. $m_X = \sum_{h=1}^{L}\frac{\omega_h}{n_h}\sum_{i=1}^{n_h} x_{h,\,i} + E[Z]$.
4. $m_X = \sum_{h=1}^{L}\frac{\omega_h}{n_h}\sum_{i=1}^{n_h}(x_{h,\,i} - z_{h,\,i}) + \frac{1}{n}\sum_{h=1}^{L}\sum_{i=1}^{n_h} z_{h,\,i}$.
5. $m_X = \sum_{h=1}^{L}\frac{\omega_h}{n_h}\sum_{i=1}^{n_h}(x_{h,\,i} - z_{h,\,i}) + E[Z]$.

PART II (FOR HIGHER GRADES)

All introduced symbols must be defined. Solutions should include sufficient detail that the argument and calculations can be easily followed.

The answer to each problem must begin on a new sheet, but answers to different parts of the same problem (a, b, c, etc.) can be written on the same sheet. The fields *Namn* (Name), *Blad nr* (Sheet number) and *Uppgift nr* (Problem number) must be filled out on every sheet.

Part II gives a total of 60 points, but this part will only be marked if the candidate has obtained at least 33 points in part I. Then the results of parts I and II and the bonus points will be added together to determine the examination grade (A, B, C, D, E).

Problem 6 (10 p)

Unionen has four member states: Aland, Beland, Celand and Deland. At the time being each member state has its own electricity market and the trading between the countries are determined by long-term contracts. Data for generation and demand are given in table 4 and data for the transmission capacity between the countries is given in table 5. The variable operation costs are assumed to be linear within the intervals; the production is zero if the price is on the lower price level and the production is maximal at the higher price level.

Assume that Unionen would introduce a commen electricity market instead. This would mean that the old long-term contracts are cancelled and that the electricity trading between the countries would then depend on supply and demand as well as the maximal transmission capability of the interconnections.

a) (7 p) How would this reform affect producers and consumers in Unionen?

NOTICE! In order to receive full score in this problem, the answer must be supported by rough calculations which show how the electricity price would be changed for different players! You may in these rough calculations assume that there is perfect competition, perfect information and that there are neither capacity nor reservoir limitations.

b) (3 p) Would a common electricity market be profitable for the society?

NOTICE! As in the previous problem your answer has to be supported by rough calculations!

Table 4 Data for generation and demand in the electricity market of Unionen.

Power source	Production capability [TWh/year]				Variable costs [¤/MWh]
	Aland	Beland	Celand	Deland	
Hydro power	60	120	–	5	5
Nuclear power	80	–	–	20	80–120
Fossil fuels	20	–	60	20	300–500
Demand	136	105	41	37	

Table 5 Data for the trading between the countries in Unionen.

Interconnection	Contracted trading	Maximal capacity [TWh/year]
Aland ↔ Beland	Aland imports 5 TWh from Beland at the price 300 ¤/MWh	10
Aland ↔ Celand	Aland exports 6 TWh to Celand at the price 380 ¤/MWh	14
Aland ↔ Deland	Aland exports 5 TWh to Deland at the price 350 ¤/MWh	10
Beland ↔ Celand	Beland exports 5 TWh to Celand at the price 300 ¤/MWh	8

Problem 7 (10 p)

The power system in Rike is divided in two areas. There are large amounts of hydro power in the northern part of the system, but the main consumption centres are in the southern part. There are eight parallel AC transmission lines between the two areas; each line has a maximal transmission of 500 MW. These lines can be assumed to have identical electrical properties, which means that a certain flow will be divided equally between the lines. If for example the flow is 3 200 MW from Northern to Souther Rike then each line will transfer 400 MW.

The primary control of Rike is divided in a normal operation reserve and a disturbance reserve. The normal operation reserve, which is used to manage normal variations in for example load and wind power generation, is available in the frequency range 49.9–50.1 Hz and has a total gain of 3 000 MW/Hz, where 2 500 MW/Hz is provided by power plants in northern Rike. The disturbance reserve is available in the frequency range 49.5–49.9 Hz and has a total gain of 2 500 MW/Hz, where 2 000 MW/Hz is provided by power plants in northern Rike. The disturbance reserve is designed to manage a so-called dimensioning fault without the frequency decreasing to less than 49.5 Hz. This requirement is also fulfilled in a situation where the normal operation reserve is fully utilised (i.e., when the system frequency is 49.9 Hz).

If a dimensioning fault should occur, Riksnät (the system operator in Rike) must immediately activate bids in the real-time balaninc market to restore the disturbance reserve; the system will then be ready to manage another large disturbance. The goal is that the frequency should be increased to at least 49.9 Hz within ten minutes after a dimensioning fault has occured. Moreover, there should be at least 800 MW unused transmission capacity between Northern and Southern Rike.

Consider an occasion when the frequency in Rike is 49.9 Hz and the transmission between Northern to Southern Rike amounts to 2 800 MW. The bids available to Riksnät are shown in table 6. The bids do not have to be accepted as a whole, but Riksnät may choose how many MW that should be activated in each bid. Which bids should be activated if one of the three dimensioning faults below occurs? Assume that Riksnät wants to minimise the cost of restoring the disturbance reserve.

a) (3 p) An outage of 800 MW generation in Northern Rike.

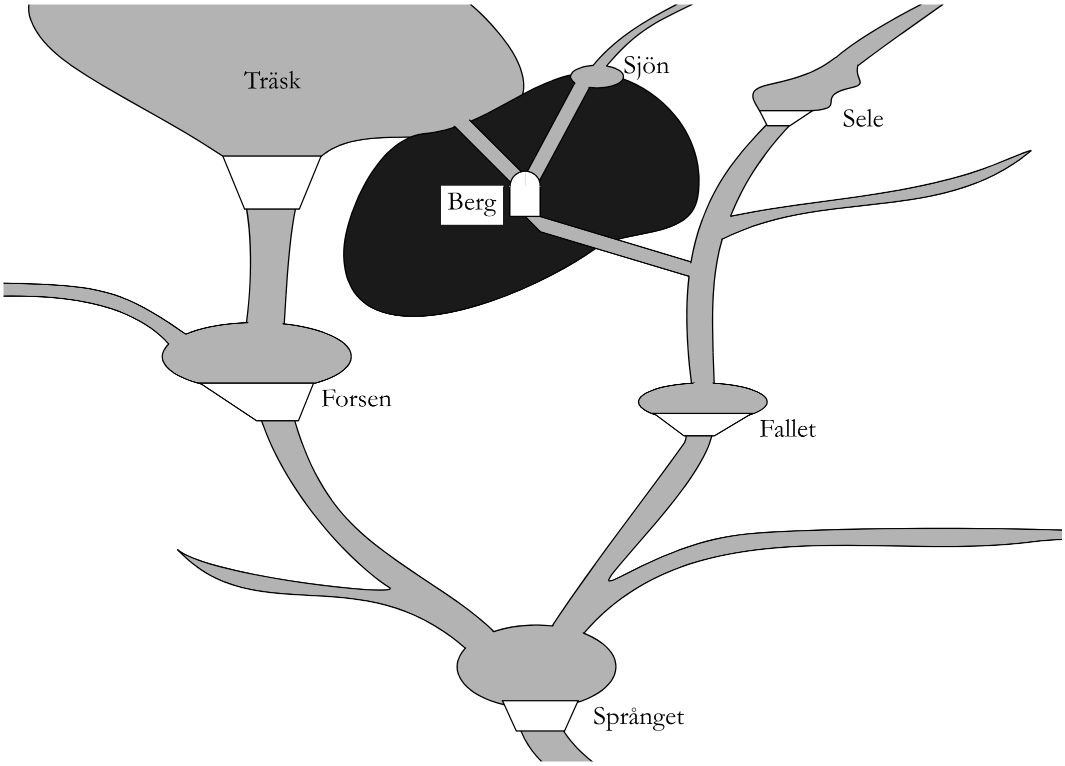
b) (3 p) An outage of 1 000 MW generation in Southern Rike.

c) (4 p) One of the transmission lines between Northern and Southern Rike is disconnected..

Table 6 Available bids to the real-time balancing market.

Bid	Up-regulation			Down-regualation		
	Maximal volume [MW]	Price [¤/MWh]	Area	Maximal volume [MW]	Price [¤/MWh]	Area
1	200	410	North	100	390	South
2	150	415	North	150	380	North
3	100	420	North	200	375	North
4	150	430	North	150	370	South
5	100	450	South	250	360	South
6	200	480	North	150	350	North
7	100	500	North	100	340	South
8	250	550	South	200	320	North
9	200	600	South	200	310	North
10	100	650	South	100	300	South

Problem 8 (20 p)



AB Vattenkraft owns five hydro power plants located as in the figure above. Berg is an underground pumped storage hydro power plants, which can be operated in three different ways:

- Water from Träsk can be discharged through the turbines in Berg.
- Water from Sjön can be discharged through the turbines in Berg.
- Water from Träsk can be pumped via Berg to Sjön.

It should be noted that water that is discharged through the turbines in Berg will be released in the reservoir of Fallet, whereas water that is spilled from Träsk passes through the natural riverbed to Forsen. The hydro reservoir Sjön does not have any spillways, i.e., the only way to lower the water level in Sjön is to dicharge water through Berg! The electricity consumption for pumping water from Träsk to Sjön is 0.5 MWh/HE and the maximal pumping is 100 HE. Additional data for the hydro power plants are given in table 7.

The company has a firm power contract of 100 MWh/h with AB Elleverantören. To deliver this quantity, AB Vattenkraft is using their own hydro power plants, but the company has also the possibility to trade at the local power exchange ElKräng. It is assumed that the company can buy and sell unlimited amounts of electricity for the prices stated in table 8. After that, the future electricity price is estimated to 275 SEK/MWh. Stored water is assumed to be used for electricity generation at the best marginal production equivalent and the water stored in Träsk is assumed to be discharged through the turbines in Berg (i.e., not pumped to Sjön). The water delay time between the power plants can be neglected.

Formulate the planning problem of AB Vattenkraft as an LP or MILP problem. Use the notation in table 9 for the parameters (it is however permitted to add further symbols if you consider it necessary).

NOTICE! The following is required to get full score for this problem:

- The symbols for the optimisation variables must be clearly defined.
- The optimisation problem should be formulated so that it is easy to determine what the objective function is, which constraints there are and which limits there are.
- The possible values for all indices should be clearly stated for each equation..

Table 7 Data for the hydro power plants of AB Vattenkraft.

Power plant	Start contents of reservoir [HE]	Maximal contents of reservoir [HE]	Marginal production equivalents [MWh/HE]		Maximal discharge [HE]		Local inflow [HE]
			Segment 1	Segment 2	Segment 1	Segment 2	
Träsk	4 000	8 000	0.50	0.45	100	20	80
Sjön	100	1 100	0.90	0.80	100	20	10
Sele	3 000	5 000	0.13	0.11	65	10	40
Forsen	1 200	2 000	0.32	0.29	40	5	25
Fallet	1 400	3 000	0.40	0.36	175	25	8
Språnget	4 000	6 000	0.64	0.56	200	40	4

Table 8 Expected prices at ElKräng.

Hour	0-1	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10	10-11	11-12
Price at ElKräng [SEK/MWh]	265	265	255	245	255	265	290	345	420	360	320	310
Hour	12-13	13-14	14-15	15-16	16-17	17-18	18-19	19-20	20-21	21-22	22-23	23-24
Price at ElKräng [SEK/MWh]	300	300	300	310	320	410	360	310	290	285	275	270

Table 9 Notation for the planning problem of AB Vattenkraft.

Symbol	Explanation	Value
$M_{i,0}$	Start contents of reservoir *i*	See table 7
$\overline{M}_i$	Maximal contents of reservoir *i*	See table 7
$\mu_{i,j}$	Marginal production equivalent of discharge from reservoir *i*, segment *j*	See table 7
$\overline{Q}_{i,j}$	Maximal discharge from reservoir *i*, segment *j*	See table 7
V_i	Local inflow to reservoir *i*	See table 7
λ_t	Expected price at ElKräng hour *t*	See table 8
γ_P	Electricity consumption of pumping	0.5
$\overline{Q}_P$	Maximal pumping	100
D	Contracted load	100
λ_f	Expected future electricity price	275

Problem 9 (20 p)

The electricity trading in Republiken is divided in a day-ahead market, a real-time balancing market and an imbalance settlement. In the day-ahead market, producers and consumers bid based on their forecast for the next day. The real-time balancing market is used by the system operator to carry out up- and down-regulation. The imbalance settlement is a purely financial transaction, where the balance responsible players settle their imbalances. The power system in Rike is mostly supplied by thermal power plants, but there is also a considerable share of wind power (2 000 MW installed capacity to be specific). A problem in the electricity market of Republiken is that it is difficult to forecast the wind power generation, which means that the system operator regularly must use the real-time balancing market to compensate wind power forecast errors. To study this issue more closely, Stads tekniska högskola has created a model of the electricity market in Republiken. The model is used to carry out Monte Carlo simulations of the electricity market in Republiken. Control variates are used in order to obtain more reliable results from the Monte Carlo simulations.

In the model it is assumed that all thermal power plants are 100% reliable. The remaining data for the thermal power plants in Republiken are shown in table 10. The variable cost in the table is the operation cost per MWh if the power plant is generating according to the result of the trading in the day-ahead market. The regulation cost is the cost to change the generation by 1 MWh/h, i.e., if a power plant, which has a variable cost of 100 ¤/MWh and a regulation cost of 5 ¤/MW, is changing its generation from 500 MWh/h to 400 MWh/h then this decreases the generation cost by 10 000 ¤, but at the same time there will be a regulation cost of 500 ¤; in total, the system operation cost is reduced by 9 500 ¤ in this case. Some power plants are not participating in the real-time balancing market; this is indicated by a dash "–" in the column for regulation costs in table 10.

Table 10 Thermal power plants in Republiken.

Fuel	Power plant	Installed capacity [MW]	Variable cost [¤/MWh]	Regulastion cost [¤/MW]
Nuclear	Strålinge 1	600	100	–
	Strålinge 2	600	100	–
	Strålinge 3	800	80	–
Combined heat and power/ industrial back-pressure	Flisinge	600	250	20
	Hamn	200	300	25
	Köping	400	250	20
	Pappersbolaget	200	300	25
	Stad	400	300	25
Coal condensing	Sotinge	800	390	100
	Röksta	1 000	400	50
Gas turbines	Bygden	100	800	5
	Ön	100	800	5

a) (6 p) Describe a detailed model of the electricity market in Republiken. The model should for a given scenario compute the total operation cost of the system including the regulation costs. The inputs for the model are indicated in table 11.

b) (6 p) Describe a simplified model of the electricity market in Republiken. The model should for a given scenario compute a control variate for the detailed model from part a. Moreover, you should compute the expectation value of the control variate.

Hint: The figure on the next page shows the duration curve of the load plus outages in the wind power generation. (This duration curve is computed based on the probability distribution of the real wind power generation, i.e., not the forecasts that are used in the day-ahead market.)

c) (6 p) Carry out a simulation of the electricity market in Republiken using the scenarios in table 11. What is the estimate of the expected operation cost of the system?

d) (2 p) Assume that exact forecasts of the wind power generation would be available in the day-ahead market. Estimate how much lower the expected operation cost of the system would be!

Table 11 Scenarios for a simulation of the electricity market in Republiken.

Scenario	Wind power generation [MWh/h]		Load [MWh/h]
	Day-ahead market forecast	Real outcome	
1	0	102	4 859
2	1 141	992	3 932
3	944	888	5 373
4	270	193	3 623
5	17	290	3 319

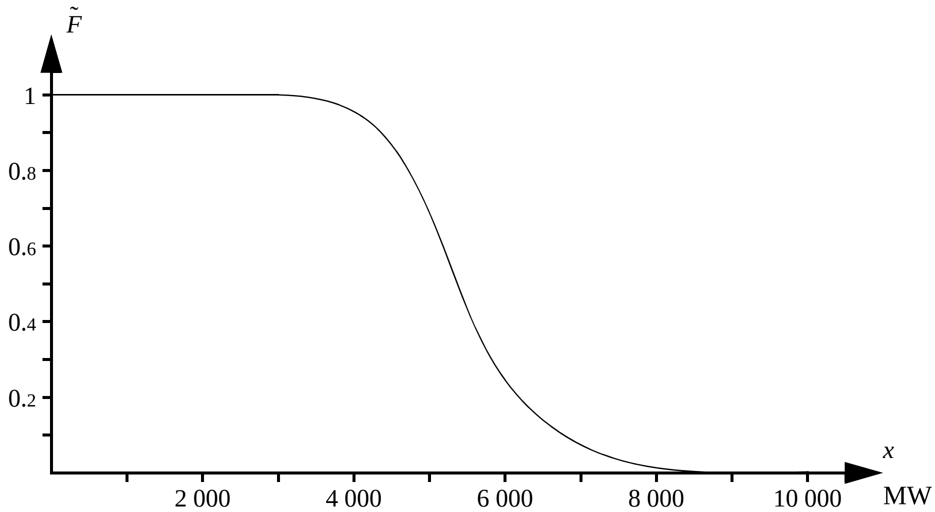


Table 12 The area below the equivalent load duration curve including outages in wind power.

a	0	2 000	3 000	4 000	4 200	4 400	4 600	4 800	5 000	5 200	5 400	5 600
b	2 000	3 000	4 000	4 200	4 400	4 600	4 800	5 000	5 200	5 400	5 600	5 800
$\int_a^b \tilde{F}(x)dx$	2 000	1 000	987.74	189.56	183.57	175.82	164.86	149.26	129.84	110.06	91.30	73.81
a	5 800	6 000	6 200	6 400	6 600	6 800	7 000	7 200	7 400	7 600	7 700	7 800
b	6 000	6 200	6 400	6 600	6 800	7 000	7 200	7 400	7 600	7 700	7 800	∞
$\int_a^b \tilde{F}(x)dx$	57.60	43.13	32.20	25.17	20.19	15.52	11.25	7.41	4.02	0.98	0.50	0.21

KTH Electrical Engineering

Answer sheet for part I

Name: ....

Personal number: ....

Problem 1

a) Alternative .... is correct.

b) Alternative .... is correct.

c) Alternative .... is correct.

Problem 2

a) .... ¤/MWh **b)** .... ¤/MWh

c) .... M¤/year

Problem 3

a) .... Hz **b)** .... Hz

c) .... Hz **d)** .... Hz

Problem 4

a) Parameters: ....

Optimisation variables: ....

b) ....

c) ....

d) .... MWh/HE

Problem 5

a) .... % **b)** .... ¤/h

c) .... % **d)** .... ¤/h

e) .... MW

f) Alternative .... is correct.

Problem 1

a) 4, **b)** 3, **c)** 3.

Problem 2

a) Assume that the electricity price, λ, is in the range 300 to 400 ¤/MWh. Hydro power and nuclear power will generate 126 TWh; thus, the other two power sources must generate 16 TWh together. The contribution from biofuel and coal condensing can be exprssed as

$$\frac{\lambda - 200}{400 - 200}20 + \frac{\lambda - 300}{500 - 300}10.$$

Setting this expression equal to 16 and solving for λ yields the electricity price λ = 340 ¤/MWh.

b) In this case biofuel and coal condensing only need to supply 8 TWh, which is less than half of the capability of biofuel. Thus, we can conclude that no coal condensing will be used, because the coal condensing is not utilised until half of the biofuel has been used. Hence, 40% of the price interval for biofuel is utilised and therefore the electricity price will be 280 ¤/MWh.

c) The income of wind power is 8 TWh/year · 280 ¤/MWh = 2 240 M¤/year, whereas the fixed costs are 4 240 M¤/year. Therefore, wind power must be subsidised by 2 000 M¤/year to make break even.

Problem 3

a) Area A is part of the same synchronous grid as areas B and E, which means that the total gain of the system is 4 500 MW/Hz. The increase in electricity consumption results in a frequency increase $\Delta f = \Delta G/R$ = 45/4 500 = 0.01 Hz, i.e., the new frequency is 50 + 0.01 = 50.01 Hz.

b) Since area C does not belong to the same synchronous grid as area A, the frequency remains the same, i.e., it is still exactly 50 Hz.

c) Since area D does not belong to the same synchronous grid as area A, the frequency remains the same, i.e., it is still exactly 50 Hz.

d) Since area E is part of the same synchronous grid as area A, the frequency must be the same, i.e., 50.01 Hz.

Problem 4

a) Parameters: β_{Gg}, C_g^+, D_t, λ_t and $u_{g,0}$. Optimisation variables: $G_{g,t}$, p_t, r_t, $s_{g,t}^+$ and $u_{g,t}$.

b) maximise $\sum_{t=1}^{24}\left(\lambda_t(r_t - p_t) - \sum_{g=1}^{3}(C_g^+ s_{g,t}^+ + \beta_{Gg}G_{g,t})\right)$.

c) $u_{g,t} - u_{g,t-1} \le s_{g,t}^+$.

d) The maximal production equivalent is obtained at the discharge where we have the best efficiency. The definition of production equivalent then gives us $\gamma_{max} = H/Q$ = 0.375 MWh/HE.

Problem 5

a) The load duration curve states the probability that the load exceeds a certain level. In this case we are looking for $\tilde{F}_0(800)$ = {read in figure} = 5%.

b) Since all power plants have 100% availability, we get $\tilde{F}_2(x) = \tilde{F}_1(x) = \tilde{F}_0(x)$, which means that

$$EG_2 = EENS_1 - EENS_2 = \int_{700}^{\infty}\tilde{F}_1(x)dx - \int_{800}^{\infty}\tilde{F}_2(x)dx = \int_{700}^{800}\tilde{F}_0(x)dx =$$
$$= (0.075 + 0.05) \cdot 100/2 = 6.25 \text{ MWh/h}.$$

We can assume that the operation cost of hydro power is negligible; hence, the expected operation cost per hour is $ETOC = 10EG_2$ = 62.5 ¤/h.

c) The risk of power deficit is given by $\tilde{F}_2(800) = 0.9\tilde{F}_1(800) + 0.1\tilde{F}_1(700)$ = 0.9 · 0.05 + 0.1 · 0.075 = 0.0525. Thus, the risk of power deficit is 5.25%.

c) $m_{TOC} = \frac{1}{2\,000}\sum_{i=1}^{2\,000} toc_i$ = 240 000/2 000 = 120 ¤/h.

e) The inverse transform method states that $D = F_D^{-1}(U)$, where U is a $U(0, 1)$-distributed random number. Since it is the duration curve that is given in the problem, we may as well use the transform $D = \tilde{F}_D^{-1}(U)$. The original random number must then have been $U = \tilde{F}_D(400)$ = 0.8. Hence, $U^* = 1 - U$ = 0.2, which results in $D^* = \tilde{F}_D^{-1}(U^*)$ = 550 MW.

f) 5.

Problem 6

a) To study how producers and consumers are affected we must first compute the electricity prices in the separated electricity markets compared to the common market. For the present conditions we get the following results:

- In Aland there is a need of 136 (consumption) – 5 (import from Beland) + 6 (export to Celand) + 5 (export to Deland) = 142 TWh generation. Hydro power and nuclear power can provide 140 TWh, which means that there is also a need for 2 TWh fossil-fuelled generation. The electricity price must therefore be 2/20 + 300 = 320 ¤/MWh.
- In Beland there is a need of 105 + 5 + 5 = 115 TWh generation, which can be covered using hydro only. If the hydro power is supplied to the electricity market based on variable costs, the electricity price would be 5 ¤/MWh. However, that price is unreasonably low and we may assume that the hydro producers in Beland use some other valuation method for hydro generation. For the sake of simplicity, we assume that the water value is equal to the electricity price stated in the contracts with Aland and Celand, i.e., 300 ¤/MWh (this assumption does however not change the continued argument, as Beland still has the lowest electricity price in Unionen).
- In Celand there is a need of 41 – 6 – 5 = 30 TWh generation, which means that half the fossil-fuelled generation must be utilised. The electricity price must therefore be 400 ¤/MWh.

- In Deland there is a need of 37 – 5 = 32 TWh generation. Hydro power and nuclear power can provide 25 TWh, which means that there is also a need for 7 TWh fossil-fuelled generation. The electricity price must therefore be 7/20 + 300 = 370 ¤/MWh.

In a common electricity market, the total consumption would be 319 TWh/year. If we disregards the transmision limitations then the electricity price would be 368 ¤/MWh, as hydro power and nuclear power can supply at most 285 TWh/year; hence, 34 TWh/year is needed from fossil fuels, which means that 34/100 of the price interval will be utilised. At this electricity price we get the following import and export between the countries:

- Aland generates 146.8 TWh/year and consumes 136 TWh/year
  $\Rightarrow$ export 10.8 TWh/year, which is possible as Aland can export in total 24 TWh/year to Celand and Deland.
- Beland generates 120 TWh/year and consumes 105 TWh/year
  $\Rightarrow$ export 15 TWh/year, which is possible as Beland can export in total 18 TWh/year to Aland and Celand.
- Celand generates 20.4 TWh/year and consumes 41 TWh/year
  $\Rightarrow$ import 20.6 TWh/year, which is possible as Celand can import in total 22 TWh/year.
- Deland generates 31.8 TWh/year and consumes 37 TWh/year
  $\Rightarrow$ import 5.2 TWh/year, which is possible as Celand can import in total 10 TWh/year.

From these results we can conclude that a common electricity market would be favourable to producers in Aland and Beland, who will sell more electricity than before and for a higher price, whereas it will be a change for the worse for producers in particularly Celand but also in Deland, as they will sell less than before and for a lower price. For consumers, it is the other way around; in Aland and Beland, the common electricity market results in higher prices, whereas the consumers in Celand and Deland will get lower prices.

b) We can study the total surplus in order to determine if the common electricity market is beneficial for the society. The total surplus is defined as

$$TS = CS + PS = B(D) - \lambda \cdot D + \lambda \cdot G - C(G) = \{D = G\} = B(D) - C(G),$$

where

CS = consumer surplus,
PS = producer surplus,
$B(D)$ = value of electricity consumption,
$C(G)$ = cost of electricity generation.

In this case is the value of electricity consumption not given, but since the consumption is the same for both alternatives; hence, we can disregards this term. This means that the alternative wehere the total generation cost is the least is preferable. The fossil fuelled generation is 39 TWh in the separated electricity markets, which can be compared to 34 TWh in the common electricity market (the difference is due to some hydro power not being utilised for the time being). Moreover, the fossil-fuelled generation is not distributed evenly in the separated markets; for example, Aland has unused capacity in fossil-fuelled power plants with a variable cost at 320 ¤/MWh, at the same time as Celand is using fossil-fuelled power plants with the variable cost 400 ¤/MWh. Thus, we can conclude that the generation costs will be lower in a common market and consequently this alternative is beneficial to the society.

Problem 7

a) To replace an outage of 800 MW Riksnät must activate the corresponding amount of up-regulation. The least cost for up-regulation is achieved if Riksnät is activating the entire bids 1–5 as well as 100 MW from bid 6. This means that 800 MW generation in Northern Rike is replaced by 700 MW from Northern Rike and 100 MW from Southern Rike. Compared to the situation before the outage the transmission from north to south will decrease by 100 MW; hence, there will not be a problem to fulfil the requirement on unused transmission capacity.

b) To replace an outage of 1 000 MW Riksnät must activate the corresponding amount of up-regulation. The least cost for up-regulation is achieved if Riksnät is activating the entire bids 1–7. This means that 1 000 MW generation in Southern Rike is replaced by 900 MW from Northern Rike and 100 MW from Southern Rike. Compared to the situation before the outage the transmission from north to south will increase by 900 MW, but in order to leave a margin of 800 MW transmission capacity, the flow may not exceed 3 200 MW, i.e., an increase by 400 MW. To fulfil the requiements at the lest cost, Riksnät should therefore activate the entire bids 1, 2, 5, 8 and 9, as well as 500 MW from bid 3 and 50 MW from bid 10. In total these bids result in an increase of 400 MW in Norther Rike and 600 MW in Southern Rike.

c) The disconnection of a transmission line does not affect the system frequency, but the problem is that there will not be sufficient margins for transmission between Northern and Southern Rike. The available transmission capacity with seven lines is 3 500 MW, which means that the transmission from north to south must decrease to 2 700 MW. One way to accomplish this is to activate 100 MW down-regulation in Northern Rike (bid 2) and 100 MW down-regulation in Southern Rike (bid 5). However, the frequency in the system is low and it would be practical for Riksnät to increase the frequency at the same time as the transmission is changed. This can be accomplished by only activating up-regulation in Southern Rike. Since 5/6 of the gain in the normal operation reserve is in Northern Rike, 120 MW up-regulation must be activated in southern Rike; this will be compensated by the primary control reducing the generation in Northern Rike by 100 MW and consequently the power flow from north to south has to decrease by 100 MW as well. In this case Riksnät should activate the entire up-regulation bid 5 and 20 MW from up-regulation bid 8.

Problem 8

The problem we want to solve is

maximise	*value of sales – cost of purchase + value of stored water*,
subject to	*hydrological balance for the reservoirs*,
	operation mode of Berg,
	load balance,
	limitations in reservoirs, discharge and spillage.

We choose to model the operation mode of Berg using three binary variables: one for discharge from Träsk, one for discharge from Sjön and one for pumping. We only allow discharge and pumping respectively if the binary variable is equal to one.

Indices for the power plants

Träsk 1, Sjön 2, Sele 3, Forsen 4, Fallet 5, Språnget 6.

Parameters

The parameters are defined in table 9 in the problem text.

Optimisation variables

$Q_{i,j,t}$ = discharge from reservoir i, segment j, during hour t,
$i = 1, \ldots, 6, j = 1, 2, t = 1, \ldots, 24,$

Q_{Pt} = pumping from Träsk to Sjön during hour t, $t = 1, \ldots, 24$,
$S_{i,t}$ = spillage from reservoir i during hour t, $i = 1, 3, \ldots, 6$, $t = 1, \ldots, 24$,
$M_{i,t}$ = contents of reservoir i at the end of hour t, $i = i = 1, \ldots, 6$, $t = 1, \ldots, 24$,
$u_{i,t}$ = discharge from reservoir i allowed during hour t, $i = 1, 2$, $t = 1, \ldots, 24$,
u_{Pt} = pumping from Träsk to Sjön allowed during hour t, $t = 1, \ldots, 24$,
p_t = purchase from ElKräg during hour t, $t = 1, \ldots, 24$,
r_t = sales to ElKräng during hour t, $t = 1, \ldots, 24$.

Objective function

$$\text{maximise} \sum_{t=1}^{24} \lambda_t(r_t - p_t) + \lambda_f((\mu_{1,1} + \mu_{5,1} + \mu_{6,1})M_{1,24} + (\mu_{2,1} + \mu_{5,1} + \mu_{6,1})M_{2,24} + (\mu_{3,1} + \mu_{5,1} + \mu_{6,1})M_{3,24} + (\mu_{4,1} + \mu_{6,1})M_{4,24} + (\mu_{5,1} + \mu_{6,1})M_{5,24} + \mu_{6,1}M_{6,24})$$

Constraints

Hydrological balance for Träsk:

$$M_{1,t} = M_{1,t-1} - Q_{1,1,t} - Q_{1,2,t} - Q_{Pt} - S_{1,t} + V_{1,t}, \qquad t = 1, \ldots, 24.$$

Hydrological balance for Sjön:

$$M_{2,t} = M_{2,t-1} - Q_{2,1,t} - Q_{2,2,t} + Q_{Pt} + V_{2,t}, \qquad t = 1, \ldots, 24.$$

Hydrological balance for Sele:

$$M_{3,t} = M_{3,t-1} - Q_{3,1,t} - Q_{3,2,t} - S_{3,t} + V_{3,t}, \qquad t = 1, \ldots, 24.$$

Hydrological balance for Forsen:

$$M_{4,t} = M_{4,t-1} + S_{1,t} - Q_{4,1,t} - Q_{4,2,t} - S_{4,t} + V_{3,t}, \qquad t = 1, \ldots, 24.$$

Hydrological balance for Fallet:

$$M_{5,t} = M_{5,t-1} + Q_{1,1,t} + Q_{1,2,t} + Q_{2,1,t} + Q_{2,2,t} + Q_{3,1,t} + Q_{3,2,t} + S_{3,t} - Q_{5,1,t} - Q_{5,2,t} - S_{5,t} + V_{5,t}, \qquad t = 1, \ldots, 24.$$

Hydrological balance for Språnget:

$$M_{6,t} = M_{6,t-1} + Q_{4,1,t} + Q_{4,2,t} + S_{4,t} + Q_{5,1,t} + Q_{5,2,t} + S_{5,t} - Q_{6,1,t} - Q_{6,2,t} - S_{6,t} + V_{6,t}, \qquad t = 1, \ldots, 24.$$

Discharge and pumping limitations in Berg:

$$Q_{i,j,t} \le \bar{Q}_{i,j} u_{i,t}, \qquad i = 1, 2, j = 1, 2, t = 1, \ldots, 24,$$

$$Q_{Pt} \le \bar{Q}_P u_{Pt}, \qquad t = 1, \ldots, 24.$$

Operation mode in Berg:

$$u_{1,t} + u_{2,t} + u_{Pt} = 1, \qquad t = 1, \ldots, 24.$$

Load balance:

$$\sum_{i=1}^{6} \sum_{j=1}^{2} \mu_{i,j} Q_{i,j,t} + p_t = D + \gamma_P Q_{Pt} + r_t, \qquad t = 1, \ldots, 24.$$

Variable limits

$0 \le Q_{i,j,t}$,	$i = 1, \ldots, 6, j = 1, 2, t = 1, \ldots, 24$,
$0 \le Q_{Pt}$,	$i = 1, \ldots, 6, j = 1, 2, t = 1, \ldots, 24$,
$0 \le S_{i,t}$,	$i = 1, 3, \ldots, 6, t = 1, \ldots, 24$,
$0 \le M_{i,t} \le \bar{M}_i$,	$i = 1, \ldots, 6, t = 1, \ldots, 24$,
$u_{i,t} \in \{0, 1\}$,	$i = 1, 2, t = 1, \ldots, 24$,
$u_{Pt} \in \{0, 1\}$,	$t = 1, \ldots, 24$,
$0 \le p_t$,	$t = 1, \ldots, 24$,
$0 \le r_t$,	$t = 1, \ldots, 24$.

Problem 9

a) The detailed model is divided in two steps corresponding to the day-ahead market and the real-time balancing market. The model of the day-ahead market can be formulated as an optimisation problem, but it can also be described in the following manner: Start by computing the need for thermal generation. We can now increase the generation in the thermal power plants according to ascending variable costs, until we have covered the load or until there is no more generation capacity available.

In the next step the system operator must compensate the wind power forecast error by activating up- or down regulation. In case of up-regulation, we sort the power plants participating in the real-time balancing market after ascending total cost (i.e., variable cost plus regulation cost) and then we increase the generation in these units until we have compensated the forecast error or until there is no more up-regulation capacity left (in that case there will be power deficit). For down-regulation, we sort the power plants participating in the real-time balancing market after descending total cost (which in this case equals the variable cost minus the regulation cost) and then decrease the generation in these power plants until we have compensated the forecast error or until there is no down-regulation capacity left (in that case it will be necessary to spill some wind power).

The total operation cost is based on the results after the real-time balancing market, i.e., the sum of the real variable costs in the thermal power plants plus the regulation costs of the power plants that have carried out up- or down-regulation.

b) In the simplified model we assume that the electricity market has access to perfect information about the wind power generation. That means that we in each scenario compute the need for thermal generation, i.e., the load minus the real wind power generation. We can now increase the generation in the thermal power plants according to ascending variable costs, until we have covered the load or until there is no more generation capacity available (in that case there will be power deficit). The total operation cost is equal to the the sum of the variable costs in the thermal power plants.

The simplified model corresponds to the model used in probabilistic production cost simulation. Thus, we can compute the expectation value of the control variate by computing the expected total operation cost according to a probabilistic production cost simulation of the electricity market in Republiken. The equivalent load duration curve including outages in wind power, i.e., $\tilde{F}_1(x)$, is given in the problem text. Since the thermal power plants are assumed to have 100% availability, we get that $\tilde{F}_2(x) = \tilde{F}_1(x)$, $\tilde{F}_3(x) = \tilde{F}_2(x)$, etc. There is no need to differentiate power plants witht the same variable costs when neglecting the risk of outages in thermal power plants, i.e., we may either consider Strålinge 1 and 2 as two 600 MW units or as one unit with the installed capacity 1 200 MW. We choose the latter alternative, because this saves some computations. The expected generation in the power plants can then be computed as follows:

Strålinge 3:

$$EG_2 = \int_{2\,000}^{2\,800} \tilde{F}_1(x)dx = \{\tilde{F}_1(x) = 1 \text{ for } x \le 3\,000\} = 800 \text{ MWh/h.}$$

Strålinge 1 and 2:

$$EG_3 = \int_{2\,800}^{4\,000} \tilde{F}_1(x)dx = \{\tilde{F}_1(x) = 1 \text{ for } x \le 3\,000\} = 200 + 987.74 = 1\,187.74 \text{ MWh/h.}$$

Flisinge and Köping:

$$EG_4 = \int_{4\,000}^{5\,000} \tilde{F}_1(x)dx = 189.56 + 183.57 + 175.82 + 164.86 + 149.26 = 863.07 \text{ MWh/h.}$$

Hamn, Pappersbolaget and Stad:

$$EG_5 = \int_{5\,000}^{5\,800} \tilde{F}_1(x)dx = 129.84 + 110.06 + 91.30 + 73.81 = 405.00 \text{ MWh/h.}$$

Sotinge:

$$EG_6 = \int_{5\,800}^{6\,600} \tilde{F}_1(x)dx = 57.60 + 43.13 + 32.20 + 25.17 = 158.11 \text{ MWh/h.}$$

Röksta:

$$EG_7 = \int_{6\,600}^{7\,600} \tilde{F}_1(x)dx = 20.19 + 15.52 + 11.25 + 7.41 + 4.02 = 58.39 \text{ MWh/h.}$$

Bygden and Ön:

$$EG_8 = \int_{7\,600}^{7\,800} \tilde{F}_1(x)dx = 0.99 + 0.50 = 1.48 \text{ MWh/h.}$$

Thus, we can compute

$$E[Z] = ETOC_{PPC} = \sum_{g=2}^{8} \beta_g EG_g = \ldots = 606\,244.40 \text{ ¤/h.}$$

c) The following results are obtained for the five scenarios:

Scenario 1: According to the day-ahead forecast we need 4 859 MWh thermal generation. This means that nuclear power, combined heat and power, industrial backpressure and Sotinge will be running at full capacity (which provides 4 600 MWh), and that 259 MWh will be generated in Röksta. However, in reality the wind power generation is 102 MWh, which means that there is a need for 102 MWh down-regulation. This down-regulation is carried out in Röksta, which then will generate 137 MWh. The variable costs are the same in both the detailed and the simplified model, but there will also be a regulation cost of 102 · 50 = 5 100 ¤ in the detailed model.

Scenario 2: According to the day-ahead forecast we need 3 932 – 1 141 = 2 791 MWh thermal generation. This means that the nuclear power will be running at full capacity and that 791 MWh will be generated in Flisinge and Köping. However, in reality wind power only generates 992 MWh, which means that there is a need for 149 MWh up-regulation. This up-regulation is carried out in Flisinge and Köping, which then will generate 940 MWh. The variable costs are the same in both the detailed and the simplified model, but there will also be a regulation cost of 149 · 20 = 2 980 ¤ in the detailed model.

Scenario 3: According to the day-ahead forecast we need 5 373 – 944 = 4 429 MWh thermal generation. This means that nuclear power, combined heat and power and industrial backpressure will be running at full capacity and that 629 MWh will be generated in Sotinge. However, in reality wind power only generates 888 MWh, which means that there is a need for 56 MWh up-regulation. This up-regulation is carried out in Röksta (the total cost per extra MWh in Röksta is 400 + 50 = 450 ¤/MWh which can be compared to 490 ¤/MWh in Sotinge), which then will generate 56 MWh. The variable costs are 800 · 80 + 1 200 · 100 + 1 000 · 250 + 800 · 300 + 629 · 390 + 56 · 400 = 941 710 ¤. In addition to that, there will be a regulation cost of 56 · 50 = 2 800 ¤. The total operation cost is thus 944 510 ¤. In the simplified model it will be Sotinge that balances the load and the real wind power generation, which yields the total operation cost 800 · 80 + 1 200 · 100 + 1 000 · 250 + 800 · 300 + 685 · 390 = 941 150 ¤. The difference between the two model is therefore 944 510 – 941 150 = 3 360 ¤ i n this scenario.

Scenario 4: According to the day-ahead forecast we need 3 623 – 270 = 3 353 MWh thermal generation. This means that nuclear power, Flisinge and Köping will be running at full capacity and that 353 MWh will be generated in Hamn, Pappersbolaget and Stad. However, in reality wind power only generates 193 MWh, which means that there is a need for 77 MWh up-regulation. This up-regulation is carried out in Hamn, Pappersbolaget and Stad, which then will generate 430 MWh. The variable costs are the same in both the detailed and the simplified model, but there will also be a regulation cost of 77 · 25 = 1 925 ¤ in the detailed model.

Scenario 5: According to the day-ahead forecast we need 3 319 – 17 = 3 302 MWh thermal generation. This means that nuclear power, Flisinge and Köping will be running at full capacity and that 302 MWh will be generated in Hamn, Pappersbolaget och Stad. However, in reality the wind power generation is 290 MWh, which means that there is a need for 273 MWh down-regulation. This down-regulation is carried out in Hamn, Pappersbolaget and Stad, which then will generate 12 MWh. The variable costs are the same in both the detailed and the simplified model, but there will also be a regulation cost of 273 · 25 = 6 825 ¤ in the detailed model.

This gives the following estimate of the expected total operation cost:

$$m_{TOC} = \frac{1}{5}(5\,100 + 2\,980 + 3\,360 + 1\,925 + 6\,825) + 606\,244.40 = 610\,282.40 \text{ ¤/h.}$$

d) The system would be operated as in the simplified model if perfect wind power forecasts were available in the day-ahead market. The difference between the detailed and the simplified model were estimated by 4 038 ¤/h—this is how much lower the expected operation costs would be with perfect wind power forecasts.